Candidate surname

Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours   June 2022

Paper reference   9MA0/01

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical
formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions
  are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers
  without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise
  stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*
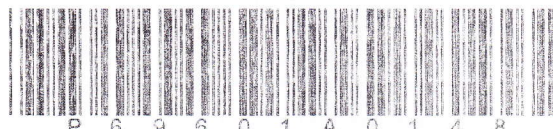
Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P 6 9 6 0 1 A 0 1 4 8

Pearson

1. The point $P(-2, -5)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

   Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

   (a) $y = f(x) + 2$

   (1)

   (b) $y = |f(x)|$

   (1)

   (c) $y = 3f(x - 2) + 2$

   (2)

   a)   shift vertical 2 units

   $$P(-2, -3)$$

   b)   -ve y coordinate reflected in
         x axis
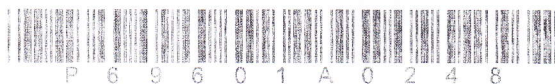
   $$P(-2, 5)$$

   c)   Shift 2 units right $(0, -5)$

   y coord × 3 $(0, -15)$

   Shift up 2 units $P(0, -13)$

2.

$$f(x) = (x - 4)(x^2 - 3x + k) - 42 \quad \text{where } k \text{ is a constant}$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of k.

(3)

as $(x + 2)$ is a factor

$f(-2) = 0$ Sub $x = -2$

$0 = (-2 - 4)(4 + 6 + k) - 42$

$0 = -6(10 + k) - 42$

$0 = -60 - 6k - 42$

$6k = -102$
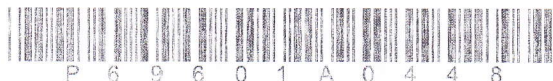
$k = -17$

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

a) $\quad x^2 - 10x + y^2 + 16y = 80$

$(x-5)^2 - 5^2 + (y+8)^2 - 8^2 = 80$

$(x-5)^2 + (y+8)^2 = 169$

(i) Centre $(5, -8)$

(ii) $r^2 = 169 \qquad r = 13$

Length OC

$= \sqrt{5^2 + 8^2}$

$= \sqrt{89}$
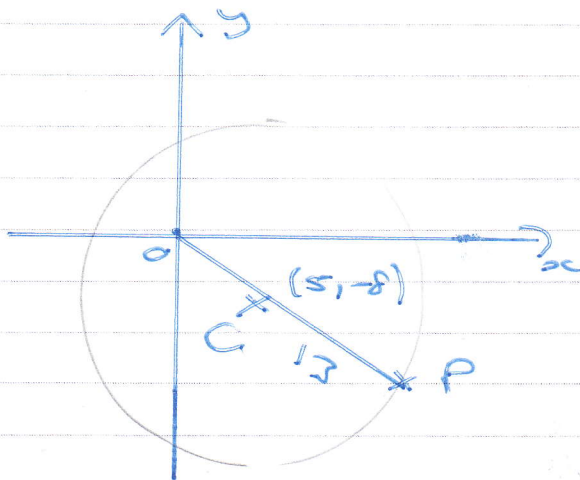
radius $CP = 13$

$OP = \sqrt{89} + 13$

4. (a) Express $\displaystyle\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \ln k$$

where k is a constant to be found.

(2)

a) $\displaystyle = \int_{2.1}^{6.3} \frac{2}{x}\, dx$

b) $\displaystyle \left[2 \ln x \right]_{2.1}^{6.3}$

$= 2 \ln(6.3) - 2 \ln(2.1)$

$= 2 \left(\ln(6.3) - \ln(2.1) \right)$

$= 2 \ln\left(\frac{6.3}{2.1} \right)$

$= 2 \ln 3$

$= \ln 3^2$

$= \ln 9$

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

* the height of the tree was 2.60 m, exactly 2 years after being planted
* the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.

(4)

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.

(2)

a) $\qquad h^2 = at + b$

$\qquad h = 2.6 \qquad t = 2$

$\qquad 2.6^2 = 2a + b$

$\qquad 6.76 = 2a + b \qquad ①$

$\qquad h = 5.1 \qquad t = 10$

$\qquad 5.1^2 = 10a + b$

$\qquad 26.01 = 10a + b \qquad ②$

Solving ① and ② simultaneously
on calculator

$a = 2.40625 = 2.41 \qquad (3sf)$
$b = 1.9475 = 1.95 \qquad (3sf)$

$\qquad h^2 = 2.41t + 1.95$

b) $\qquad t = 20$

$\qquad h^2 = 2.41 \times 20 + 1.95$
$\qquad h^2 = 50.15$
$\qquad h = 7.08 \, m \qquad (3sf)$

This is close to the actual 7m
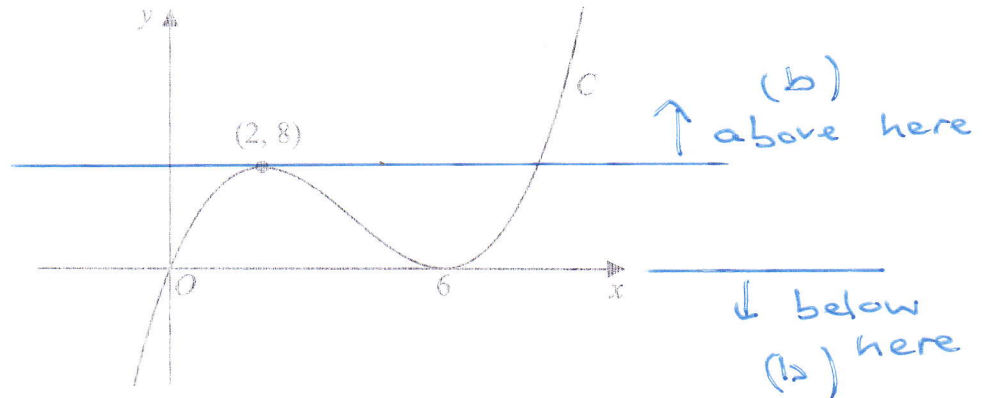so the model is accurate

6.

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

a) $f'(x) < 0$ for $2 < x < 6$

b) $k > 8$ or $k < 0$

$\{k : k > 8\} \cup \{k : k < 0\}$

c) $+ x^3$ curve

$y = ax(x - 6)^2$

goes through $(0,0)$ touches at $x = 6$

then at $x = 2, y = 8$

$8 = ax \, 2 \, (-4)^2$

$a = \frac{8}{32} = \frac{1}{4}$

$y = \frac{1}{4}x(x - 6)^2$

(b) ↑ above here

↓ below (b) here

12
P 6 9 6 0 1 A 0 1 2 4 8

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

(i) There exist integers p and q such that pq is even and both p and q are odd

let $p = 2m + 1$
$q = 2n + 1$

$\therefore pq = (2m + 1)(2n + 1)$
$= \underbrace{4mn + 2m + 2n}_{even} + 1$

as $4mn + 2m + 2n$ is even
then $4mn + 2m + 2n + 1$ is odd

This is a contradiction, so if pq is even, at least one of p and q must be even

(ii) $(x + y)^2 < 9x^2 + y^2$
$x^2 + 2xy + y^2 < 9x^2 + y^2$

$2xy < 8x^2$
$\div x \qquad \div x$

as $x < 0$
when
$\div x$
switch
inequality
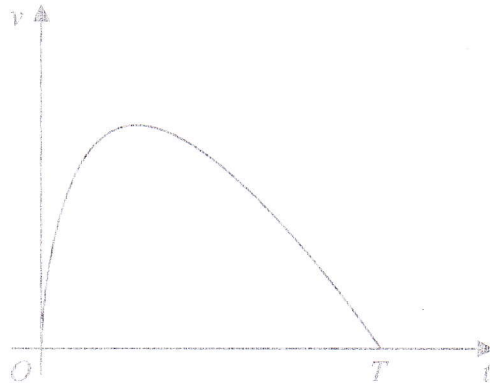sign
round

$2y > 8x$
$y > 4x$ \qquad as required

8.

Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v\,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leqslant t \leqslant T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

$v = 0$ when $10 - 0.4t = 0$

or $\ln(t+1) = 0 \qquad t = 25$

$t = 0 \qquad T = 25$

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

b) $\quad V = (10 - 0.4t) \times \ln(t+1)$

$$\frac{dv}{dt} = (10 - 0.4t) \times \frac{1}{t+1}$$

$$+ \ln(t+1)(-0.4)$$

maximum speed when $\frac{dv}{dt} = 0$

$$0 = \frac{10 - 0.4t}{t+1} - 0.4 \ln(t+1)$$

$$\frac{10 - 0.4t}{t+1} = 0.4 \ln(t+1)$$

$$10 - 0.4t = (t+1) \, 0.4 \ln(t+1)$$

$$10 - 0.4t = 0.4t \ln(t+1) + 0.4 \ln(t+1)$$

$$10 - 0.4 \ln(t+1) = 0.4t + 0.4t \ln(t+1)$$
$$10 - 0.4 \ln(t+1) = 0.4t (1 + \ln(t+1))$$
$$0.4(25 - \ln(t+1)) = 0.4t (1 + \ln(t+1))$$

$$\frac{25 - \ln(t+1)}{1 + \ln(t+1)} = t$$

$$\frac{(26 - 1) - \ln(t+1)}{1 + \ln(t+1)} = t$$

$$\frac{26 - (1 + \ln(t+1))}{1 + \ln(t+1)} = t$$

$$\frac{26}{1 + \ln(t+1)} - 1 = t$$

as required

c) $t_1 = 7$ on   calculator

$$t_2 = \frac{26}{1 + \ln(Ans + 1)} - 1 = 7.443089$$

$t_3 = 7.297832711$

(i) $t_3 = 7.298$ (3 dp)

(ii)   repeated iteration stabilises
         at $t = 7.332800321$

         $t = 7.33$ seconds (3sf
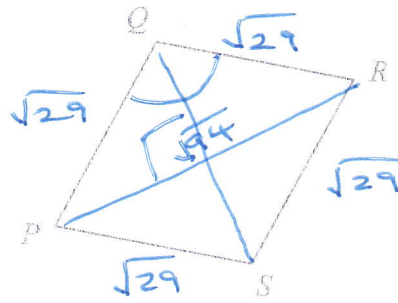   to reach max speed

9.

Figure 3

Figure 3 shows a sketch of a parallelogram $PQRS$.

Given that

- $\overrightarrow{PQ} = 2i + 3j - 4k$

- $\overrightarrow{QR} = 5i - 2k$

(a) show that parallelogram $PQRS$ is a rhombus.

(2)

(b) Find the exact area of the rhombus $PQRS$.

(4)

a) $|\overrightarrow{PQ}| = \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{29}$

$|\overrightarrow{QR}| = \sqrt{5^2 + (-2)^2} = \sqrt{29}$

AS parallelogram

$|\overrightarrow{QR}| = |\overrightarrow{PS}|$ and $|\overrightarrow{PQ}| = |\overrightarrow{SR}|$

as $|\overrightarrow{PQ}| = |\overrightarrow{QR}| = \sqrt{29}$

PQRS is a rhombus

b) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 7i + 3j - 6k$

$|\overrightarrow{PR}| = \sqrt{7^2 + 3^2 + (-6)^2} = \sqrt{94}$

$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{QR}$

$= -2i - 3j + 4k$
$+ 5i \qquad - 2k$

$= +3i - 3j + 2k$

$|\overrightarrow{QS}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{22}$

$$\text{Area } \triangle PQR = \frac{1}{2} \times \sqrt{94} \times \left(\frac{1}{2}\sqrt{22}\right)$$

$$= \frac{\sqrt{517}}{2}$$

$$\therefore \text{ Area rhombus} = 2 \times \frac{\sqrt{517}}{2}$$

$$= \sqrt{517}$$

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study.

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a rate of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

(4)

a) $N_b = 45 + 220 e^{0.05t}$

at $t = 0$, $N_b = 45 + 220$

$= 265$ thousand

$= 265000$

b) $\dfrac{dN_b}{dt} = 0.05 \times 220\, e^{0.05t}$

$= 11 e^{0.05t}$

at $t = 10$

$\dfrac{dN_b}{dt} = 11 \times e^{0.05 \times 10}$

$= 18.13593398$

≈ 18 thousand per year

c) $\quad 45 + 220e^{0.05t} = 10 + 800e^{-0.05t}$

$$220e^{0.05t} - 800e^{-0.05t} + 35 = 0$$

$\times$ through by $e^{0.05t}$

$$220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$$

Quadratic equation solver
on calculator

$$e^{0.05t} = 1.829038082 \quad ①$$

or $\quad e^{0.05t} = -1.988128991 \quad ②$

① gives $\quad 0.05t = \ln(1.829038082)$

$$t = \frac{\ln(1.829038082)}{0.05}$$

$$t = 12.07580381$$

② gives $\quad t = \frac{\ln(-1.988128991)}{0.05}$

Math error
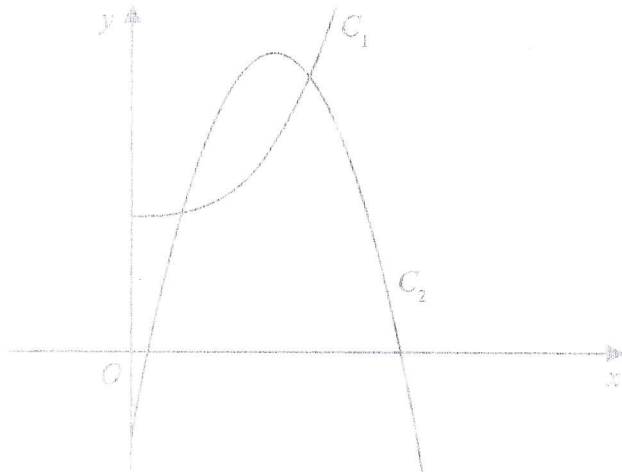
$\therefore \quad T = 12.08 \quad (2dp)$

11.

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \dfrac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

a) $\quad 2x^3 + 10 = 42x - 15x^2 - 7$
$\qquad$ at $x = \dfrac{1}{2}$

$C_1 \quad y = 2 \times \left(\dfrac{1}{2}\right)^3 + 10 = \dfrac{41}{4}$

$C_2 \quad y = 42 \times \dfrac{1}{2} - 15 \times \left(\dfrac{1}{2}\right)^2 - 7 = \dfrac{41}{4}$

$\qquad \therefore$ meet at $\left(\dfrac{1}{2}, \dfrac{41}{4}\right)$

b) $\qquad 2x^3 + 10 = 42x - 15x^2 - 7$
$\qquad 2x^3 + 15x^2 - 42x + 17 = 0$

from part a) $\quad 2x - 1$ is a factor
$\qquad$ if meet at $x = \dfrac{1}{2}$

$$\begin{array}{r} x^2 + 8x - 17 \\ 2x - 1 \overline{\smash{\big)}\ 2x^3 + 15x^2 - 42x + 17} \\ \underline{-2x^3 - x^2} \quad\downarrow \\ 16x^2 - 42x \\ \underline{-16x^2 - 8x} \\ -34x + 17 \\ \underline{-34x + 17} \\ 0 \end{array}$$

$$(2x - 1)(x^2 + 8x - 17) = 0$$

polynomial solver

$$x = \tfrac{1}{2} \quad , \quad \boxed{x = -4 + \sqrt{33}} \quad , \quad x = -4 - \sqrt{33}$$

From diagram this value
of x as must
be positive value

not P

12.

Show that

$$\int^{e^2} x^3 \ln x\, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

integrate by parts $\quad u = \ln x \quad v = \frac{1}{4}x^4$

$\qquad\qquad\qquad\qquad u' = \frac{1}{x} \qquad v' = x^3$

$$= \left[\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\, dx \right]^{e^2}_1$$

$$= \left[\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 \right]^{e^2}_1$$

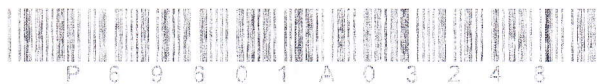$$= \left(\frac{1}{4}(e^2)^4 \ln(e^2) - \frac{1}{16}(e^2)^4 \right)$$

$$\qquad - \left(\frac{1}{4}\ln 1 - \frac{1}{16} \right)$$

$$= \frac{1}{4}e^8 \times 2\ln e - \frac{1}{16}e^8 + \frac{1}{16}$$

$$= \frac{1}{4}e^8 \times 2 - \frac{1}{16}e^8 + \frac{1}{16}$$

$$= \frac{7}{16}e^8 + \frac{1}{16}$$

$$\text{where} \quad a = \frac{7}{16}, \quad b = \frac{1}{16}$$

13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2} \left[2a + (n-1)d \right]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

(i)

① $S_n = a + (a+d) + (a+2a) + \cdots + (a + (n-1)d)$

reversing

② $S_n = (a + (n-1)d) + \cdots + (a+2a) + (a+d) + a$

adding ① + ② we get

$2 S_n = n \times (2a + (n-1)d)$

$S_n = \frac{n}{2} (2a + (n-1)d)$

(ii)
week	1	2	3
	10	9.20	8.40

-0.8 -0.8

$a = 10$ $d = -0.8$

$$S_n = 64 = \frac{n}{2}(2a + (n-1)d$$

$$128 = n(20 + (-0.8)(n-1)$$

$$128 = n(20 - 0.8n + 0.8)$$
$$128 = n(20.8 - 0.8n)$$
$$128 = 20.8n - 0.8n^2$$
$$0.8n^2 - 20.8n + 128 = 0$$
$$\div \text{ through by } 0.8$$

$$n^2 - 26n + 160 = 0 \text{ (as required)}$$

b) Use polynomial solver on calculator
$$n = 16 \text{ or } n = 10$$

c) 10 weeks   He has saved
the amount by 10 weeks
so would not save for
another 6 weeks.

14.

(a) Given that

$$2\sin(x - 60°) = \cos(x + 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

a)

$$2(\sin x \cos 60 - \cos x \sin 60)$$
$$= \cos x \cos 30 + \sin x \sin 30$$

$$2\sin x \times \frac{1}{2} - 2\cos x \times \frac{\sqrt{3}}{2}$$
$$= \cos x \times \frac{\sqrt{3}}{2} + \frac{1}{2}\sin x$$

$$\sin x - \sqrt{3}\cos x = \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$$

$$\sin x - \frac{1}{2}\sin x = \frac{\sqrt{3}}{2}\cos x + \sqrt{3}\cos x$$

$$\frac{1}{2}\sin x = \frac{\sqrt{3}}{2}\cos x (1 + 2)$$

$$\sin x = 3\sqrt{3}\cos x$$

$$\frac{\sin x}{\cos x} = 3\sqrt{3}$$

$$\tan x = 3\sqrt{3} \quad (\text{as required})$$

b)

$$2\sin(x - 60) = \cos(x - 30)$$

Let $x = 2\theta + 60$

will transform to
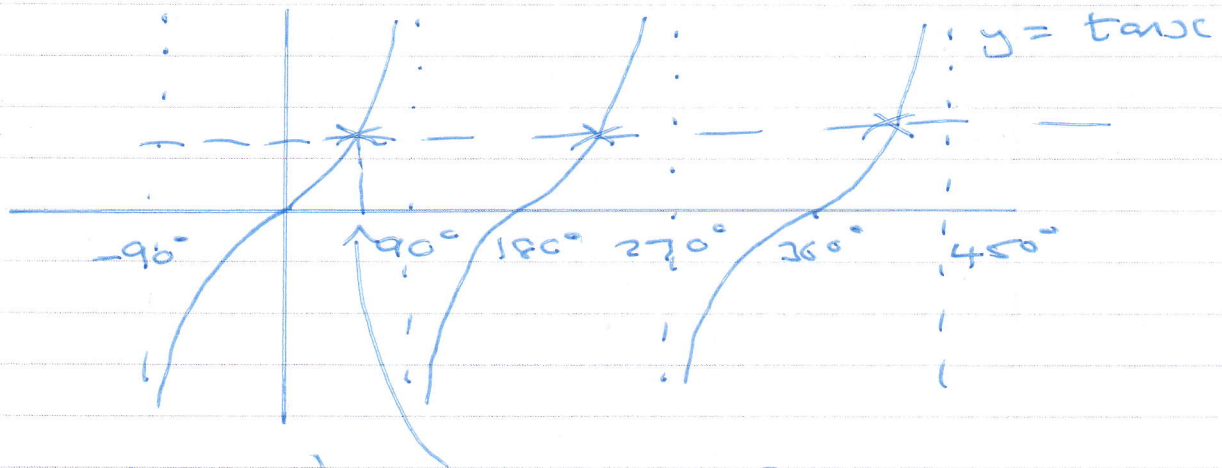
$$2\sin(2\theta + 60 - 60) = \cos(2\theta + 60 - 30)$$
$$2\sin 2\theta = \cos(2\theta + 30)$$

$$0 \leq \theta \leq 180$$

$$60 \leq 2\theta + 60 \leq 420$$

$$\tan^{-1}(3\sqrt{3}) = 79.1066° \quad \text{primary solution}$$

$$+180 = 259.1066°$$
$$+180 = 439.1066° \text{ (out of range)}$$

$$\therefore \quad \theta = \frac{79.1066 - 60}{2} = 9.5533°$$

$$\theta = \frac{259.1066 - 60}{2} = 99.5533°$$

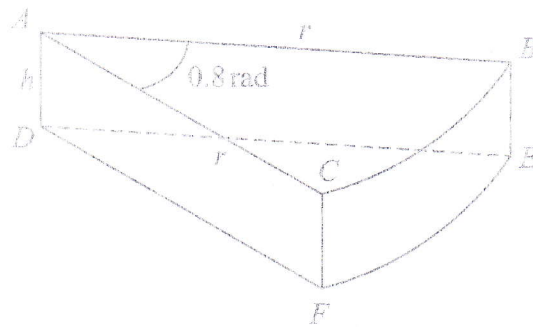$$\theta = 9.6° \text{ or } 99.6° \quad (\text{to 1dp})$$

15.

Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240 \, \text{cm}^3$

(a) show that the surface area of the toy, $S \, \text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation.

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

a) area of constant cross section
$$= \frac{1}{2} r^2 \theta = \frac{1}{2} \times r^2 \times 0.8 = 0.4r^2 \quad \text{①}$$

∴ Volume $= 0.4r^2 \times h$

$V = 0.4r^2 h$

$240 = 0.4r^2 h$

$h = \frac{240}{0.4r^2} = \frac{600}{r^2}$

Surface area = 2 × constant cross-section ①
 + 2 × side rectangle ②
 + curved end piece ③

side rectangle = $r \times h = r \times \dfrac{600}{r^2} = \dfrac{600}{r}$ ②

curved end = $h \times r \times 0.8$
 $= \dfrac{600}{r^2} \times r \times 0.8 = \dfrac{480}{r}$ ③

∴ $S = 2 \times 0.4r^2 + \dfrac{2 \times 600}{r} + \dfrac{480}{r}$

$S = 0.8r^2 + \dfrac{1680}{r}$

(as required)

b) $S = 0.8r^2 + 1680r^{-1}$

$\dfrac{dS}{dr} = 1.6r - \dfrac{1680}{r^2}$

stationary when $\dfrac{dS}{dr} = 0$

$\dfrac{1680}{r^2} = 1.6r$

$\sqrt[3]{\dfrac{1680}{1.6}} = r$

$r = 10.1639$

$r = 10.1 \quad (3sf)$

c) $\dfrac{dS}{dr} = 1.6r - 1680r^{-2}$

$\dfrac{d^2S}{dr^2} = 1.6 + \dfrac{3360}{r^3}$

when $r = 10.1 \dots \quad \dfrac{d^2S}{dr^2} = 4.8$

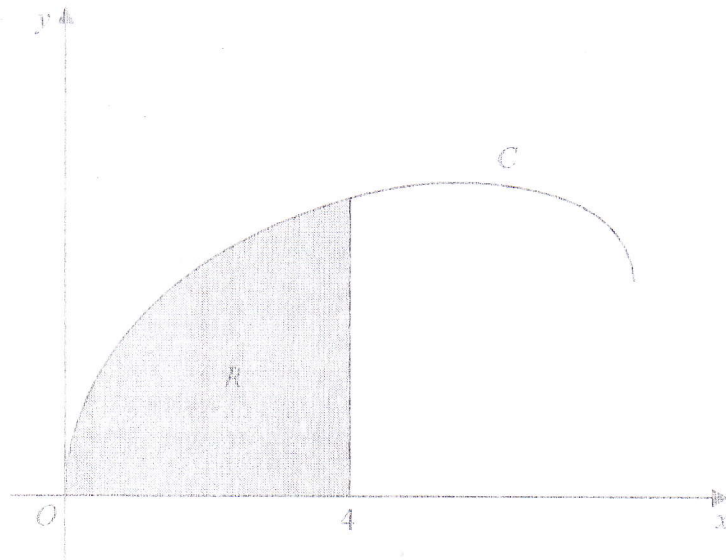as $\dfrac{d^2S}{dr^2} > 0$; then this is a minimum area

16.

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_a^{\cdot}(8 - 8\cos 4t + 48\sin^2 t \cos t)\,dt$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

a) $\text{Area} = \int_0^4 y \cdot \frac{dx}{dt} \, dt$

$\frac{dx}{dt} = 16 \sin t \cos t$

limits $\quad x = 0, \quad t = 0$

$\qquad\qquad x = 4 \qquad 4 = 8\sin^2 t$

$\qquad\qquad\qquad\qquad \sin^2 t = \frac{1}{2}$

$\int_0^{\frac{\pi}{4}} (2\sin 2t + 3\sin t) \times 16 \sin t \cos t \, dt \qquad \sin t = \pm\sqrt{\frac{1}{2}}$

$\qquad\qquad\qquad\qquad\qquad\qquad t = \frac{\pi}{4} \text{ or } -\frac{\pi}{4}$

$= \int_0^{\frac{\pi}{4}} 32 \sin 2t \sin t \cos t + 48 \sin^2 t \cos t \, dt \qquad t = \frac{\pi}{4}$

need to write as $\cos 4t$!!

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P 6 9 6 0 1 A 0 4 6 4 8

$$32 \sin 2t \sin t \cos t$$

$$= 16 \times \sin 2t \times 2 \times \sin t \cos t$$
$$= 16 \times \sin 2t \times \sin 2t$$
$$= 16 \sin^2 2t$$

$$\cos 2t = 1 - 2\sin^2 t$$
$$\cos 4t = 1 - 2\sin^2 2t$$
$$\therefore \sin^2 2t = \frac{1 - \cos 4t}{2}$$

$$= 16 \times \frac{(1 - \cos 4t)}{2} \qquad = 8 - 8\cos 4t$$

$$\therefore R = \int_0^{\frac{\pi}{4}} \left(8 - 8\cos 4t + 48 \sin^2 t \cos t\right) dt$$

as required

b)

reverse chain rule
$$\frac{d}{dt}\left(\sin^3 t\right) = 3\sin^2 t \cos t$$

Integrating R $\qquad \therefore \frac{d}{dt}\left(16\sin^3 t\right) = 48\sin^2 t \cos t$

$$= \left[8t - 8 \times \frac{1}{4}\sin 4t + 16\sin^3 t\right]_0^{\frac{\pi}{4}}$$

$$= \left(8 \times \frac{\pi}{4} - 2 \times \sin \pi + 16 \times \left(\sin \frac{\pi}{4}\right)^3\right)$$

$$\longrightarrow \left(0 - 2\sin 0 + 16(\sin 0)^3\right)$$

$$= \left(2\pi - 0 + 16 \times \left(\frac{\sqrt{2}}{2}\right)^3\right) - (0 + 0 + 0)$$

$$= 2\pi + 16 \times \frac{2\sqrt{2}}{8}$$

$$= 2\pi + 4\sqrt{2} \qquad \text{square units}$$