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OCR
Oxford Cambridge and RSA

Monday 19 October 2020 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 Triangle ABC has $AB = 8.5\,\text{cm}$, $BC = 6.2\,\text{cm}$ and angle $B = 35°$.

Calculate the area of the triangle. **[2]**

$$\text{Area} = \frac{1}{2} \times 8.5 \times 6.2 \times \sin(35)$$

$$= 15.1\,\text{cm}^2 \quad (3\text{ sf})$$

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2 A sequence of transformations maps the curve $y = e^x$ to the curve $y = e^{2x+3}$.

Give details of these transformations. **[3]**

Translation $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ or 3 units left along x-axis

Stretch by scale factor $\frac{1}{2}$ parallel to x-axis

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3 The functions f and g are defined for all real values of x by

$f(x) = 2x^2 + 6x$ and $g(x) = 3x + 2$.

(a) Find the range of f. **[3]**

$f'(x) = 4x + 6$

min when $4x + 6 = 0$

$x = -\frac{6}{4} = -\frac{3}{2} = -1.5$

$f(x) = 2 \times (-1.5)^2 + 6 \times -1.5 = -\frac{9}{2}$

range is $f(x) \geq -\frac{9}{2}$

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(b) Give a reason why f has no inverse. **[1]**

Function is not one to one

(c) Given that $fg(-2) = g^{-1}(a)$, where a is a constant, determine the value of a. **[4]**

$g(-2) = 3 \times -2 + 2 = -4$

$f(-4) = 2 \times (-4)^2 + 6 \times -4 - 8$

$\therefore \quad 8 = g^{-1}(a)$

for g $\quad y = 3x + 2$

$y - 2 = 3x$

$\frac{y-2}{3} = x$

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$g^{-1}(x) = \frac{x-2}{3}$

$\frac{a-2}{3} = 8$

$a - 2 = 24$

$a = 26$

(d) Determine the set of values of x for which $f(x) > g(x)$. Give your answer in set notation. **[3]**

$2x^2 + 6x > 3x + 2$

$2x^2 + 3x - 2 > 0$

$(2x - 1)(x + 2) > 0$

$x = \frac{1}{2}$ $x = -2$

$$-2 \leq x \leq \frac{1}{2}$$

$$\{x : x > \frac{1}{2}\} \cup \{x : x < -2\}$$

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4 A curve has equation $y = 2\ln(k-3x) + x^2 - 3x$, where k is a positive constant.

(a) Given that the curve has a point of inflection where $x = 1$, show that $k = 6$. **[5]**

$$y = 2\ln(k-3x) + x^2 - 3x$$

$$\frac{dy}{dx} = 2 \times \frac{1 \times -3}{k-3x} + 2x - 3$$

$$\frac{dy}{dx} = -6(k-3x)^{-1} + 2x - 3$$

$$\frac{d^2y}{dx^2} = 6(k-3x)^{-2} \times -3 + 2$$

$$\frac{d^2y}{dx^2} = \frac{-18}{(k-3x)^2} + 2$$

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at point of inflection $\frac{d^2y}{dx^2} = 0$

$$0 = \frac{-18}{(k-3x)^2} + 2$$

$$\frac{18}{(k-3x)^2} = 2$$

$$18 = 2(k-3x)^2$$

at $x = 1$

$$18 = 2(k-3)^2$$

$$9 = (k-3)^2$$

$3 = (k-3)$ or $-3 = k-3$

$k = 6$ or $k = 0$

$\therefore k = 6$ as k is positive

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(b) Show by calculation that the x-coordinate of this point lies between 0.5 and 1.5. **[2]**

$f(0.5) = 2 \times \ln(6 - 3 \times 0.5) + (0.5)^2 - 3 \times 0.5$

$= 1.758$

$f(1.5) = 2 \times \ln(6 - 3 \times 1.5) + (1.5) - 3 \times 1.5$

$= -1.439$

as there is a change of sign between $f(0.5)$ and $f(1.5)$ there is a solution between 0.5 and 1.5

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(c) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x-coordinate of the point where the curve intersects the x-axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. **[3]**

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f(x) = 2\ln(6-3x) + x^2 - 3x$

$f'(x) = -6(6-3x)^{-1} + 2x - 3$

$x_0 = 1$ Press 1 = on calculator

$$\text{Ans} - \frac{2\ln(6-3\times\text{Ans}) + \text{Ans}^2 - 3\times\text{Ans}}{-6(6-3\times\text{Ans})^{-1} + 2\times\text{Ans} - 3}$$

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$x_1 = 1.065741526$

$x_2 = 1.065675303$

$x_3 = 1.065675303$

$x = 1.06568$ (5 dp)

(d) By choosing suitable bounds, verify that your answer to part **(c)** is correct to 5 decimal places.

[1]

$f(1.065675) = 2\ln(6 - 3 \times 1.065675) + 1.065675^2 - 3 \times 1.065675$

$= 9.11 \times 10^{-7}$

$$f(1.0655685) = 2\ln(6 - 3\times 1.0655685) + 1.0655685^2 - 3\times 1.0655685$$

$$\simeq -2.91\times 10^{-5}$$

Change of sign between $f(1.0655675)$ and $f(1.0655685)$ indicates that 1.06568 is correct to 5 dp

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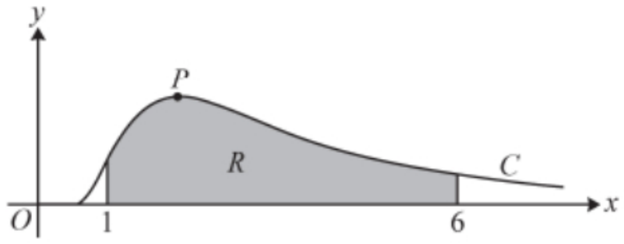
5



The diagram shows the curve C with parametric equations

$x = \frac{3}{t}$, $y = t^3 e^{-2t}$, where $t > 0$.

The maximum point on C is denoted by P.

(a) Determine the exact coordinates of P. **[4]**

$\frac{dx}{dt} = 3 \ln t$

$y = t^3 e^{-2t}$

$\frac{dy}{dt} = 3t^2 e^{-2t} - 2e^{-2t} \times t^3$

Max when $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2e^{-2t} - 2e^{-2t} \times t^3}{3 \ln t}$$

$\therefore \quad 0 = 3t^2e^{-2t} - 2e^{-2t} \times t^3$

$0 = e^{-2t} t^2 (3 - 2t)$

as $t > 0$ then $3 - 2t = 0$

$t = 1.5$

$x = \frac{3}{1.5} = 2$

$y = 1.5^3 \times e^{-2 \times 1.5} = \frac{27}{8} e^{-3}$

coords of $P\left(2, \frac{27}{8} e^{-3}\right)$

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The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

(b) Show that the area of R is given by

$$\int_a^b 3te^{-2t}\,dt,$$

where a and b are constants to be determined. **[3]**

$$\text{Area} = \int y \frac{dx}{dt}\,dt$$

$$x = \frac{3}{t} = 3t^{-1} \qquad \frac{dx}{dt} = -3t^{-2} = \frac{3}{t^2} \qquad y = t^3 e^{-2t}$$

$$\int_a^b t^3 e^{-2t} \times -\frac{3}{t^2}\,dt = \int_a^b 3te^{-2t}\,dt$$

limits $t = 1$, $x = \frac{3}{1} = 3$ $\qquad$ $t = 6$, $x = \frac{3}{6} = 0.5$

$$= \int_{0.5}^{3} 3te^{-2t}\,dt$$

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(c) Hence determine the exact area of R. **[5]**

$$\int_{0.5}^{3} 3te^{-2t}\,dt$$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

$u = 3t \qquad v = -\frac{1}{2}e^{-2t}$

$u' = 3 \qquad v' = e^{-2t}$

$$= \left[-\frac{1}{2}\times 3t\times e^{-2t} + \frac{3}{2}\int e^{-2t}\,dt\right]_{0.5}^{3}$$

$$= \left[-\frac{1}{2}\times 3t\times e^{-2t} - \frac{3}{4}e^{-2t}\right]_{0.5}^{3}$$

$$= \left(-\frac{1}{2}\times 3\times 3\times e^{-6} - \frac{3}{4}e^{-6}\right) - \left(-\frac{3}{4}e^{-1} - \frac{3}{4}e^{-1}\right)$$

$$= -\frac{21}{4}e^{-6} + \frac{3}{2}e^{-1}$$

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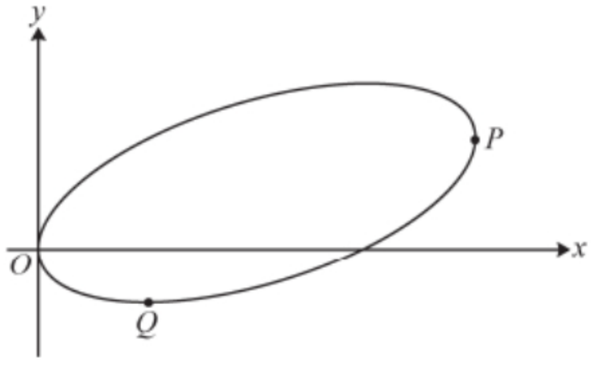
6 In this question you must show detailed reasoning.



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

$4xy = 2x^2 + 8y^2 - 9x$

implicit differentiation

$4x\frac{dy}{dx} + 4y = 4x + 16y\frac{dy}{dx} - 9$

$$4x\frac{dy}{dx} + 4y = 4x + 16y\frac{dy}{dx} - 9$$

$$4x\frac{dy}{dx} - 16y\frac{dy}{dx} = 4x - 4y - 9$$

$$\frac{dy}{dx}(4x - 16y) = 4x - 4y - 9$$

$$\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$$ as required

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(b) Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**

at P $\frac{dy}{dx}$ is infinite

$\therefore 4x - 16y = 0$ (denominator in $\frac{dy}{dx} = 0$)

$x = 4y$

$4xy = 2x^2 + 8y^2 - 9x$

$4 \times 4y \times y = 2(4y)^2 + 8y^2 - 9 \times 4y$

$16y^2 = 32y^2 + 8y^2 - 36y$

$0 = 24y^2 - 36y$

$0 = 12y(2y - 3)$

$y = 0$ or $y = \frac{3}{2}$ $\therefore x = 4 \times \frac{3}{2} = 6$

P is $(6, \frac{3}{2})$

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at Q , $\frac{dy}{dx} = 0$

$\therefore$ numerator of $\frac{dy}{dx} = 0$

$$4x - 4y - 9 = 0$$

$$4x = 4y + 9$$

$$x = y + \frac{9}{4}$$

$$4xy = 2x^2 + 8y^2 - 9x$$

$$4x\left(y + \frac{9}{4}\right)xy = 2\left(y + \frac{9}{4}\right)^2 + 8y^2 - 9\left(y + \frac{9}{4}\right)$$

$$4y^2 + 9y = 2\left(y^2 + \frac{9}{2}y + \frac{81}{16}\right) + 8y^2 - 9y - \frac{81}{4}$$

$$4y^2 + 9y = 2\left(y^2 + \frac{9}{2}y + \frac{81}{16}\right) + 8y^2 - 9y - \frac{81}{4}$$

$$4y^2 + 9y = 2y^2 + 9y + \frac{81}{8} + 8y^2 - 9y - \frac{81}{4}$$

$$0 = 6y^2 - 9y - \frac{81}{8}$$

$$y = \frac{9}{2} \quad \text{or} \quad y = -\frac{3}{4}$$

from diagram Q below x-axis

$$\therefore y = -\frac{3}{4}$$

$$x = -\frac{3}{4} + \frac{9}{4} = \frac{6}{4} = \frac{3}{2}$$

$$Q\left(\frac{3}{2}, -\frac{3}{4}\right)$$

$P\left(6, \frac{3}{2}\right)$ $Q\left(\frac{1}{2}, -\frac{3}{4}\right)$

$$\text{length } PQ = \sqrt{\left(\frac{3}{2} - -\frac{3}{4}\right)^2 + \left(6 - \frac{3}{2}\right)^2}$$

$$= \frac{9\sqrt{5}}{4}$$

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Section B: Mechanics

Answer **all** the questions.

7 A particle P moves with constant acceleration $(-4\mathbf{i}+2\mathbf{j})\,\text{m s}^{-2}$. At time $t = 0$ seconds, P is moving with velocity $(7\mathbf{i}+6\mathbf{j})\,\text{m s}^{-1}$.

(a) Determine the speed of P when $t = 3$. **[4]**

(b) Determine the change in displacement of P between $t = 0$ and $t = 3$. **[2]**

$\underset{\sim}{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

integrating

$\underset{\sim}{v} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} t + \begin{pmatrix} a \\ b \end{pmatrix}$

at $t = 0$ $\quad \underset{\sim}{v} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$

$\therefore \quad a = 7 \quad b = 6$

$\underset{\sim}{v} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} t + \begin{pmatrix} 7 \\ 6 \end{pmatrix}$

$$\underset{\sim}{v} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} t + \begin{pmatrix} 7 \\ 6 \end{pmatrix}$$

when $t = 3$

$$\underset{\sim}{v} = 3\begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} -12 \\ 6 \end{pmatrix} + \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$$

Speed is $\sqrt{(-5)^2 + (12)^2} = 13 \text{ m s}^{-1}$

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(b) Determine the change in displacement of P between $t = 0$ and $t = 3$. **[2]**

$$\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2$$

$\underset{\sim}{s}$

$\underset{\sim}{u} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$ at $t = 0$

$\underset{\sim}{v}$ =

$\underset{\sim}{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

$t = 3$

$$\underset{\sim}{s} = 3\begin{pmatrix} 7 \\ 6 \end{pmatrix} + \frac{1}{2} \times 3^2 \times \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\underset{\sim}{s} = \begin{pmatrix} 21 \\ 18 \end{pmatrix} + \frac{9}{2}\begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\underset{\sim}{s} = \begin{pmatrix} 21 \\ 18 \end{pmatrix} + \begin{pmatrix} -18 \\ 9 \end{pmatrix} = \begin{pmatrix} 3 \\ 27 \end{pmatrix} \text{ m}$$

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8 A car is travelling on a straight horizontal road. The velocity of the car, $v\,\text{ms}^{-1}$, at time t seconds as it travels past three points, P, Q and R, is modelled by the equation

$$v = at^2 + bt + c,$$

where a, b and c are constants.

The car passes P at time $t = 0$ with velocity $8\,\text{ms}^{-1}$.

(a) State the value of c. **[1]**

$v = at^2 + bt + c$

at $t = 0$, $v = 8$

$8 = c$

The car passes Q at time $t = 5$ and at that instant its deceleration is $0.12\,\text{ms}^{-2}$. The car passes R at time $t = 18$ with velocity $2.96\,\text{ms}^{-1}$.

(b) Determine the values of a and b. **[4]**

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Q at $t = 5$, $a = -0.12\ ms^{-2}$

R at $t = 18$, $v = 2.96\ ms^{-1}$

$v = at^2 + bt + 8$

differentiating

$acc = 2at + b$

at Q $-0.12 = 10a + b$ (1)

at R $2.96 = a \times 18^2 + 18b + 8$

$2.96 = 324a + 18b + 8$

$-5.04 = 324a + 18b$ (2)

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use calculator to solve simultaneous equations (1) and (2)

$a = -0.02 \qquad b = 0.08$

(c) Find, to the nearest metre, the distance between points P and R. **[2]**

$v = -0.02t^2 + 0.08t + 8$

distance travelled

$$= \int_0^{18} (-0.02t^2 + 0.08t + 8)\,dt$$

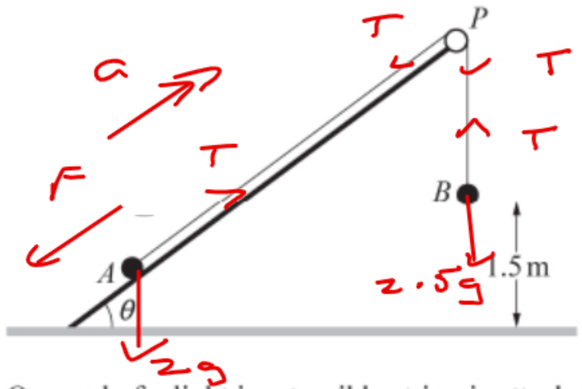
$$= \left[\frac{-0.02}{3} t^3 + \frac{0.08t^2}{2} + 8t \right]_0^{18}$$

$$\left[\frac{-0.02}{3}t^3 + \frac{0.08t^2}{2} + 8t\right]_0^{18}$$

$$= \left(\frac{-0.02}{3} \times 18^3 + \frac{0.08}{2} \times 18^2 + 18 \times 8\right) - (0 + c + 0)$$

$$= 118.08 \text{ m}$$

9



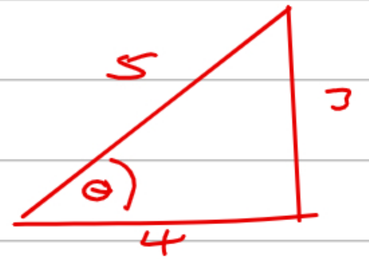
One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 2.5 kg. Particle A is in contact with a rough plane inclined at θ to the horizontal, where $\cos\theta = \frac{4}{5}$. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. Particle B hangs freely below P at a distance 1.5 m above horizontal ground, as shown in the diagram.

The coefficient of friction between A and the plane is μ. The system is released from rest and in the subsequent motion B hits the ground before A reaches P. The speed of B at the instant that it hits the ground is $1.2\,\text{m s}^{-1}$.

(a) For the motion before B hits the ground, show that the acceleration of B is $0.48\,\text{m s}^{-2}$. **[1]**

$\mu =$

$\downarrow a$



$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

a) $s = 1.5\,\text{m}$

$u = 0\,\text{m s}^{-1}$

$v = 1.2\,\text{m s}^{-1}$

$a =$

t

$v^2 = u^2 + 2as$

$1.2^2 = 0^2 + 2 \times a \times 1.5$

$a = \frac{1.2^2}{3} = 0.48\,\text{m s}^{-2}$

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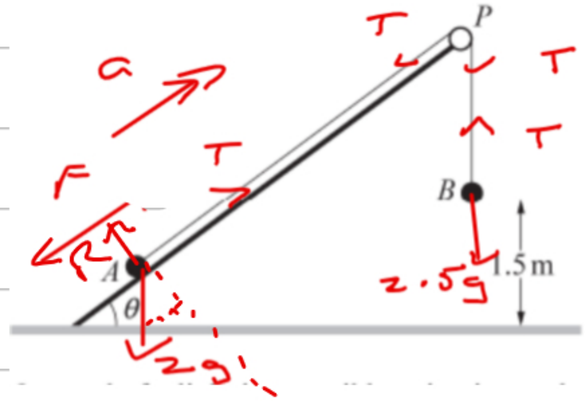


(b) For the motion before B hits the ground, show that the tension in the string is 23.3 N. **[3]**

For B (↓)

$2.5 \times 0.48 = 2.5g - T$

$T = 2.5 \times 9.8 - 2.5 \times 0.48$

$T = 23.3$ N (as required)

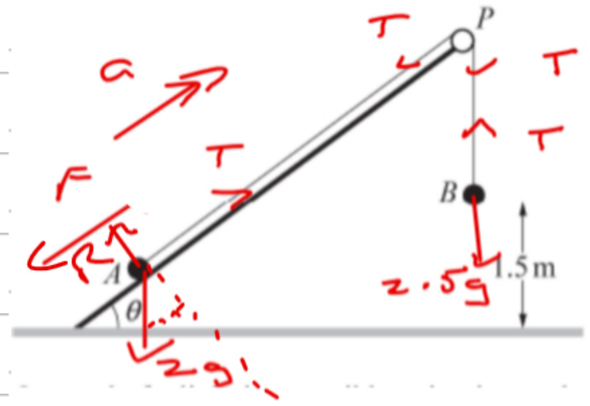
(c) Determine the value of μ. **[5]**

Motion A (→) plane

$2 \times 0.48 = T - F - 2g \sin\theta$ (1)

Limiting Friction

$F = \mu R$ (2)



Motion A (↖) perpendicular to plane

$2 \times 0 = R - 2g\cos\theta$

$0 = R - 2 \times 9.8 \times \frac{4}{5}$

$R = 15.68\ N$

$2 \times 0.48 = T - F - 2g\sin\theta$ (1)

$0.96 = 23.3 - \mu R - 2 \times 9.8 \times \frac{3}{5}$

$0.96 = 23.3 - \mu \times 15.68 - 11.76$

$15.68\mu = 23.3 - 11.76 - 0.96$

$\mu = \frac{23.3 - 11.76 - 0.96}{15.68} = \frac{529}{784} = 0.675$ (3sf)

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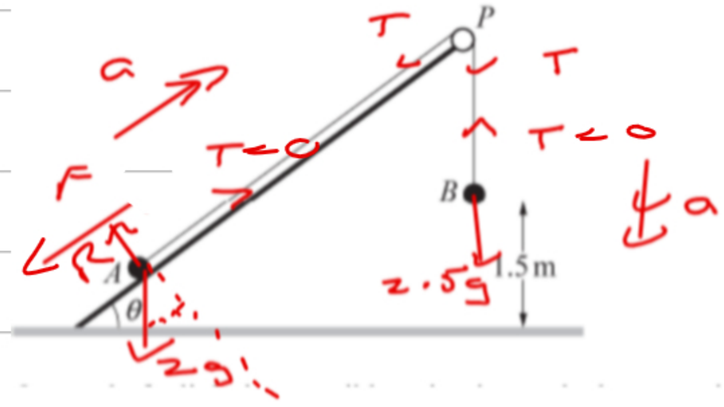

After B hits the ground, A continues to travel up the plane before coming to instantaneous rest before it reaches P.

(d) Determine the distance that A travels from the instant that B hits the ground until A comes to instantaneous rest. **[4]**

Equation of motion for B (↗)
with not T as string slack

$$2 \times a = -F - 2g \sin\theta$$

$$2 \times a = \mu R - 2 \times 9.8 \times \frac{3}{5}$$

$$2a = -0.675 \times 15.68 - 11.76$$

$$a = \frac{-0.675 \times 15.68 - 11.76}{2}$$

$$a = -11.172 \text{ m s}^{-2}$$

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For A

$s = ?$

$u = 1.2\ ms^{-1}$

$v = 0$

$a = -11.172$

t

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - 1.2^2}{2 \times -11.172} = 0.0644\ m \text{ (3 sf)}$$

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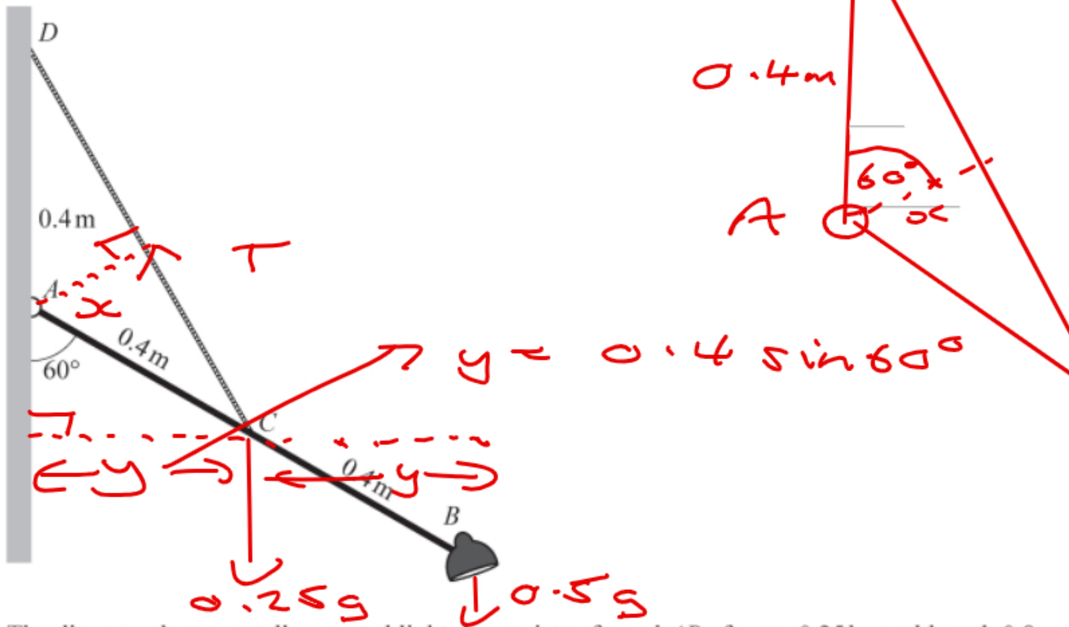

The diagram shows a wall-mounted light. It consists of a rod AB of mass 0.25 kg and length 0.8 m which is freely hinged to a vertical wall at A, and a lamp of mass 0.5 kg fixed at B. The system is held in equilibrium by a chain CD whose end C is attached to the midpoint of AB. The end D is fixed to the wall a distance 0.4 m vertically above A. The rod AB makes an angle of 60° with the downward vertical.

The chain is modelled as a light inextensible string, the rod is modelled as uniform and the lamp is modelled as a particle.

(a) By taking moments about A, determine the tension in the chain. **[4]**

$x = 0.4\cos 60°$ perpendicular from A to T

$y = 0.4\sin 60°$

m(A)

$0.4\cos 60 \times T = y \times 0.25g + 2y \times 0.5g$

$0.4\cos 60\,T = 0.4\sin 60 \times 0.25g + 2 \times 0.4 \times \sin 60 \times 0.5g$

$T = \dfrac{0.4\sin 60 \times 0.25g + 0.8\sin 60 \times 0.5g}{0.4\cos 60}$

$T = 21.2\ \text{N}$ (3sf)

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(b) (i) Determine the magnitude of the force exerted on the rod at A. **[4]**

(ii) Calculate the direction of the force exerted on the rod at A. **[2]**

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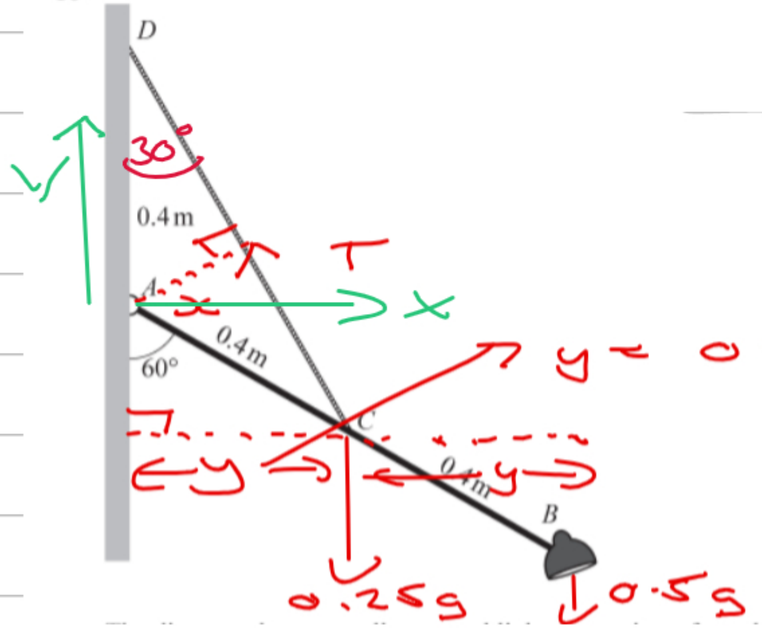

X horizontal force at A

Y vertical force at A

$y = 0.4 \sin 60°$

$R(\uparrow)\quad Y + T\cos 30° = 0.75g$

$R(\rightarrow)\quad X = T\sin 30°$

$X = 21.217622239 \times \frac{1}{2}$

$X = 10.6088112\ N$

$Y = 0.75g - 21.217622239 \times \cos 30°$

$Y = -11.025\ N$

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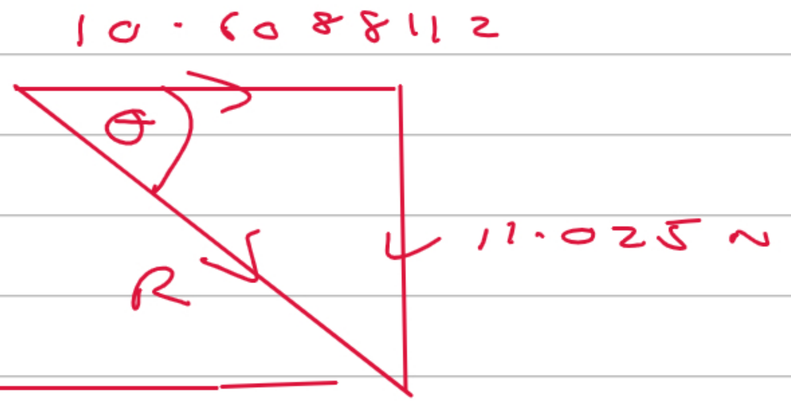
$X = 10.6088112\ N$

$Y = 0.75g - 21.2176229 \times \cos 30°$

$Y = -11.025\ N$



(i) $R = \sqrt{10.6088112^2 + 11.025^2}$

$= 15.358 = 15.4\ N$ (3 sf)

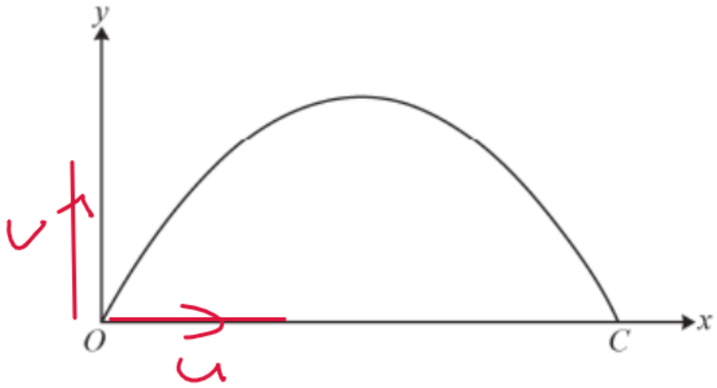
(ii) $\theta = \tan^{-1}\left(\frac{11.025}{10.6088112}\right) \approx 46.1°$ (3 sf)

below horizontal

(c) Suggest one improvement that could be made to the model to make it more realistic. **[1]**

Consider the weight of the chain

11



A particle P moves freely under gravity in the plane of a fixed horizontal axis Ox, which lies on horizontal ground, and a fixed vertical axis Oy. P is projected from O with a velocity whose components along Ox and Oy are U and V, respectively. P returns to the ground at a point C.

(a) Determine, in terms of U, V and g, the distance OC. **[4]**

↓ g

a) Horizontal (→)

$OC = Ut$ (1)

Vertical (OC)

$s = 0$
$u = V$
$v = ?$
$a = -g$
$t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = Vt - \frac{1}{2}gt^2$

$0 = t(V - \frac{1}{2}gt)$

Solve $t = 0$ or $V = \frac{1}{2}gt$

$t = \frac{2V}{g}$

in (1) $OC = \frac{2UV}{g}$

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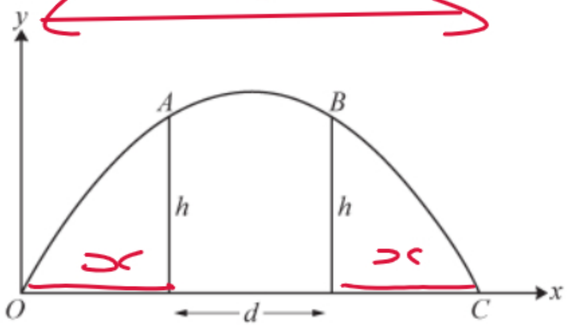
$\frac{2uv}{g}$



P passes through two points A and B, each at a height h above the ground and a distance d apart, as shown in the diagram.

(b) Write down the horizontal and vertical components of the velocity of P at A. **[2]**

horizontal velocity = U

vertical

$s = h$ $\quad v^2 = u^2 + 2as$

$u = V$ $\quad v = \sqrt{u^2 + 2as}$

$v = ?$

$a = -g$ $\quad v = \sqrt{V^2 - 2gh}$

t

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(c) Hence determine an expression for d in terms of U, V, g and h. **[3]**

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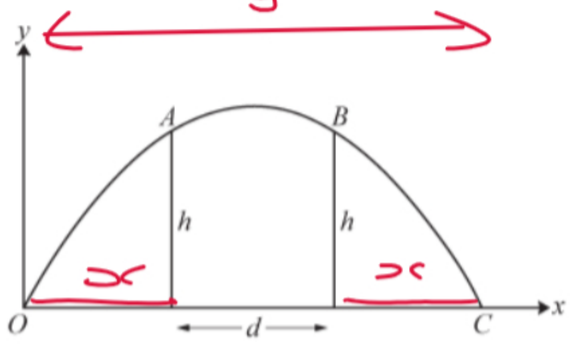
$\frac{2UV}{g}$



From A to B (horizontally)

$t = \frac{d}{U}$ (1)

Vertical from A to B

$s = 0$

$u = \sqrt{V^2 - 2gh}$

$v =$

$a = -g$

t

$s = ut + \frac{1}{2}at^2$

$0 = \sqrt{V^2 - 2gh}\,t - \frac{1}{2} \times g \times t$

(1) gives $t = \frac{d}{U}$

$0 = \sqrt{V^2 - 2gh} - \frac{1}{2}\frac{gd}{U}$

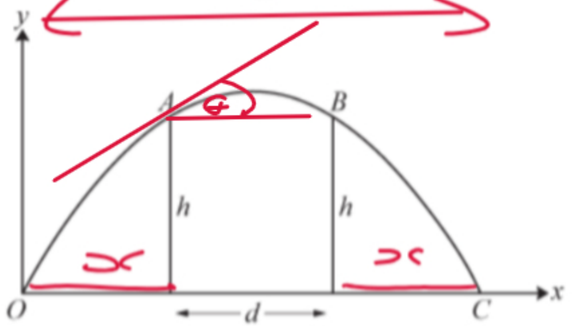
$$0 = \sqrt{v^2 - 2gh} - \frac{1}{2}\frac{gd}{u}$$

$$\frac{gd}{2u} = \sqrt{v^2 - 2gh}$$

$$d = \frac{2u}{g}\sqrt{v^2 - 2gh} \quad (1)$$

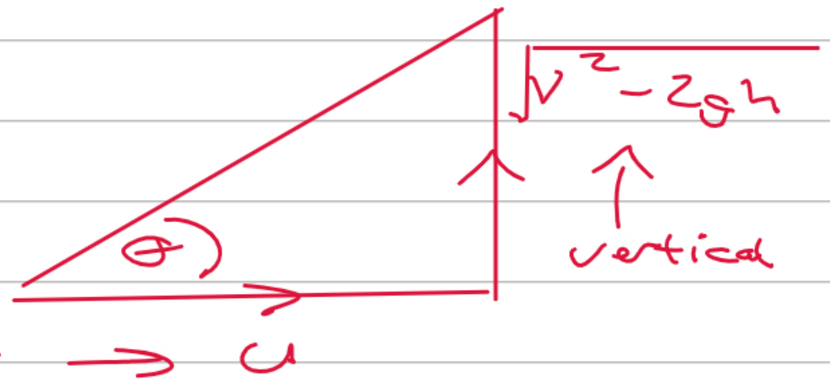
(d) Given that the direction of motion of P as it passes through A is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{1}{2}$, determine an expression for V in terms of g, d and h. **[4]**

$\frac{2uv}{g}$



$$\tan\theta = \frac{\sqrt{v^2 - 2gh}}{u} = \frac{1}{2}$$

or $\tan\theta = \frac{1}{2}$



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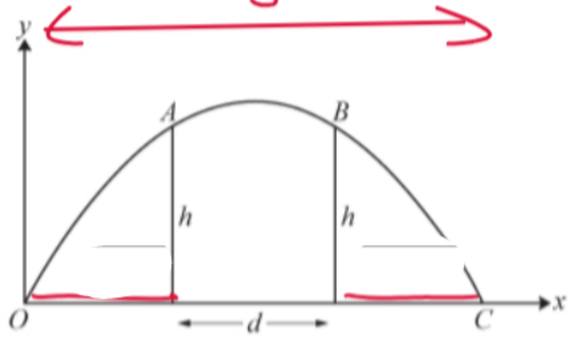
(c) He

$\frac{2uv}{g}$



$$\frac{\sqrt{v^2 - 2gh}}{u} = \frac{1}{2} \quad (2)$$

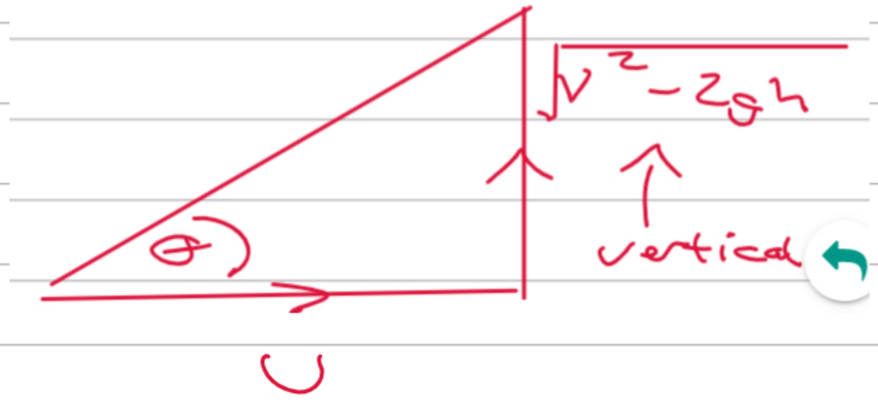
$$\frac{v^2 - 2gh}{u^2} = \frac{1}{4} \quad (3)$$

need to replace u in the above

part c) gives

$$d = \frac{2u}{g}\sqrt{v^2 - 2gh} \quad (1)$$

$$\therefore \frac{dg}{2u} = \sqrt{v^2 - 2gh}$$



$$\therefore \quad \tan\theta = \frac{\frac{dg}{2v}}{v} = \frac{1}{2}$$

$$\frac{dg}{2v^2} = \frac{1}{2}$$

$$v^2 = dg$$

$$v = \sqrt{dg}$$

Replacing $v = \sqrt{dg}$ in (3)

$$\frac{v^2 - 2gh}{(\sqrt{ag})^2} = \frac{1}{4} \quad (3)$$

$$\frac{v^2 - 2gh}{ag} = \frac{1}{4}$$

$$v^2 = \frac{ag}{4} + \frac{2gh}{1}$$

$$v^2 = \frac{ag + 8gh}{4}$$

$$v^2 = \frac{1}{4} g(a + 8h)$$

$$v = \sqrt{\frac{1}{4} g(a + 8h)}$$