

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number

Candidate Number

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Wednesday 14 October 2020

Afternoon (Time: 2 hours)

Paper Reference **9MA0/02**

Mathematics

Advanced

Paper 2: Pure Mathematics 2



You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical formulae
stored in them.**

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 - *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets
 - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ➤

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P 6 6 7 8 6 A 0 1 5 2



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- 1 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

- (a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}} dx$$

giving your answer to 3 significant figures.

(3)

- (b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} dx = 4.535 \text{ to 4 significant figures}$$

- (c) comment on the accuracy of your answer to part (b).

(1)

a) $= \frac{1}{2} \times 0.5 \times [0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)]$
 $= 1.50475$
 $= 1.50 \text{ (3sf)}$

b) $\sqrt{9} = 3$ can be taken outside
as a constant

$$3 \times 1.50 = 4.50$$

c) 4.50 is accurate to
2 significant figures



2. Relative to a fixed origin, points P , Q and R have position vectors \mathbf{p} , \mathbf{q} and \mathbf{r} respectively.

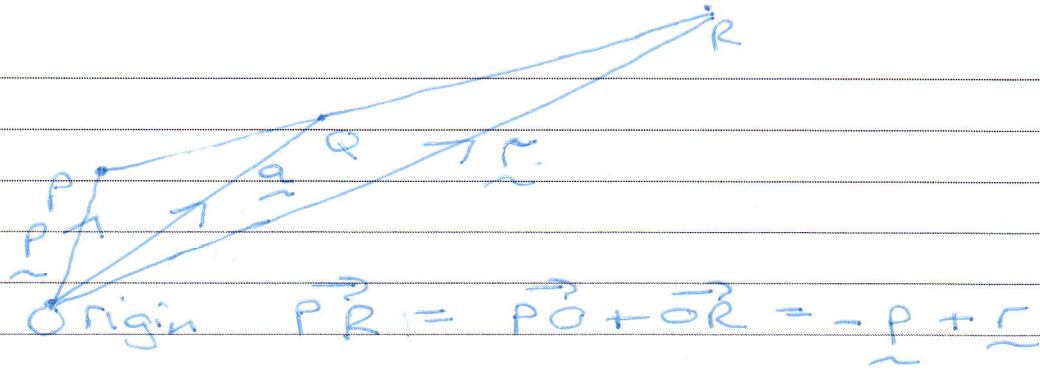
Given that

- P , Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)



$$\tilde{\vec{PQ}} = \frac{1}{3} \tilde{\vec{PR}} = \frac{1}{3} (-\underline{\vec{P}} + \underline{\vec{R}})$$

$$\tilde{\vec{PQ}} = \tilde{\vec{PO}} + \tilde{\vec{OQ}}$$

$$\frac{1}{3} (-\underline{\vec{P}} + \underline{\vec{R}}) = -\underline{\vec{P}} + \underline{\vec{q}}$$

$$-\frac{1}{3} \underline{\vec{P}} + \underline{\vec{P}} + \frac{1}{3} \underline{\vec{R}} = \underline{\vec{q}}$$

$$\frac{2}{3} \underline{\vec{P}} + \frac{1}{3} \underline{\vec{R}} = \underline{\vec{q}}$$

$$\frac{1}{3} (2\underline{\vec{P}} + \underline{\vec{R}}) = \underline{\vec{q}}$$

as required



3. (a) Given that

$$2 \log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

- (b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

- (ii) State which of the roots in (b)(i) is **not** a solution of

$$2 \log(4-x) = \log(x+8)$$

giving a reason for your answer.

$$2 \log(4-x) - \log(x+8) = 0 \quad (2)$$

$$\log(4-x)^2 - \log(x+8) = 0$$

$$\log \frac{(4-x)^2}{x+8} = 0$$

$$\frac{(4-x)^2}{x+8} = 10^0$$

$$(4-x)^2 = x+8$$

$$\begin{aligned} 16 - 8x + x^2 &= x+8 \\ x^2 - 9x + 8 &= 0 \quad (\text{as required}) \end{aligned}$$

b) (i) $(x-8)(x-1) = 0$
 $x = 8, x = 1$

(ii) $x = 8$ is not a solution
as $\log(4-8)$ gives a
math error



4. In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15120

Find the value of a .

(3)

$$7C_0 a^7 + 7C_1 a^6 \times 2x + 7C_2 a^5 (2x)^2 \\ + 7C_3 a^4 (2x)^3 + 7C_4 a^3 (2x)^4 \\ + \dots$$

$$7C_4 \times a^3 \times 2^4 = 15120$$

$$35 \times 16 \times a^3 = 15120 \\ a^3 = \frac{15120}{35 \times 16}$$

$$a = \sqrt[3]{\frac{15120}{35 \times 16}}$$

$$a = 3$$



5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P .

Find, using algebra, the exact x coordinate of P .

(4)

$$3 \times 2^x = 15 - 2^{x+1}$$

$$3 \times 2^x = 15 - 2^x \times 2^1$$

$$3 \times 2^x + 2 \times 2^x = 15$$

$$5 \times 2^x = 15$$

$$2^x = \frac{15}{5}$$

$$2^x = 3$$

$$\log_2 2^x = \log_2 3$$

$$x = \log_2 3$$



6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \quad x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants A , B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} dx$$

giving your answer in the form $a + b \ln 2$ where a and b are integers to be found.

(4)

a)

$$\begin{array}{r} x+6 \\ x+2 \longdiv{)x^2 + 8x - 3} \\ -x^2 - 2x \\ \hline 6x - 3 \\ -6x + 12 \\ \hline -15 \end{array}$$

$$\frac{x^2 + 8x - 3}{x + 2} = x + 6 - \frac{15}{x + 2}$$

$$A = 1, B = 6, C = -15$$

b)

$$\int_0^6 x + 6 - \frac{15}{x + 2} dx$$

$$= \left[\frac{x^2}{2} + 6x - 15 \ln(x+2) \right]_0^6$$

$$= (18 + 36 - 15 \ln 8) - (0 + 0 - 15 \ln 2)$$

$$= 54 - 15 \ln 2^8 + 15 \ln 2$$

$$= 54 - 45 \ln 2 + 15 \ln 2$$

$$= 54 - 30 \ln 2$$



7.

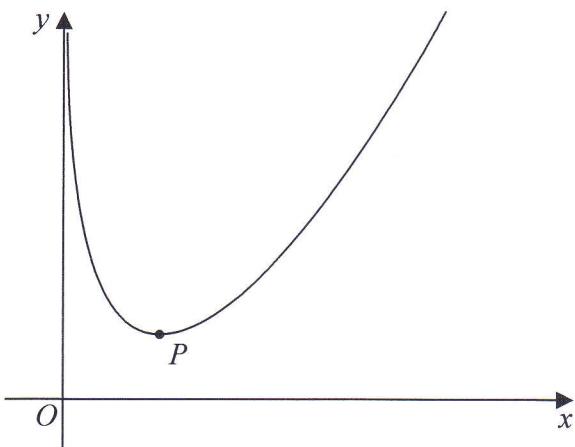
**Figure 1**

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4 \ln x \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} \quad (4)$$

The point P , shown in Figure 1, is the minimum turning point on C .

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}} \quad (3)$$

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)



Question 7 continued

a) $y = \frac{4x^2 + x}{2\sqrt{x}}$ ← u quotient rule

$$\begin{aligned} u &= 4x^2 + x \\ \frac{du}{dx} &= 8x + 1 \end{aligned}$$

$$\begin{aligned} v &= 2x^{\frac{1}{2}} \\ \frac{dv}{dx} &= x^{-\frac{1}{2}} \end{aligned}$$

$$\frac{dy}{dx} = v \frac{du}{dx} - u \frac{dv}{dx}$$

$$= \frac{2x^{\frac{1}{2}}(8x+1) - x^{-\frac{1}{2}}(4x^2+x)}{4x}$$

$$= \frac{16x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 4x^{\frac{3}{2}} - x^{\frac{1}{2}}}{4x}$$

$$= \frac{12x^{\frac{3}{2}} + x^{\frac{1}{2}}}{4x} \quad ①$$

$$\frac{d}{dx} (-4\ln x) = -\frac{4}{x} \quad ②$$

Adding ① and ②

$$\frac{12x^{\frac{3}{2}} + x^{\frac{1}{2}}}{4x} - \frac{4}{x}$$

$$= \frac{12x^{\frac{3}{2}} + x^{\frac{1}{2}}}{4x} - \frac{4 \times 4}{x}$$

$$= \frac{12x^{\frac{3}{2}} + x^{\frac{1}{2}} - 16}{4x}$$

$$= \frac{4x^{-\frac{1}{2}}(12x^2 + x - 16x^{\frac{1}{2}})}{4x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

as required ¹⁷

Turn over ▶



Question 7 continued

b) at $P \frac{dy}{dx} = 0$

$$0 = 12x^2 + x - 16 + \sqrt{x}$$

$$0 = x^{\frac{1}{2}}(12x^{\frac{3}{2}} + x^{\frac{1}{2}} - 16)$$

as $x^{\frac{1}{2}} \neq 0$

equals zero when

$$12x^{\frac{3}{2}} + x^{\frac{1}{2}} - 16 = 0$$

$$12x^{\frac{3}{2}} + x^{\frac{1}{2}} = 16$$

$$12x^{\frac{3}{2}} = 16 - \sqrt{x}$$

$$x^{\frac{3}{2}} = \frac{16}{12} - \frac{\sqrt{x}}{12}$$

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

as required

c) $x_1 = 2$

(i) $x_2 = \left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^{\frac{2}{3}} = 1.13894$

(ii) $x_3 = 1.138935342$

$$x_3 = 1.156928424$$

$$x_4 = 1.156494678$$

$$x_5 = 1.156505095$$

$$x_6 = 1.156504845$$

$$x_7 = 1.156504851$$

rounded to 5dp

$$1.15650$$



8. A curve C has equation $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$

$$f'(x) = 6x^2 + ax - 23 \quad (6)$$

$$f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$$

at y intercept -12

$$-12 = c$$

$$f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x - 12$$

if $(x + 4)$ is a factor, $f(-4) = 0$

$$2(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$$

$$-128 + 8a + 92 - 12 = 0$$

$$8a = 48$$

$$a = 6$$

$$f(x) = x^3 + 3x^2 - 23x - 12$$



9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, $\theta^\circ\text{C}$, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18°C
- after 10 seconds the temperature of the ethanol was 44°C

- (a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78°C

- (b) Use this information to evaluate the model.

(2)

a) $\theta = A - Be^{-0.07t}$
at $t = 0, \theta = 18$

$$18 = A - Be^0$$

$$18 = A - B \quad \textcircled{1}$$

$$\text{at } t = 10, \theta = 44$$

$$44 = A - Be^{-0.07 \times 10} \quad \textcircled{2}$$

$\textcircled{2} - \textcircled{1}$ gives

$$26 = -Be^{-0.7} + B$$

$$26 = B(1 - e^{-0.7})$$

$$\frac{26}{1 - e^{-0.7}} = B$$

$$B = 51.6 \quad (3 \text{ sf})$$

$$\text{in } \textcircled{1} \quad A = 18 + 51.6e^{-0.7} \\ A = 69.6 \quad (3 \text{ sf})$$

b) The maximum temperature is 69.6°C
From model.

The model is not appropriate as ethanol has
a boiling point of 78°C



10.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4 \cos^3 A - 3 \cos A$$

(4)

(b) Hence solve, for $-90^\circ \leq x \leq 180^\circ$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

a)

$$\begin{aligned}
 \cos(3A) &= \cos(2A + A) \\
 &= \cos 2A \cos A - \sin 2A \sin A \\
 &= (2\cos^2 A - 1)\cos A - (2\sin A \cos A)\sin A \\
 &\quad \uparrow \quad \uparrow \\
 &\quad \cos 2A \quad \sin 2A \\
 &= 2\cos^3 A - \cos A - 2\cos A \sin^2 A \\
 &= 2\cos^3 A - \cos A - 2\cos A(1 - \cos^2 A) \\
 &= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A \\
 &= 4\cos^3 A - 3\cos A
 \end{aligned}$$

(as required)

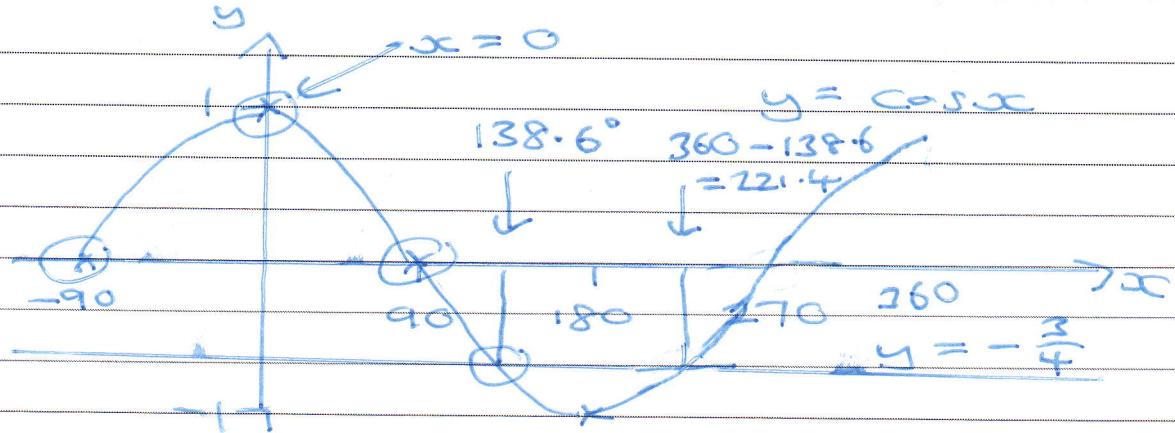
b)

$$1 - \cos 3x = 1 - 4\cos^3 x + 3\cos x$$

$$\begin{aligned}
 \therefore 1 - 4\cos^3 x + 3\cos x &= \sin^2 x \\
 1 - 4\cos^3 x + 3\cos x &= 1 - \cos^2 x \\
 0 &= 4\cos^3 x - \cos^2 x - 3\cos x \\
 0 &= \cos x (4\cos^2 x - \cos x - 3) \\
 0 &= \cos x (4\cos x + 3)(\cos x - 1) \\
 \text{Either } \cos x &= 0, \cos x = -\frac{3}{4}, \cos x = 1
 \end{aligned}$$



Question 10 continued



solutions in range are circled

$$-90^\circ \leq x \leq 180^\circ$$

$$x = -90, 0, 90, 138.6$$



11.

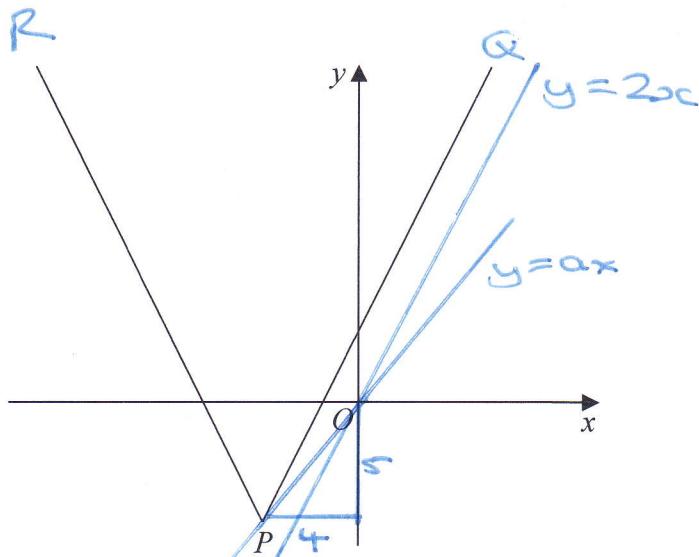


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P , shown in Figure 2.

- (a) Find the coordinates of P .

(2)

- (b) Solve the equation

(2)

$$3x + 40 = 2|x + 4| - 5$$

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

- (c) find the range of possible values of a , writing your answer in set notation.

(3)

$$\begin{aligned} a) \quad -2(x+4)-5 &= 2(x+4)-5 \\ -2x-8-5 &= 2x+8-5 \end{aligned}$$

$$-16 = 4x$$

$$x = -4$$

$$y = 2(-4+4)-5 = -5$$

$$P(-4, -5)$$



Question 11 continued

b) $3x + 40 = 2(x + 4) - 5$
 $3x + 40 = 2x + 8 - 5$
 $x = 8 - 5 - 40$
 $x = -37$ (not possible
as minimum pt
is P(-4, -5))

$$3x + 40 = -2(x + 4) - 5$$

$$3x + 40 = -2x - 8 - 5$$

$$5x = -53$$

$$x = -10.6$$

c) $y = ax$ must go through origin

Gradient $PQ = \frac{5}{4} = 1.25$

This line would have 1 solution.

Any gradient less than or equal
to $\frac{5}{4}$ has 1 or 2 solutions

Gradient of PQ is 2

Line $y = 2x$ is parallel to PQ.
Any line with a gradient > 2
would intersect with PQ or PR

$$\therefore a \leq 1.25 \text{ or } a > 2$$

in set notation

$$\{a : a \leq 1.25\} \cup \{a : a > 2\}$$



12.

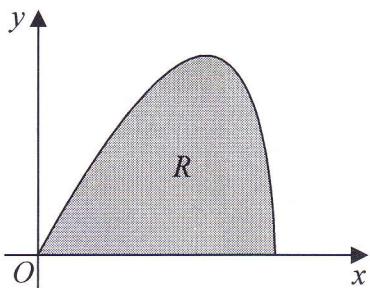


Figure 3

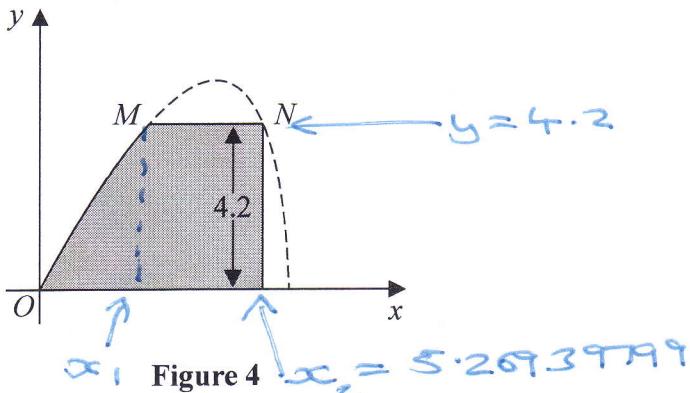
The curve shown in Figure 3 has parametric equations

$$x = 6 \sin t \quad y = 5 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

- (a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$ (3)

- (ii) Hence show, by algebraic integration, that the area of R is exactly 20 (3)



Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4. Using the model and given that

- x and y are in metres
 - the vertical wall of the dam is 4.2 metres high
 - there is a horizontal walkway of width MN along the top of the dam
- (b) calculate the width of the walkway. (5)



Question 12 continued

$$\text{a) (i)} \quad x = 6 \sin t \quad y = 5 \sin 2t$$

$$\textcircled{1} \quad \frac{dx}{dt} = 6 \cos t \quad \frac{dy}{dt} = 10 \cos 2t$$

$$\text{Area } R = \int_{\textcircled{1}} y \, dx$$

$$\textcircled{1} \text{ gives } dx = 6 \cos t \, dt$$

$$\begin{aligned} \text{Area } R &= \int_0^{\frac{\pi}{2}} 5 \sin 2t \times 6 \cos t \, dt \\ &= \int_0^{\frac{\pi}{2}} 5 \times 2 \sin t \cos t \times 6 \cos t \, dt \\ &= \int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt \\ &\quad \text{as required} \end{aligned}$$

$$\text{(ii)} \quad \frac{d}{dt} (\cos^3 t) = -3 \cos^2 t \sin t$$

$$\therefore \frac{d}{dt} (-20 \cos^3 t) = 60 \sin t \cos^2 t$$

$$\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt = [-20 \cos^3 t]_0^{\frac{\pi}{2}}$$

$$R = (-20 \times 0^3) - (-20 \times 1^3) = 20$$

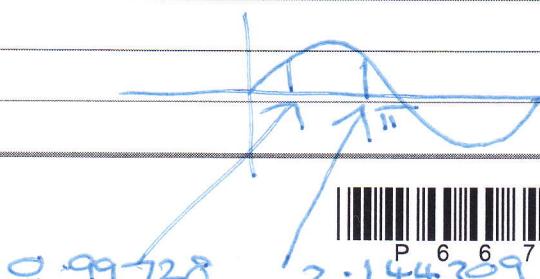
b) Find x -coordinate where line $y=4.2$ meets the curve

$$y = 5 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

$$4.2 = 5 \sin 2t$$

$$\sin 2t = \frac{4.2}{5} = \frac{21}{25} \quad 0 \leq 2t \leq \pi$$

$$2t = 0.99728 \quad \text{or } 2.144309$$



Question 12 continued

$$\therefore \div \text{ by } 2 \quad t = 0.4986416 \text{ or } t = 1.0721547$$

$$x = 6 \sin t$$

$$x_1 = 6 \sin(0.4986416) = 2.86939799$$

$$\text{or } x_2 = 6 \sin(1.0721547) = 5.26939799$$

$$MN = x_2 - x_1$$

$$MN = 5.26939799 - 2.86939799$$

$$MN = 2.4 \text{ m}$$

(Total for Question 12 is 11 marks)



P 6 6 7 8 6 A 0 3 7 5 2

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \quad x > 0 \quad x \neq k$$

where k is a constant.

(a) Deduce the value of k .

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g .

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

$$\begin{aligned} a) \quad \ln(x) - 2 &= 0 \\ \ln x &= 2 \\ x &= e^2 \\ R &= e^2 \end{aligned}$$

b) Quotient rule

$$u = 3\ln(x) - 7 \quad v = \ln(x) - 2$$

$$u' = \frac{3}{x} \quad v' = \frac{1}{x}$$

$$g'(x) = v \frac{du}{dx} - u \frac{dv}{dx}$$

\sqrt{x}

$$= -2 \times \frac{3}{x} - (-7) \times \frac{1}{x}$$

$$\frac{(-2) \times 3 + 7}{(\ln(x) - 2)^2}$$

$$g'(x) = \frac{\frac{1}{x}}{(\ln(x) - 2)^2}$$

as $x > 0$, $\frac{1}{x}$ must be positive,
and as denominator is squared, it must
be positive, so $g'(x)$ must be > 0



Question 13 continued

c) $g(a) > 0$ when

$$3 \ln(x) - 7 > 0$$

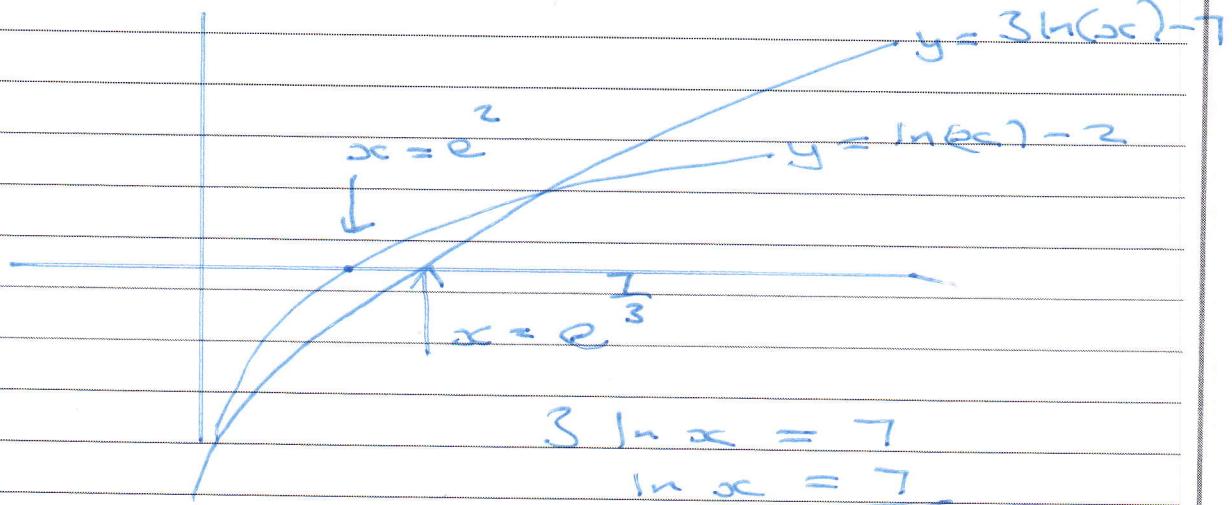
and

$$\ln(x) - 2 > 0$$

$$\text{or } 3 \ln(x) - 7 < 0$$

and

$$\ln(x) - 2 < 0$$



$$\ln x = 2$$

$$x = e^2$$

$$3 \ln x = 7$$

$$\ln x = \frac{7}{3}$$

$$x = e^{\frac{7}{3}}$$

Range of values of a for
 $g(a) > 0$

$$0 < a < e^2 \quad \text{or} \quad a > e^{\frac{7}{3}}$$

(both < 0)

(both > 0)



14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x -axis and touches the y -axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

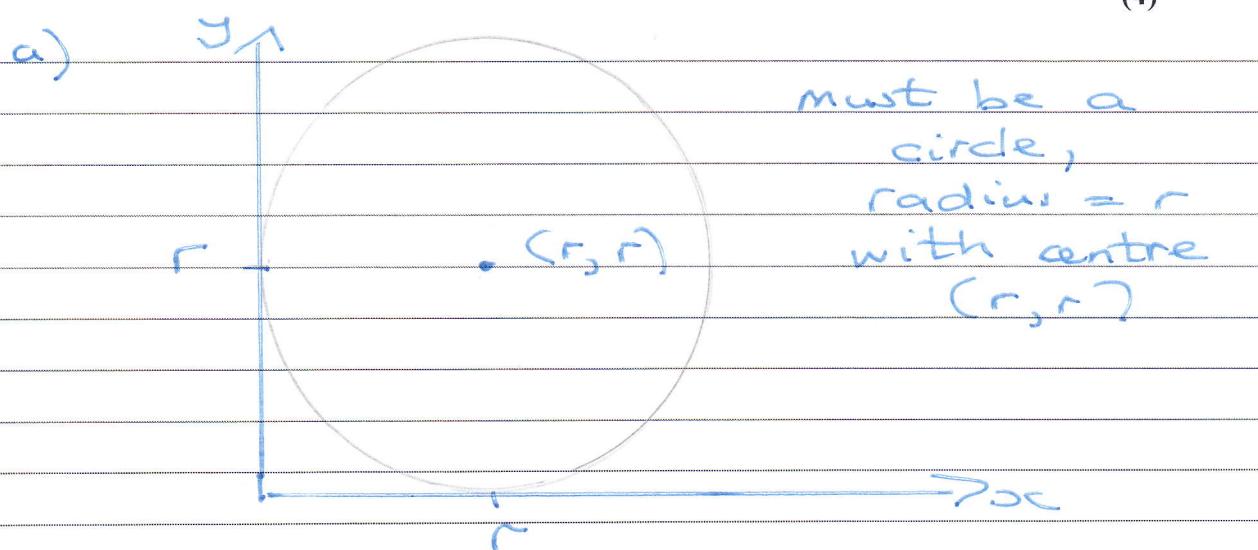
$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C ,

(b) find the two possible values of r , giving your answers as fully simplified surds.

(4)



$$y = -2x + 12 \quad \textcircled{1}$$

$$\text{Circle } (x-r)^2 + (y-r)^2 = r^2 \quad \textcircled{2}$$

sub \textcircled{1} in \textcircled{2}

$$(x-r)^2 + (-2x+12-r)^2 = r^2$$

$$(x-r)(x-r) + (-2x+12-r)(-2x+12-r) = r^2$$

$$\begin{aligned} x^2 - 2rx + r^2 + 4x^2 - 24x + 144 - 12r &= r^2 \\ -24x + 2rx &+ 144 - 12r \\ &= -12r + r^2 \\ &= r^2 \end{aligned}$$



Question 14 continued

$$5x^2 + 2rx - 48x + r^2 - 24r + 144 = r^2$$

$$5x^2 + 2rx - 48x + r^2 - 24r + 144 = 0$$

$$5x^2 + (2r-48)x + (r^2-24r+144) = 0$$

as required

b) IF L is a tangent to C
it only touches at one point $\therefore b^2 - 4ac = 0$

$$a = 5, b = 2r-48, c = r^2-24r+144$$

$$b^2 - 4ac = 0$$

$$(2r-48)^2 - 4 \times 5 \times (r^2 - 24r + 144) = 0$$

$$4r^2 - 192r + 2304 - 20r^2 + 480r - 2880 = 0$$

$$-16r^2 + 288r - 576 = 0$$

solve quadratic using calculator

$$r = 9 + 3\sqrt{5} \text{ or } 9 - 3\sqrt{5}$$



P 6 6 7 8 6 A 0 4 3 5 2

15.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r} \quad (4)$$

Given also that S_{10} is four times S_5

(b) find the exact value of r . (4)

a) ① $S_n = a + ar + ar^2 + \dots + ar^{n-1}$

② $r S_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$

① - ② gives

$$S_n - r S_n = a - ar^n$$

$$S_n (1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

b) $S_{10} = 4 S_5$

$$\frac{a(1-r^{10})}{1-r} = \frac{4a(1-r^5)}{1-r}$$

$$a - ar^{10} = 4a - 4ar^5$$

$$4ar^5 - ar^{10} = 3ar^5$$

$$0 = r^{10} - 4r^5 + 3$$

$$0 = (r^5 - 3)(r^5 - 1)$$

Either $r^5 - 3 = 0$ or $r^5 - 1 = 0$

$$r^5 = 3 \quad r = \sqrt[5]{3}$$

$$r = 1 \quad \text{but } r \neq 1$$



16. Use algebra to prove that the square of any natural number is either a multiple of 3 or one more than a multiple of 3

(4)

Natural numbers are

$$3k, 3k+1, 3k+2$$

$$(3k)^2 = 9k^2 = 3 \times 3k^2$$

a multiple of 3

$$(3k+1)^2 = 9k^2 + 6k + 1$$
$$= 3(3k^2 + 2k) + 1$$

one more than a
multiple of 3

$$(3k+2)^2 = 9k^2 + 12k + 4$$
$$= 9k^2 + 12k + 3 + 1$$
$$= 3(3k^2 + 4k + 1) + 1$$

one more than a
multiple of 3

Each of the 3 cases above is
either a multiple of 3 or
one more than a multiple of 3

