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Candidate surname

Other names

Pearson Edexcel
Level 3 GCE

Centre Number

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Candidate Number

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Wednesday 7 October 2020

Morning (Time: 2 hours)

Paper Reference **9MA0/01**

Mathematics
Advanced
Paper 1: Pure Mathematics 1



You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

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Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

- (b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

$$a) \quad 1 + \frac{\left(\frac{1}{2}\right)(8x)}{1} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(8x)^2}{1 \times 2} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(8x)^3}{1 \times 2 \times 3} + \dots$$

$$\approx 1 + 4x - 2x^2 + 32x^3 + \dots$$

$$b) \quad \left(1 + \frac{8}{32}\right)^{\frac{1}{2}} = \left(1 + \frac{1}{4}\right)^{\frac{1}{2}} = \left(\frac{5}{4}\right)^{\frac{1}{2}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$$

$$\therefore \text{when } x = \frac{1}{32}$$

$$(1 + 8x)^{\frac{1}{2}} = \frac{\sqrt{5}}{2}$$

$$\therefore 2(1 + 8x)^{\frac{1}{2}} = \sqrt{5}$$

Substitute $x = \frac{1}{32}$ in

$$2\left(1 + 4 \times \frac{1}{32} - 2 \times \frac{1}{32^2} + \frac{32}{32^3} + \dots\right)$$

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2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

$$\ln 4^{3p-1} = \ln 5^{210}$$

$$(3p-1) \ln 4 = 210 \ln 5$$

$$3p \ln 4 - \ln 4 = 210 \ln 5$$

$$3p \ln 4 = 210 \ln 5 + \ln 4$$

$$p = \frac{210 \ln 5 + \ln 4}{3 \ln 4}$$

$$p = 81.6008$$

$$p = 81.6 \text{ (1dp)}$$

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3. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

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(a) Find \vec{AB} (2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer. (2)

$$a) \vec{AB} = (3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$$

$$= \mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$$

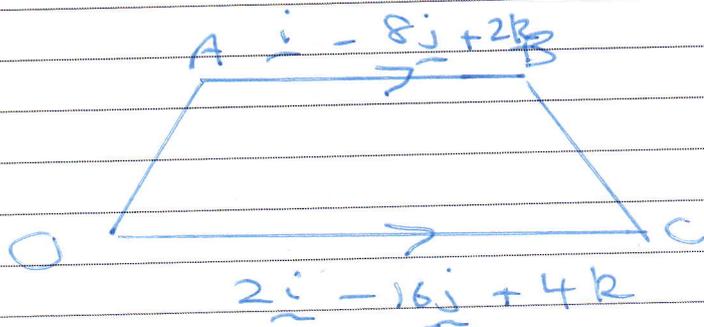
$$b) \vec{BC} = \mathbf{c} - \mathbf{b} =$$

$$(2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}) - (3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k})$$

$$= -\mathbf{i} - 13\mathbf{j} + 8\mathbf{k}$$

$$\vec{OC} = 2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$$

this is an exact multiple of \vec{AB} $\therefore \vec{OC}$ and \vec{AB} are parallel



$$\text{but } |\vec{OC}| = \sqrt{2^2 + 16^2 + 4^2} = 2\sqrt{69}$$

$$|\vec{AB}| = \sqrt{1^2 + 8^2 + 2^2} = \sqrt{69}$$

as parallel but $|\vec{OC}|$ is double size of $|\vec{AB}|$ they are parallel but



4. The function f is defined by

$$f(x) = \frac{3x-7}{x-2} \quad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$ (2)

(b) Show that $ff(x) = \frac{ax+b}{x-3}$ where a and b are integers to be found. (3)

$$\begin{aligned} \text{a)} \quad y &= \frac{3x-7}{x-2} \\ y(x-2) &= 3x-7 \\ xy-2y &= 3x-7 \\ xy-3x &= 2y-7 \\ x(y-3) &= 2y-7 \\ x &= \frac{2y-7}{y-3} \end{aligned}$$

$$f^{-1}(x) = \frac{2x-7}{x-3}$$

$$f^{-1}(7) = \frac{2 \times 7 - 7}{7 - 3} = \frac{7}{4}$$

$$\text{b)} \quad ff(x) = 3 \left(\frac{3x-7}{x-2} \right) - 7$$

$$\frac{3x-7}{x-2} - 2$$

$$= \frac{9x-21}{x-2} - \frac{7(x-2)}{x-2}$$

$$\frac{3x-7}{x-2} - \frac{2(x-2)}{x-2}$$

$$= \frac{9x-21-7x+14}{x-2}$$

$$\frac{3x-7-2x+4}{x-2}$$

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Question 4 continued

$$= \frac{9x - 21 - 7x + 14}{3x - 7 - 2x + 4}$$

$$= \frac{2x - 7}{x - 3}$$

(Total for Question 4 is 5 marks)



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6. (a) Express $\sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, $\theta^\circ\text{C}$, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2 \cos\left(\frac{\pi t}{12} - 3\right) \quad 0 \leq t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

- (b) deduce the maximum temperature of the room during this day,

(1)

- (c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

$$R \sin(x + \alpha) = R \sin x \cos \alpha + R \cos x \sin \alpha$$

$$\sin x + 2 \cos x$$

$$R \cos \alpha = 1$$

$$R \sin \alpha = 2$$

$$R = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\tan \alpha = 2$$

$$\therefore \alpha = 1.107148718$$

$$\alpha = 1.107 \text{ (3dp)}$$

$$\sin x + 2 \cos x = \sqrt{5} \sin(x + 1.107)$$

$$\text{b) } \theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2 \cos\left(\frac{\pi t}{12} - 3\right)$$

$$\text{Max value of } \sin x + 2 \cos x = \sqrt{5}$$

$$\therefore \text{max } \theta = 5 + \sqrt{5} = 7.24^\circ\text{C} \text{ (2dp)}$$



Question 6 continued

$$c) \quad \sin x + 2 \cos x = \sqrt{5} \sin(x + 1.107)$$

$$\Theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2 \cos\left(\frac{\pi t}{12} - 3\right)$$

$$\frac{\pi t}{12} - 3$$

\therefore maximum when

$$\sin(x + 1.107) = 1$$

$$\sin\left(\frac{\pi t}{12} - 3 + 1.107\right) = 1$$

$$\Rightarrow \frac{\pi t}{12} - 3 + 1.107 = \frac{\pi}{2}$$

rearranging

$$\text{as } \sin \frac{\pi}{2} = 1$$

$$t = \frac{(3 + \frac{\pi}{2} - 1.107) \times 12}{\pi}$$

$$t = 13.23072737 \text{ minutes}$$

$$0.23072737 \times 60 = 13.84 \text{ minutes}$$

$$\approx 14 \text{ minutes}$$

$$\text{Time} = 1.14 \text{ pm}$$

(Total for Question 6 is 7 marks)



7.

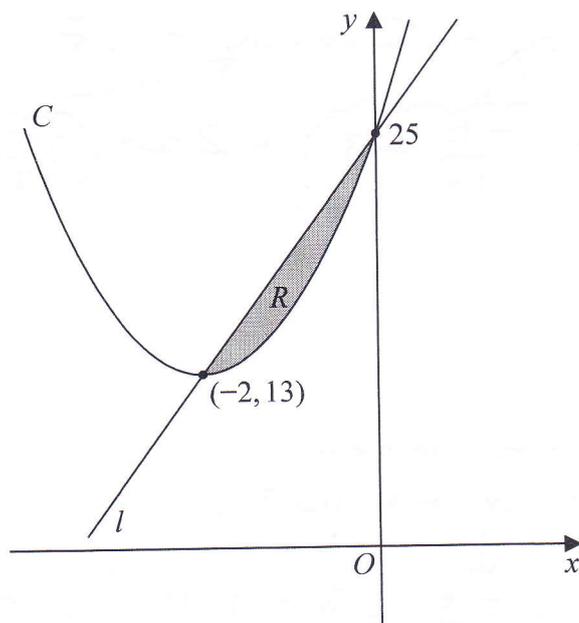


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ and a straight line l .

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $f(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = f(x)$

use inequalities to define R .

$$\text{line } l \quad \begin{matrix} (0, 25) \\ (-2, 13) \end{matrix} \quad m = \frac{25-13}{0-(-2)} = \frac{12}{2} = 6 \quad (5)$$

$$l \text{ is } y = 6x + 25$$

Curve, Using minimum pt of curve is $(-2, 13)$
 $y = a(x+2)^2 + 13$
 format for completing the square

$$\begin{aligned} \therefore y &= a(x^2 + 4x + 4) + 13 \\ &= ax^2 + 4ax + 4a + 13 \end{aligned}$$

when $x = 0, y = 25$
 $\therefore 4a + 13 = 25, a = 3$



Question 7 continued

Inequality for R is

$$3(x+2)^2 + 13 < y < 6x + 25$$



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8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n , at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t .

(You do not need to evaluate any unknown constants in your equation.)

(2)

$$n = Ae^{kt}$$

where A and k are positive constants



9.

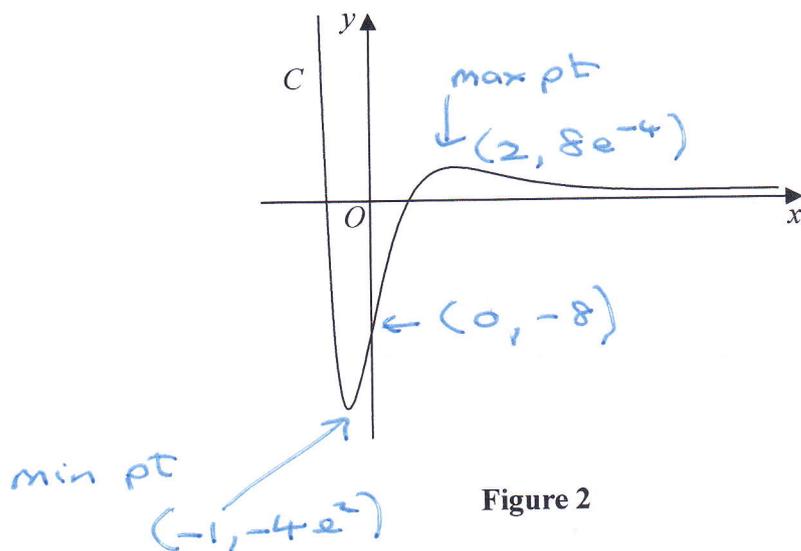


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = 4(x^2 - 2)e^{-2x} \quad x \in \mathbb{R}$$

- (a) Show that $f'(x) = 8(2 + x - x^2)e^{-2x}$ (3)
- (b) Hence find, in simplest form, the exact coordinates of the stationary points of C . (3)

The function g and the function h are defined by

$$g(x) = 2f(x) \quad x \in \mathbb{R}$$

$$h(x) = 2f(x) - 3 \quad x \geq 0$$

- (c) Find (i) the range of g
(ii) the range of h (3)

$$a) \quad f(x) = 4(x^2 - 2)e^{-2x}$$

$$u = 4x^2 - 8$$

$$u' = 8x$$

$$v = e^{-2x}$$

$$v' = -2e^{-2x}$$

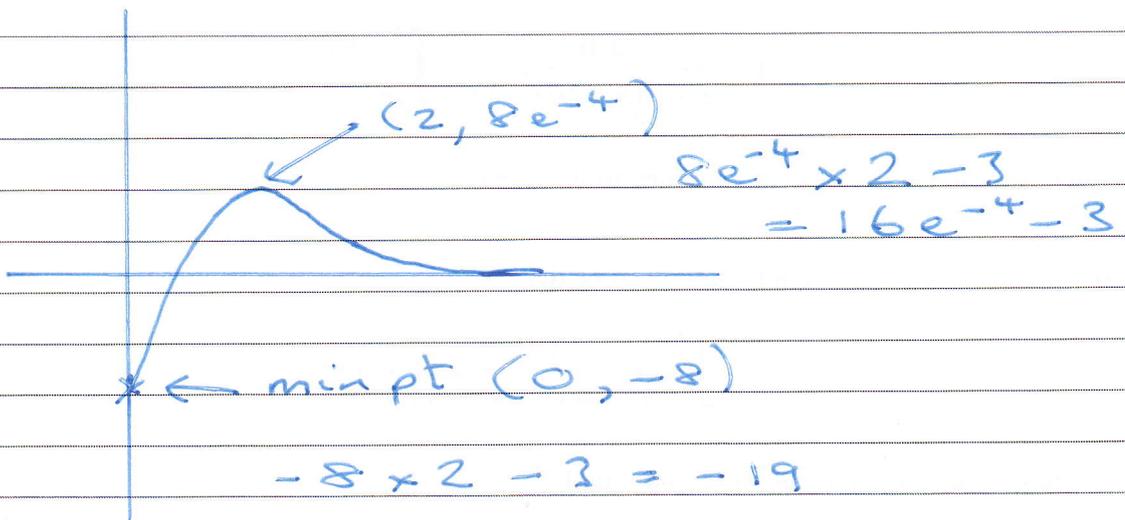
$$f'(x) = 8x e^{-2x} - 8x^2 e^{-2x} + 16e^{-2x}$$

$$= 8e^{-2x}(x - x^2 + 2)$$

as required



Question 9 continued



∴ Range of h is $[-19, 16e^{-4} - 3]$

↑ ↑
min max)

(Total for Question 9 is 9 marks)



10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3 dx}{(x-1)(3+2\sqrt{x-1})} = \int_p^q \frac{6 du}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3 dx}{(x-1)(3+2\sqrt{x-1})} = \ln a$$

where a is a rational constant to be found.

(6)

a) $x = u^2 + 1$ limits
 $\frac{dx}{du} = 2u$ $x = 5 \quad 5 = u^2 + 1$
 $dx = 2u du$ $5 - 1 = u^2$
 $4 = u^2$
 $u = 2$

$x = 10 \quad 10 = u^2 + 1$
 $10 - 1 = u^2$
 $9 = u^2$
 $u = 3$

$$\int_5^{10} \frac{3 dx}{(x-1)(3+2\sqrt{x-1})}$$

\uparrow $u^2 + 1$ \uparrow $u^2 + 1$
 $\leftarrow 2u du$

$$= \int_2^3 \frac{3 \times 2u du}{(u^2 + 1 - 1)(3 + 2\sqrt{u^2 + 1 - 1})}$$

$$= \int_2^3 \frac{6u du}{u^2(3 + 2u)}$$

$$= \int_2^3 \frac{6 du}{u(3 + 2u)} \quad \text{as required}$$



Question 10 continued

$$b) \int_2^3 \frac{6 \, du}{u(3+2u)}$$

Partial fraction

$$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u}$$

$$6 = A(3+2u) + Bu$$

$$u=0, \quad 6 = 3A \Rightarrow A = 2$$

$$u = -\frac{3}{2}, \quad 6 = -\frac{3}{2}B \Rightarrow B = -4$$

$$\int_2^3 \left(\frac{2}{u} + \frac{-4}{3+2u} \right) du$$

$$= \left[2 \ln u - 2 \ln(3+2u) \right]_2^3 du$$

$$= (2 \ln 3 - 2 \ln 9) - (2 \ln 2 - 2 \ln 7)$$

$$= \ln 3^2 - \ln 9^2 - \ln 2^2 + \ln 7^2$$

$$= \ln 9 - \ln 81 - \ln 4 + \ln 49$$

$$= \ln \frac{9 \times 49}{4 \times 81} = \ln \frac{49}{36}$$

$$= \ln a$$

where $a = \frac{49}{36}$



11.

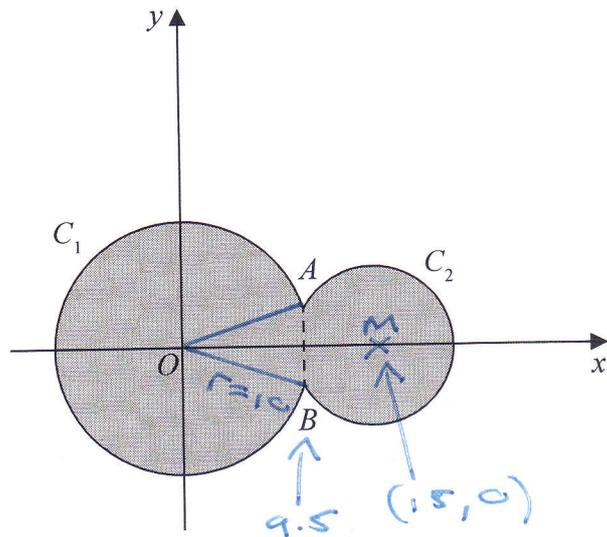


Figure 3

- ① Circle C_1 has equation $x^2 + y^2 = 100$ $r = 10$
- ② Circle C_2 has equation $(x - 15)^2 + y^2 = 40$ $r = \sqrt{40}$
centre $(15, 0)$

The circles meet at points A and B as shown in Figure 3.

- (a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin. (4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

- (b) Find the perimeter of the shaded region, giving your answer to one decimal place. (4)

a) ① and ② gives

$$x^2 + 40 - (x - 15)^2 = 100$$

$$x^2 + 40 - (x^2 - 30x + 225) = 100$$

$$x^2 + 40 - x^2 + 30x - 225 = 100$$

$$30x = 100 + 225 - 40$$

$$30x = 285$$

$$x = 9.5$$

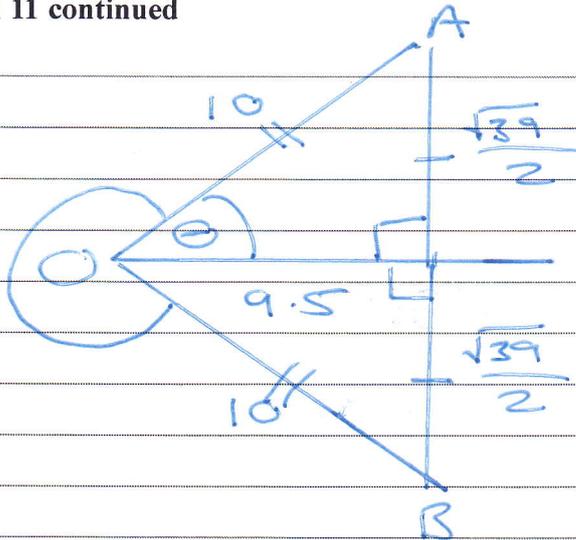
in ① $(9.5)^2 + y^2 = 100$

$$y^2 = 100 - 9.5^2$$

$$y = \frac{+\sqrt{39}}{2}$$



Question 11 continued



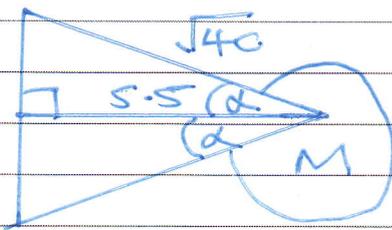
$$\cos \theta = \frac{9.5}{10}$$

$$\theta = 0.31756^\circ$$

$$\begin{aligned} \angle AOB &= 2\theta \\ &= 0.635^\circ \end{aligned} \quad (3 \text{ sf})$$

$$\begin{aligned} \text{Reflex angle at } O &= 2\pi - 0.635120 \\ &= 5.648064449^\circ \end{aligned}$$

$$\begin{aligned} \text{Arc length } C_1 &= 10 \times 5.648064449 \\ &= 56.48064449 \end{aligned} \quad (3)$$



$$2\alpha = 2\cos^{-1}\left(\frac{5.5}{\sqrt{40}}\right)$$

$$= 1.032702637^\circ$$

$$\begin{aligned} \text{reflex angle} &= 2\pi - 1.032702637 \\ &= 5.250482671^\circ \end{aligned}$$

$$\begin{aligned} C_2 \text{ arc length} &= \sqrt{40} \times 5.250482671^\circ \\ &= 33.20696811 \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Perimeter} &= (3) + (4) \\ &= 89.6876126 \\ &= 89.7 \text{ units (1 dp)} \end{aligned}$$



12.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta \quad \theta \neq (180n)^\circ \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ) \quad (5)$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\therefore \frac{\cos \theta \times \cos \theta}{\sin \theta} = \cos \theta \cot \theta$$

$$\operatorname{cosec} \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta$$

$$= \frac{1 - \sin^2 \theta}{\sin \theta}$$

$$= \frac{\cos^2 \theta}{\sin \theta} = \cos \theta \times \frac{\cos \theta}{\sin \theta}$$

$$= \cos \theta \cot \theta \quad (\text{as required})$$

$$\begin{aligned} \text{Identity} \\ \sin^2 \theta + \cos^2 \theta &= 1 \\ 1 - \sin^2 \theta &= \cos^2 \theta \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \operatorname{cosec} x - \sin x &= \cos x \cot(3x - 50^\circ) \\ \therefore \frac{1}{\sin x} - \sin x &= \cos x \cot(3x - 50^\circ) \\ \therefore \frac{1 - \sin^2 x}{\sin x} &= \cos x \cot(3x - 50^\circ) \end{aligned}$$

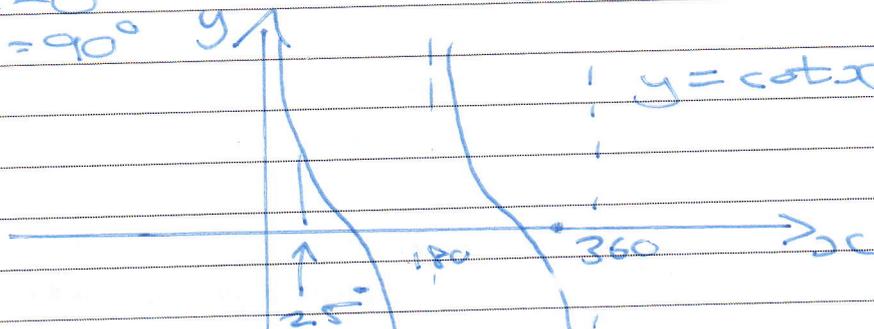
$$2x = 50$$

$$x = 25$$

$$0 < x < 180^\circ$$

$$\infty x = 0$$

$$\Rightarrow x = 90^\circ$$



Question 12 continued

$$\cot x = \cot (3x - 50)$$

next solution has a period of 180°

$$x + 180 = 3x - 50$$

$$230 = 2x$$

$$x = 115^\circ \text{ in}$$

$$0 < x < 180$$

try $x + 360 = 3x - 50$

$$410 = 2x$$

$$x = 205^\circ \text{ outside}$$

$$0 < x < 180$$

also when we divided both sides initially by $\cos x$,

\therefore another solution when $\cos x = 0$

$$\therefore x = 90^\circ$$

Solutions are $x = 25^\circ, 90^\circ, 115^\circ$



13. A sequence of numbers a_1, a_2, a_3, \dots is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0 \quad (3)$$

(b) For this sequence explain why $k \neq 1$ (1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r \quad (3)$$

$$a) \quad a_1 = 2$$

$$a_2 = \frac{k(2+2)}{2} = 2k$$

$$a_3 = \frac{k(2k+2)}{2k} = k+1$$

$$a_4 = \frac{k(k+1+2)}{k+1} = \frac{k(k+3)}{k+1}$$

If periodic sequence of order 3

$$a_1 = a_4$$

$$2 = \frac{k(k+3)}{k+1}$$



Question 13 continued

$$2(k+1) = k^2 + 3k$$

$$2k + 2 = k^2 + 3k$$

$$0 = k^2 + k - 2$$

$$0 = (k+2)(k-1)$$

Either $k = -2$ or $k = 1$

b) If $k = 1$

$$a_1 = 2$$

$$a_2 = 2$$

$$a_3 = 2$$

$$a_4 = \frac{1 \times 4}{2} = 2$$

All terms the same, so the sequence would not have period 3

c) $\sum_{r=1}^{\infty} a_r$

$$26 \times 3 = 78$$

If $k = -2$

lots of

$$\left. \begin{array}{l} a_1 = 2 \quad a_2 = -4 \quad a_3 = -1 \\ a_4 = 2 \quad a_5 = -4 \quad a_6 = -1 \\ \dots \quad \dots \quad \dots \\ a_{76} = 2 \quad a_{77} = -4 \quad a_{78} = -1 \\ a_{79} = 2 \quad a_{80} = -4 \end{array} \right\}$$

$$\sum_{r=1}^{80} a_r = 26 \times (2 - 4 - 1) + 2 - 4 = -80$$



Variables
 V r t

14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm³

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t .

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

a) $\frac{dV}{dt} = -c$

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} \quad \textcircled{1}$$

$$V = \frac{4}{3} \pi r^3 \quad (\text{volume of sphere})$$

$$\frac{dV}{dr} = 4\pi r^2 \quad \therefore \frac{dr}{dV} = \frac{1}{4\pi r^2}$$

in $\textcircled{1}$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2} \times -c = -\frac{c}{4\pi} \times \frac{1}{r^2}$$

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

where k replaces
constant $\frac{c}{4\pi}$

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Question 14 continued

$$b) \quad \begin{aligned} t=0 \quad r &= 40 \\ t=5 \quad r &= 20 \end{aligned}$$

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

$$\int r^2 dr = \int -k dt$$

$$\frac{r^3}{3} = -kt + c$$

$$t=0, r=40$$

$$\frac{64000}{3} = c$$

$$\frac{8000}{3} = -5k + \frac{64000}{3} \quad t=5, r=20$$

$$5k = \frac{56000}{3}$$

$$k = \frac{11200}{3}$$

$$\therefore \frac{r^3}{3} = -\frac{11200}{3}t + \frac{64000}{3}$$

$$r^3 = -11200t + 64000$$

$$c) \quad \text{valid when } 64000 - 11200t \geq 0$$

$$11200t \leq 64000$$

$$t \leq \frac{40}{7} \text{ seconds}$$



15. The curve C has equation

$$x^2 \tan y = 9 \quad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81} \quad (4)$$

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$ (3)

a) $x^2 \tan y = 9$ (1)

implicitly differentiating

$$x^2 \times \sec^2 y \frac{dy}{dx} + \tan y \times 2x = 0$$

$$x^2 \sec^2 y \frac{dy}{dx} = -2x \tan y$$

$$\frac{dy}{dx} = \frac{-2x \tan y}{x^2 \sec^2 y}$$

rearranging (1) gives $\tan y = \frac{9}{x^2}$ (3)

Identity $\sec^2 y = 1 + \tan^2 y$ (4)
 $= 1 + \frac{81}{x^4}$

Substituting (3) and (4) in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-2x \times \frac{9}{x^2}}{x^2 \left(1 + \frac{81}{x^4}\right)}$$

$$\frac{dy}{dx} = \frac{-18}{x^3 \left(\frac{x^4 + 81}{x^4}\right)}$$

$$\frac{dy}{dx} = \frac{-18}{x^4 + 81} = \frac{-18x}{x^4 + 81}$$

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Question 15 continued

$$b) \quad \begin{array}{l} u = -18x \\ u' = -18 \end{array} \quad \begin{array}{l} v = x^4 + 81 \\ v' = 4x^3 \end{array}$$

$$\frac{d^2y}{dx^2} = \frac{-18(x^4 + 81) + 72x^4}{(x^4 + 81)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-18x^4 - 1458 + 72x^4}{(x^4 + 81)^2}$$

$$= \frac{54x^4 - 1458}{(x^4 + 81)^2}$$

$$= \frac{54(x^4 - 27)}{(x^4 + 81)^2}$$

We have a point of inflection
if $\frac{d^2y}{dx^2} = 0$

$$\therefore x^4 - 27 = 0$$

$$x^4 = 27$$

$$x = \sqrt[4]{27} \quad \text{as required}$$



16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

Factorising, then assume that there are positive integers p and q such that $(2p+q)(2p-q) = 25$

Factors of 25 are 1 and 25
5 and 5

$$\therefore 2p+q = 25 \quad \textcircled{1} \quad \text{First pair}$$

$$2p-q = 1 \quad \textcircled{2}$$

$\textcircled{1} + \textcircled{2}$

$$4p = 26$$

$$p = 6.5 \quad \text{in } \textcircled{1} \text{ gives } q = 12$$

or

$$2p+q = 1 \quad \textcircled{1}$$

$$2p-q = 25 \quad \textcircled{2}$$

$\textcircled{1} + \textcircled{2}$

$$4p = 26$$

$$p = 6.5, \quad q = 12$$

or

$$2p+q = 5 \quad \textcircled{1}$$

$$2p-q = 5 \quad \textcircled{2}$$

$\textcircled{1} + \textcircled{2}$

$$4p = 10$$

$$p = 2.5, \quad q = 0$$

This is a contradiction as there are no integer solutions for any of the combinations

Hence there are no positive integers p and q such that $4p^2 - q^2 = 25$

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