OCR
Oxford Cambridge and RSA

Friday 14 June 2019 – Afternoon

A Level Mathematics A

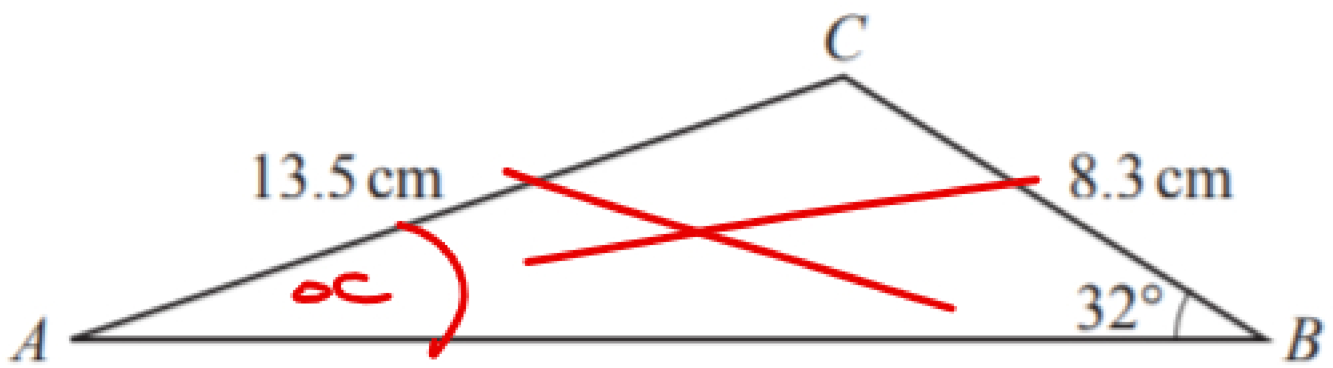
H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

1



The diagram shows triangle ABC, with $AC = 13.5\text{cm}$, $BC = 8.3\text{cm}$ and angle $ABC = 32°$.

Find angle CAB. **[2]**

$$\frac{\sin x}{8.3} = \frac{\sin(32)}{13.5}$$

$$x = \sin^{-1}\left(\frac{8.3\sin(32)}{13.5}\right)$$

$$\angle CAB = 19.0° \ (3sf)$$

2 A circle with centre C has equation $x^2 + y^2 - 6x + 4y + 4 = 0$.

(a) Find

(i) the coordinates of C, **[2]**

(ii) the radius of the circle. **[1]**

(b) Determine the set of values of k for which the line $y = kx - 3$ does not intersect or touch the circle. **[5]**

a) $x^2 - 6x + y^2 + 4y + 4 = 0$

$(x-3)^2 - 3^2 + (y+2)^2 - 2^2 + 4 = 0$

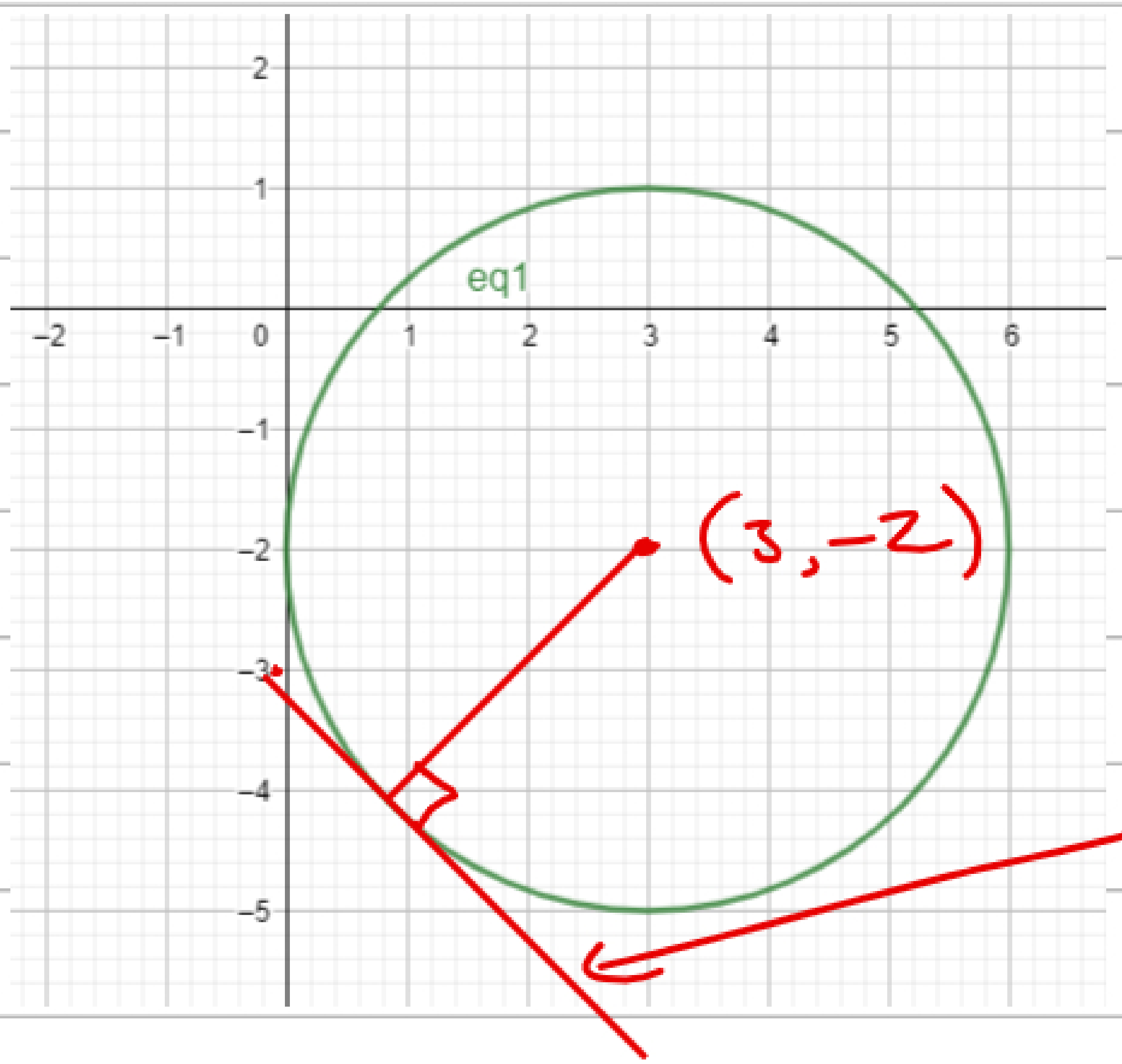
$(x-3)^2 + (y+2)^2 - 9 - 4 + 4 = 0$

$(x-3)^2 + (y+2)^2 = 9$

(i) Centre $(3, -2)$

(ii) radius $= 3$

(b) Determine the set of values of k for which the line $y = kx - 3$ does not intersect or touch the circle. **[5]**



find equation of this tangent $y = kx - 3$

$$(x-3)^2+(y+2)^2=9$$

let $y = kx - 3$

$$(x-3)^2+(kx-3+2)^2=9$$

$$x^2-6x+9+(kx-1)^2=9$$

$$x^2-6x+9+k^2x^2-2kx+1-9=0$$

$$x^2(1+k^2)+x(-6-2k)+1=0$$

$$b^2-4ac=0$$

$$(-6-2k)^2-4(1+k^2)=0$$

$$36+24k+\cancel{4k^2}-4-\cancel{4k^2}=0$$

$$24k=-32$$

$$k=-\frac{4}{3} \quad \therefore \quad k<-\frac{4}{3}$$

3 (a) In this question you must show detailed reasoning.

Solve the inequality $|x-2| \leqslant |2x-6|$. **[4]**

a) $(x-2)^2 \le (2x-6)^2$

$x^2 - 4x + 4 \le 4x^2 - 24x + 36$

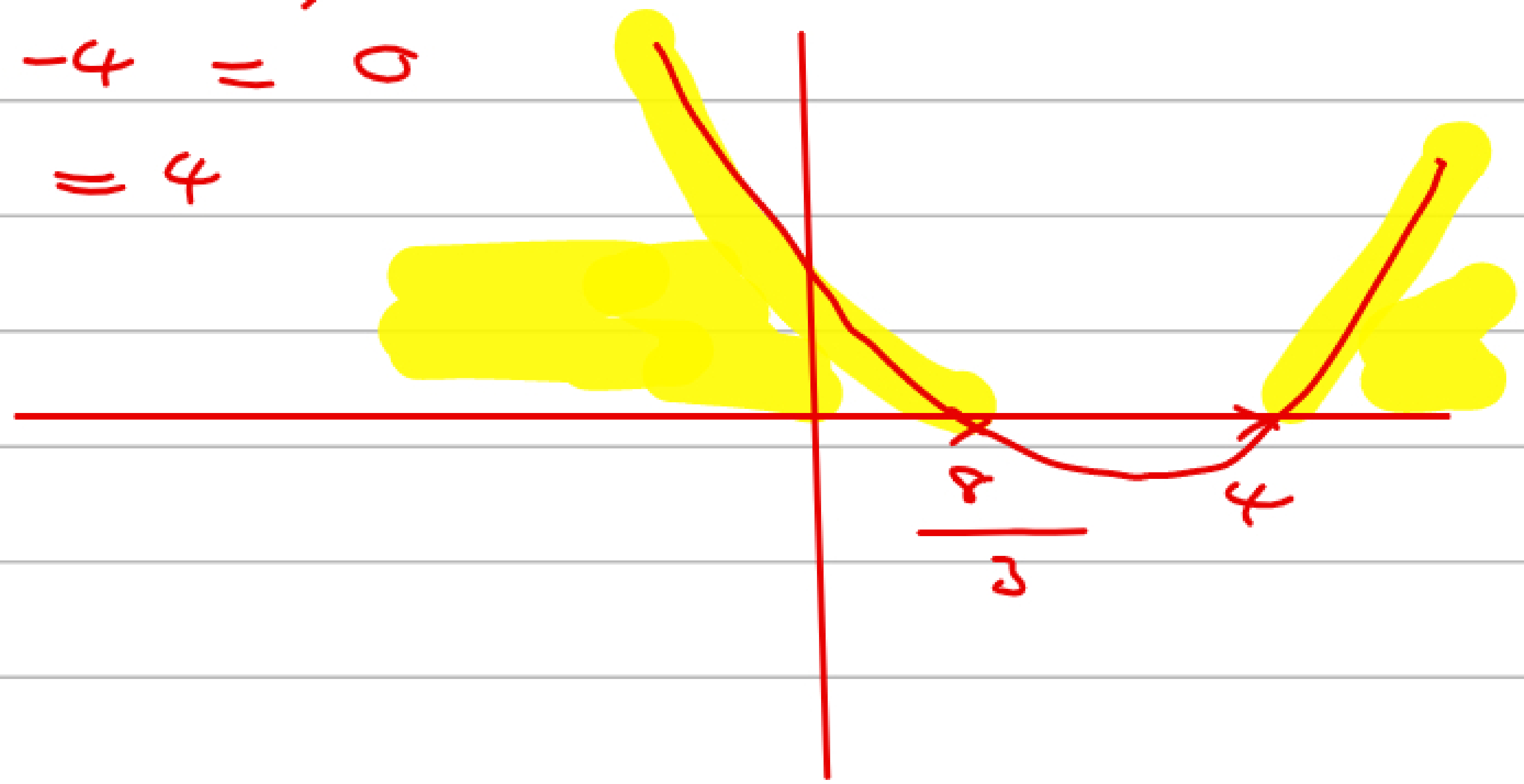
$0 \le 3x^2 - 20x + 32$

$0 \le (3x-8)(x-4)$

critical points $3x - 8 = 0$ or $x - 4 = 0$

$x = \frac{8}{3}$ $\quad$ $x = 4$

$x \le \frac{8}{3}$ or $x \ge 4$



(b) Give full details of a sequence of two transformations needed to transform the graph of $y = |x-2|$ to the graph of $y = |2x-6|$. **[3]**

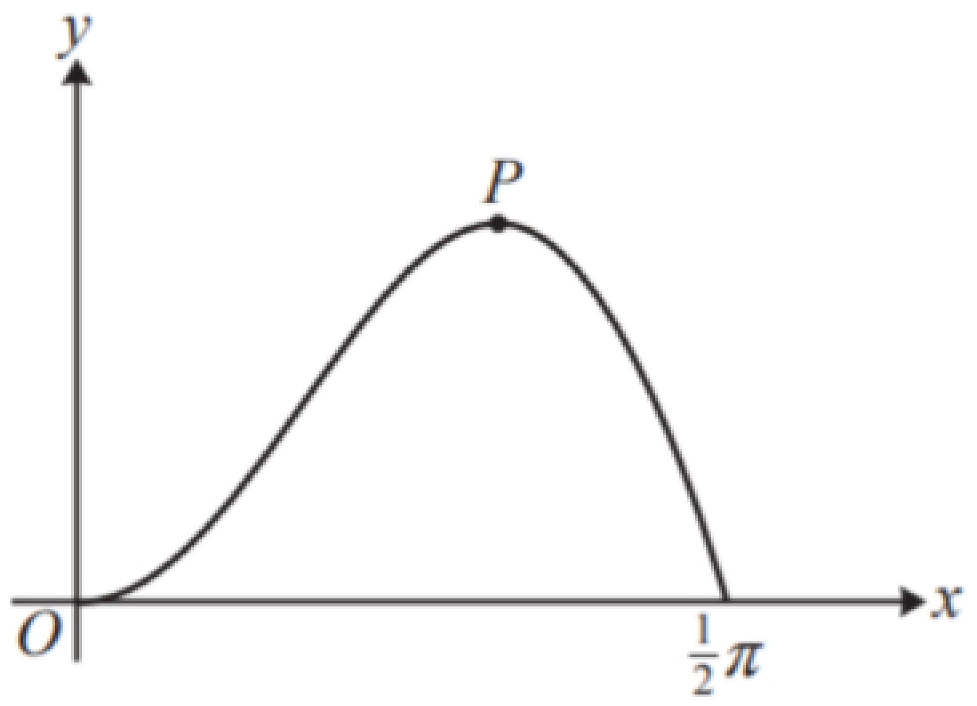
1st for $y = |x-2|$ to $y = |x-6|$

translation $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$, 4 units in +ve x direction

2nd for $y = |x-6|$ to $y = |2x-6|$

Stretch by scale factor $\frac{1}{2}$ in x direction

4



The diagram shows the part of the curve $y = 3x\sin 2x$ for which $0 \leqslant x \leqslant \frac{1}{2}\pi$.

The maximum point on the curve is denoted by P.

(a) Show that the x-coordinate of P satisfies the equation $\tan 2x + 2x = 0$. **[3]**

(b) Use the Newton-Raphson method, with a suitable initial value, to find the x-coordinate of P, giving your answer correct to 4 decimal places. Show the result of each iteration. **[4]**

a) $y = 3x \quad \sin 2x$

$u = 3x \qquad v = \sin 2x$

$u' = 3 \qquad v' = 2\cos 2x$

$\frac{dy}{dx} = 6x\cos 2x + 3\sin 2x$

at P, $\frac{dy}{dx} = 0$

$\therefore 6x\cos 2x + 3\sin 2x = 0$

$\div$ thru by $3\cos 2x$

$\frac{6x\cos 2x}{3\cos 2x} + \frac{3\sin 2x}{3\cos 2x} = 0$

$2x + \tan 2x = 0$

as required

(b) Use the Newton-Raphson method, with a suitable initial value, to find the x-coordinate of P, giving your answer correct to 4 decimal places. Show the result of each iteration. **[4]**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = 2x + \tan(2x)$$

$$f'(x) = 2 + 2\sec^2(2x)$$

On calculator

let $x_0 = 1$ (use ANS)

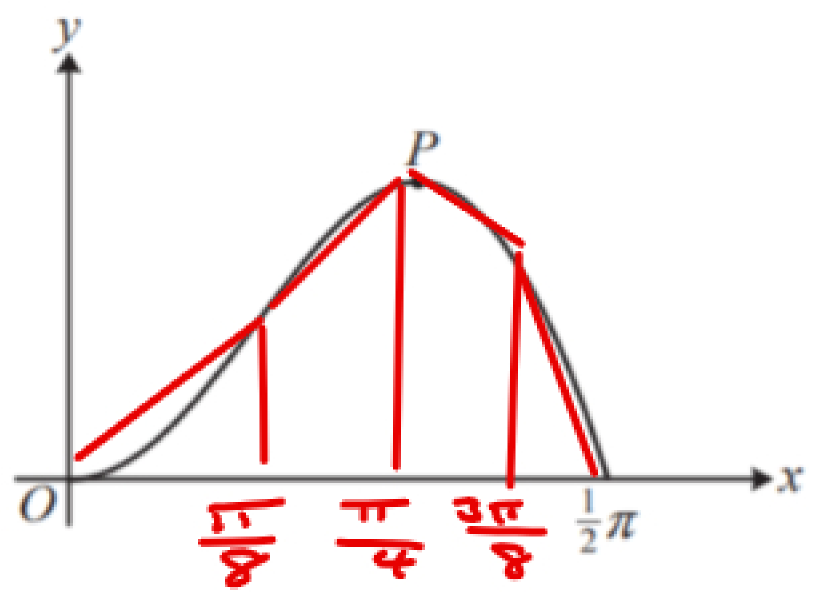
(calculator in radians)

$$x_1 = ANS - \frac{2 \times ANS + \tan(2 \times ANS)}{2 + \dfrac{2}{(\cos(2 \times ANS))^2}}$$

$x_1 = 1.01365729$

$x_2 = 1.01437149$

$x = 1.0144$ (4dp)



(c) The trapezium rule, with four strips of equal width, is used to find an approximation to $\int_0^{\frac{1}{2}\pi} 3x \sin 2x \, dx$.

Show that the result can be expressed as $k\pi^2(\sqrt{2}+1)$, where k is a rational number to be determined. **[4]**

$x = 0 \qquad 3 \times 0 \times \sin(0) = 0$

$x = \frac{\pi}{8} \qquad 3 \times \frac{\pi}{8} \times \sin\left(\frac{2\pi}{8}\right) = \frac{3}{8}\pi \times \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}\,\pi}{16}$

$x = \frac{\pi}{4} \qquad 3 \times \frac{\pi}{4} \times \sin\left(\frac{\pi}{2}\right) = \frac{3}{4}\pi \times 1 = \frac{3}{4}\pi$

$x = \frac{3\pi}{8} \qquad 3 \times \frac{3\pi}{8} \times \sin\left(2 \times \frac{3\pi}{8}\right) = \frac{9\pi}{8} \times \frac{\sqrt{2}}{2} = \frac{9\sqrt{2}\,\pi}{16}$

$x = \frac{\pi}{2} \qquad 3 \times \frac{\pi}{2} \times \sin\left(2 \times \frac{\pi}{2}\right) = 0$

$\text{Area} = \frac{1}{2} \times \frac{\pi}{8}\left(0 + 0 + 2\left(\frac{3\sqrt{2}\pi}{16} + \frac{3\pi}{4} + \frac{9\sqrt{2}\pi}{16}\right)\right)$

$= \frac{\pi}{16}\left(2\left(\frac{12\sqrt{2}\pi}{16} + \frac{3\pi}{4}\right)\right)$

$= \frac{\pi}{16}\left(\frac{3}{2}\sqrt{2}\pi + \frac{3}{2}\pi\right) = \frac{3}{2} \times \pi \times \frac{\pi}{16}(\sqrt{2}+1)$

$= \frac{3}{32}\pi^2(\sqrt{2}+1)$

where $k = \frac{3}{32}$

(d) **(i)** Evaluate $\int_0^{\frac{1}{2}\pi} 3x\sin 2x\,dx$. **[1]**

(ii) Hence determine whether using the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region enclosed by the curve $y = 3x\sin 2x$ and the x-axis for $0 \leqslant x \leqslant \frac{1}{2}\pi$. **[1]**

(iii) Explain briefly why it is not easy to tell from the diagram alone whether the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region in this case. **[1]**

(i) using integration function on calculator

$= \frac{3}{4}\pi$

(ii) Exact $= \frac{3}{4}\pi = 2.35619449$

Trapezium rule $= \frac{3}{32}\pi^2(\sqrt{2}+1) = 2.2338125$

which is < 2.35619449 $\therefore$ under estimate

(iii) Left hand trapezium would be over the curve, but other trapezia below the curve, so overall approximation is not clear.

5 In this question you must show detailed reasoning.

(a) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **[4]**

$$(\cot\theta + \operatorname{cosec}\theta)^2$$

$$= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\cos\theta + 1}{\sin\theta}\right)^2$$

$$= \frac{(\cos\theta+1)(\cos\theta+1)}{\sin^2\theta}$$

$$= \frac{(\cos\theta+1)(\cos\theta+1)}{1-\cos^2\theta}$$

$$= \frac{\cancel{(\cos\theta+1)}(\cos\theta+1)}{\cancel{(1+\cos\theta)}(1-\cos\theta)}$$

$$= \frac{1+\cos\theta}{1-\cos\theta}$$

as required

(a) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **[4]**

(b) Hence solve, for $0 < \theta < 2\pi$, $3(\cot\theta + \operatorname{cosec}\theta)^2 = 2\sec\theta$. **[5]**

$$3(\cot\theta + \operatorname{cosec}\theta)^2 = 2\sec\theta$$

$$3\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = \frac{2}{\cos\theta}$$

$$3\cos\theta(1+\cos\theta) = 2(1-\cos\theta)$$

$$3\cos\theta + 3\cos^2\theta = 2 - 2\cos\theta$$

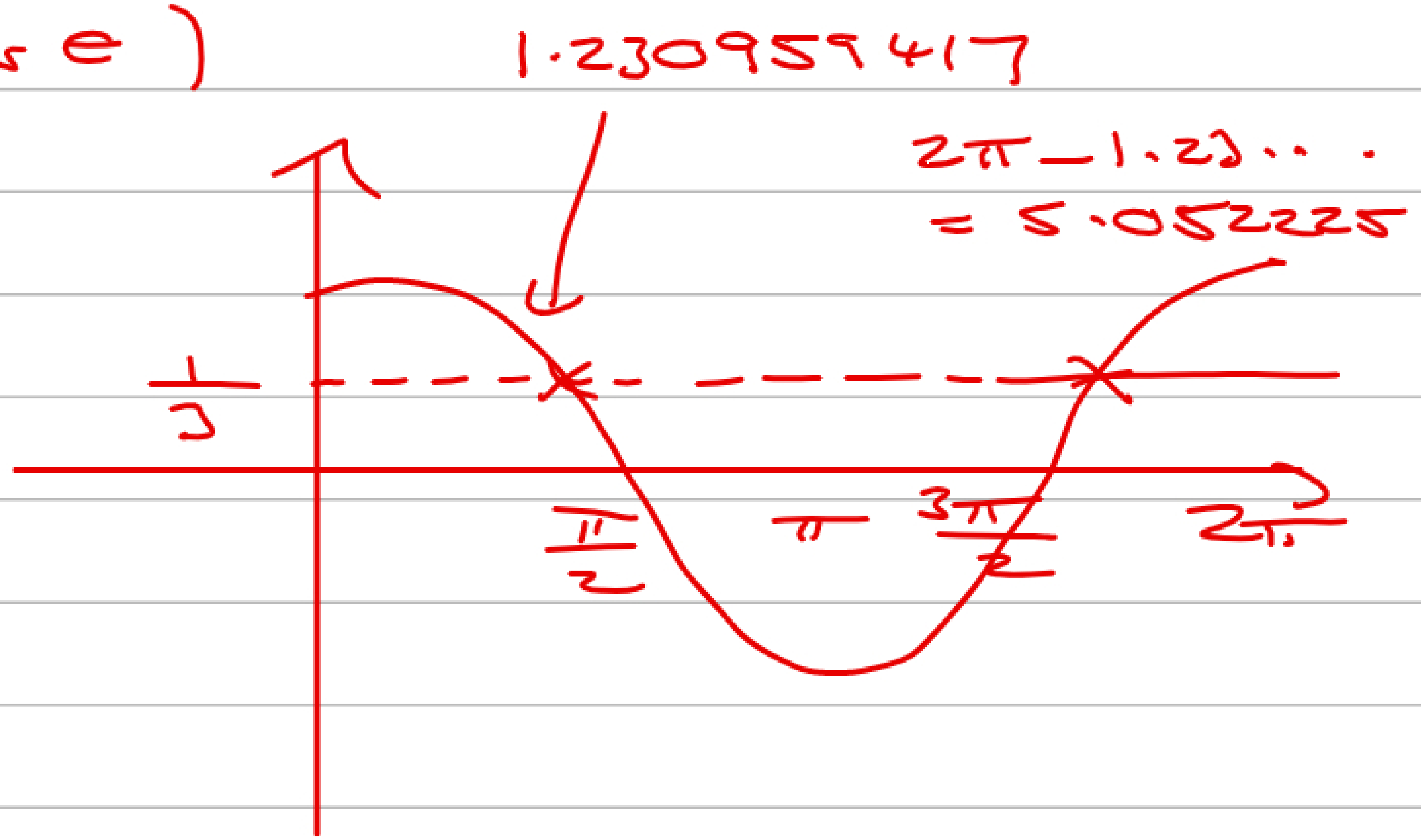
$$3\cos^2\theta + 5\cos\theta - 2 = 0$$

$$(3\cos\theta - 1)(\cos\theta + 2) = 0$$

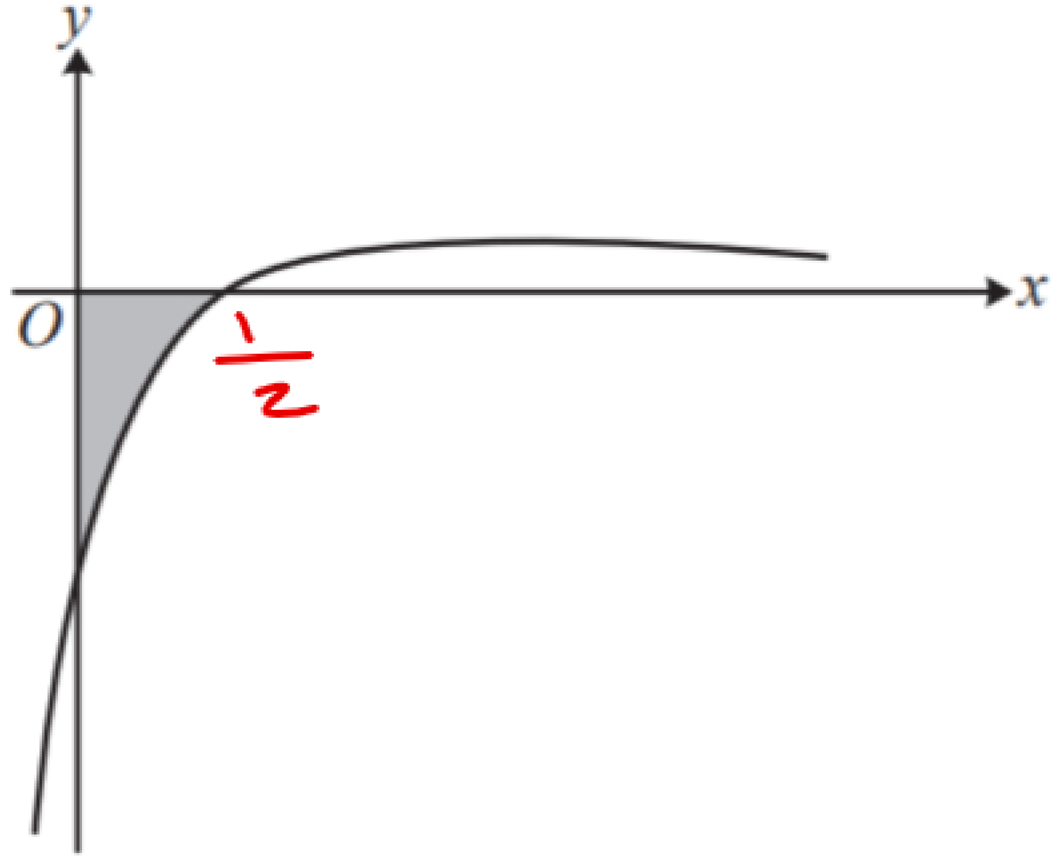
$\cos\theta = \frac{1}{3}$ $\quad\quad$ $\cos\theta = -2$

impossible

$\theta = 1.23, 5.05$ (3sf)



6



curve meets x-axis when $y = 0$

$\therefore\ 2x - 1 = 0$

$x = \frac{1}{2}$

The diagram shows part of the curve $y = \dfrac{2x-1}{(2x+3)(x+1)^2}$.

Find the exact area of the shaded region, giving your answer in the form $p + q\ln r$, where p and q are positive integers and r is a positive rational number. **[10]**

Partial fractions

$$\frac{2x-1}{(2x+3)(x+1)^2} = \frac{A}{2x+3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$2x - 1 = A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$$

$$2x-1 = A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$$

$x = -1 \qquad -3 = C(-2+3)$

$$C = -3$$

$x = -\frac{3}{2} \qquad \cancel{2} \times -\frac{3}{\cancel{2}} - 1 = A\left(-\frac{3}{2}+1\right)^2$

$$-4 = \frac{1}{4}A$$

$$A = -16$$

Equate coefficients of x^2

$$0 = A + 2B$$

$$0 = -16 + 2B$$

$$B = 8$$

$$\int_0^{\frac{1}{2}} \frac{-16}{2x+3} + \frac{8}{x+1} - 3(x+1)^{-2} \, dx$$

$$= \left[-16 \times \frac{1}{2} \times \ln(2x+3) + 8\ln(x+1) + 3(x+1)^{-1} \right]_0^{\frac{1}{2}}$$

$$= \left[-8\ln(2x+3) + 8\ln(x+1) + \frac{3}{x+1} \right]_0^{\frac{1}{2}}$$

$$x = 0$$

$$= \left(-8\ln 4 + 8\ln\left(\frac{3}{2}\right) + 2 \right) - \left(-8\ln 3 + 8\ln 1 + 3 \right)$$

$$= -8\ln 4 + 8\ln\left(\frac{3}{2}\right) + 8\ln 3 + 2 - 3$$

$$= 8\ln\left(\frac{\frac{3}{2} \times 3}{4}\right) - 1$$

$$= 8\ln\left(\frac{9}{8}\right) - 1$$ (this is -ve value as area below curve)

$$\therefore \text{Area} = -\left(8\ln\left(\frac{9}{8}\right) - 1\right) = 1 - 8\ln\frac{9}{8}$$

Section B: Mechanics

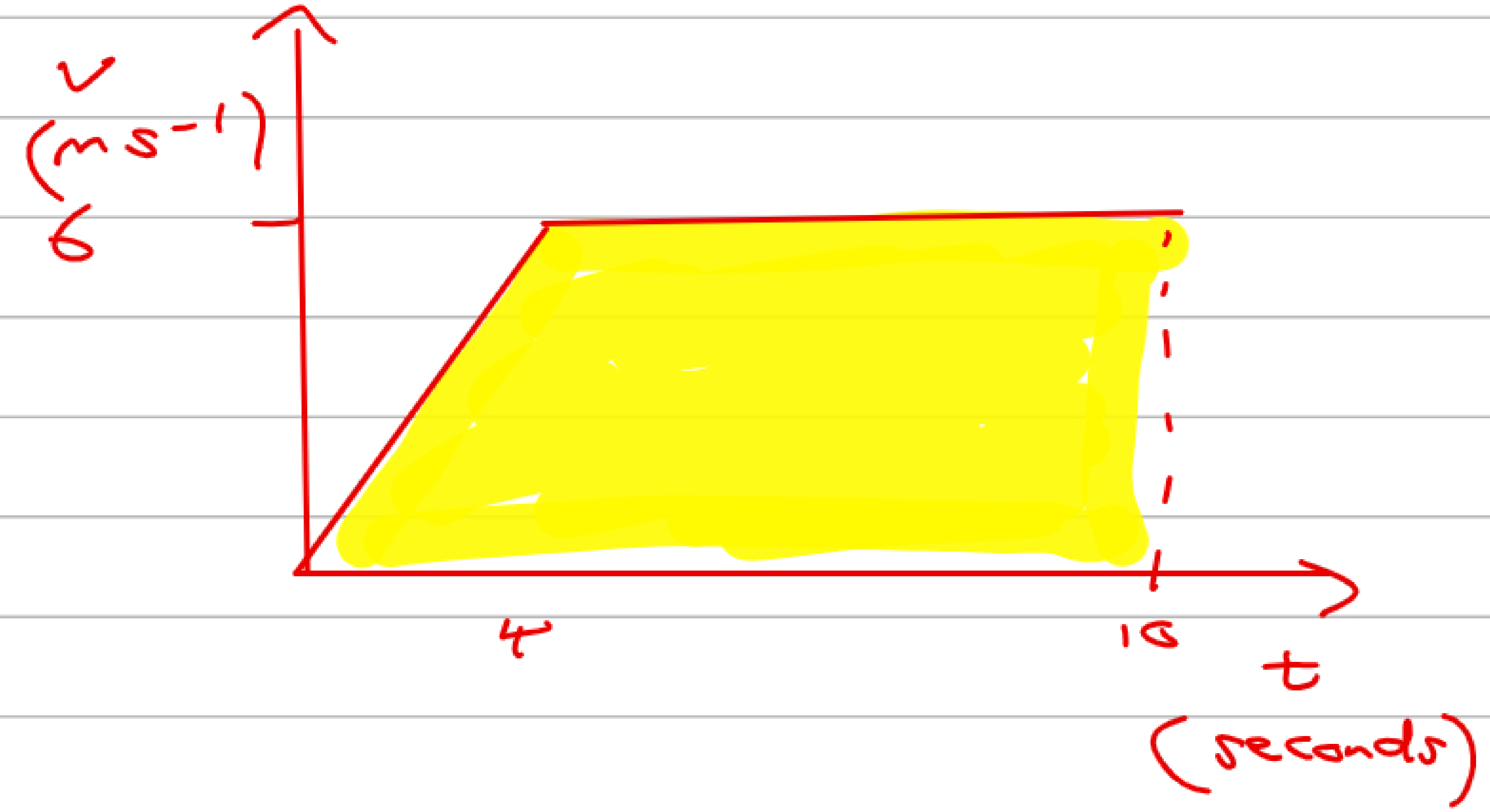
Answer **all** the questions.

7 A cyclist starting from rest accelerates uniformly at $1.5\,\mathrm{m\,s^{-2}}$ for 4 s and then travels at constant speed.

(a) Sketch a velocity-time graph to represent the first 10 seconds of the cyclist's motion. **[2]**

(b) Calculate the distance travelled by the cyclist in the first 10 seconds. **[2]**

a)



b)

s
$u = 0$
$v = ?$
$a = 1.5$
$t = 4$

$v = u + at$
$v = 1.5 \times 4 = 6\,\mathrm{ms^{-1}}$

Distance travelled is area of shaded trapezium

$= \frac{1}{2} \times (6 + 10) \times 6 = 48\,\mathrm{m}$

8 A particle P projected from a point O on horizontal ground hits the ground after 2.4 seconds.

The horizontal component of the initial velocity of P is $\frac{5}{3}d\ \mathrm{m\,s^{-1}}$.

(a) Find, in terms of d, the horizontal distance of P from O when it hits the ground. **[1]**

(b) Find the vertical component of the initial velocity of P. **[2]**

P just clears a vertical wall which is situated at a horizontal distance d m from O.

(c) Find the height of the wall. **[3]**

The speed of P as it passes over the wall is $16\ \mathrm{m\,s^{-1}}$.

(d) Find the value of d correct to 3 significant figures. **[4]**

($\longrightarrow$) motion

speed $= \frac{5}{3}d\ \mathrm{m\,s^{-1}}$

a) distance $= \frac{5}{3}d \times 2.4$

$= 4d$ m

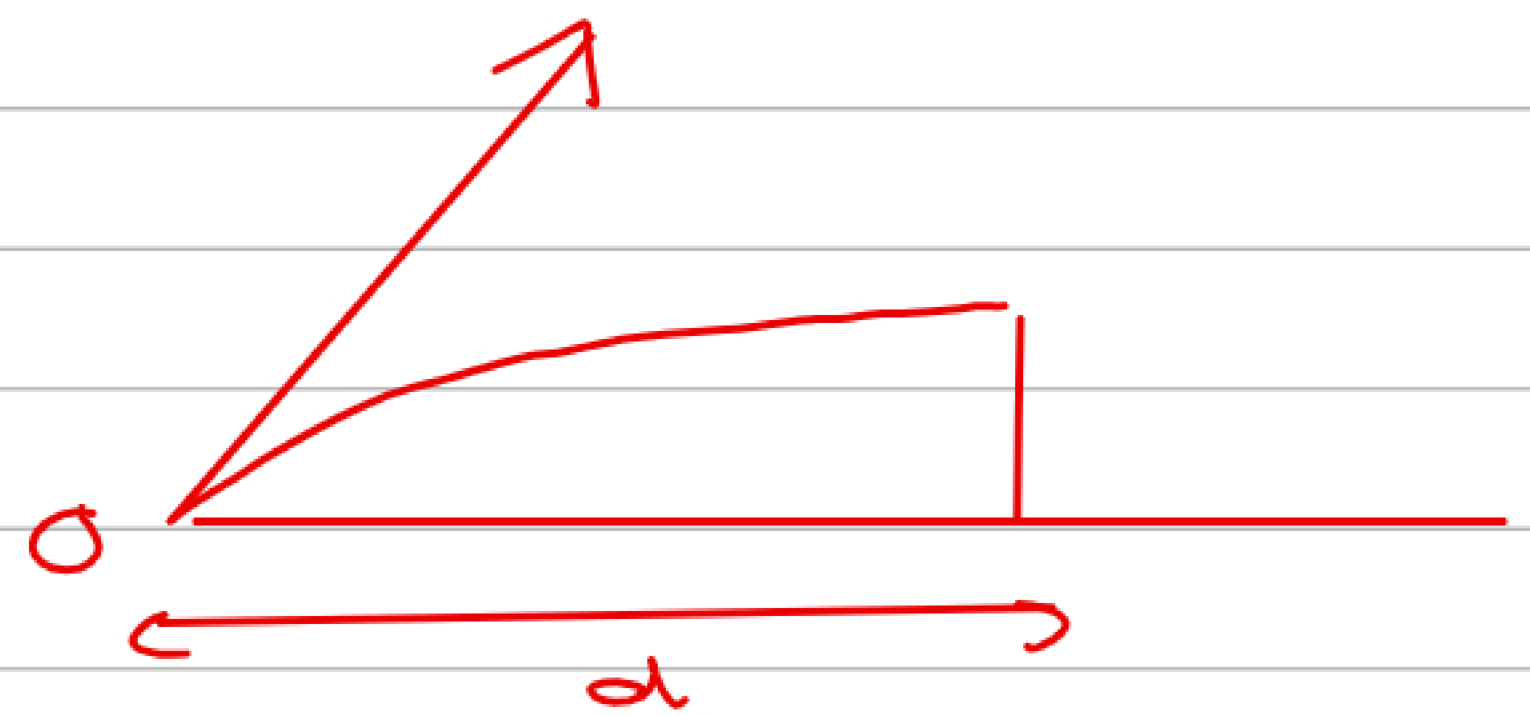
b) $s = 0$

$u = ?$

$v = ?$

$a = -9.8$

$t = 2.4$



$s = ut + \frac{1}{2}at^2$

$0 = 2.4u - \frac{1}{2} \times 9.8 \times 2.4^2$

$0 = 2.4u - 28.224$

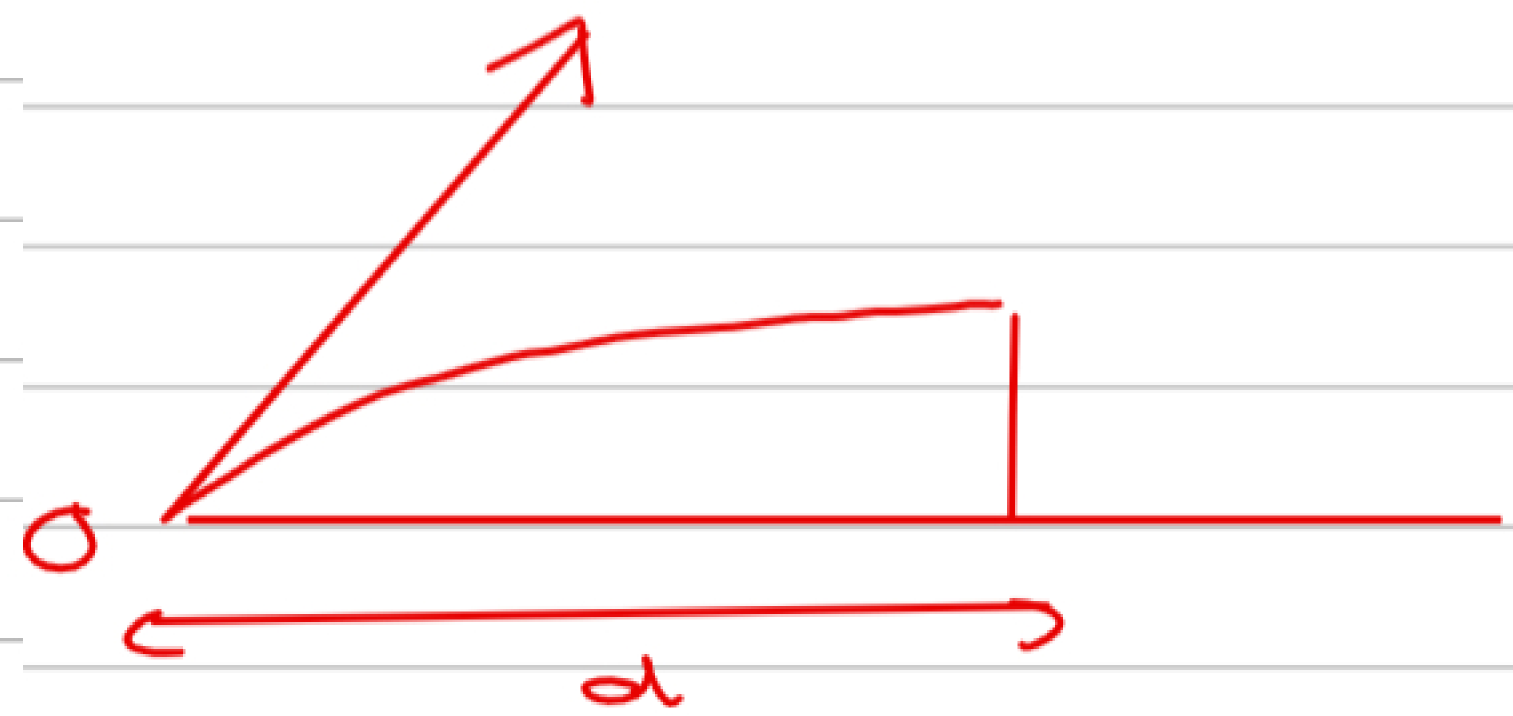
$u = \frac{28.224}{2.4} = 11.76\ \mathrm{ms^{-1}}$

P just clears a vertical wall which is situated at a horizontal distance d m from O.

(c) Find the height of the wall. **[3]**

The speed of P as it passes over the wall is $16\,\text{m s}^{-1}$.

(d) Find the value of d correct to 3 significant figures. **[4]**



c) (↑) motion

$s = ?$

$u = 11.76$

$v = ?$

$a = -9.8$

$t = \frac{3}{5}$

(→) motion

$d = \frac{5}{3} d \times t$

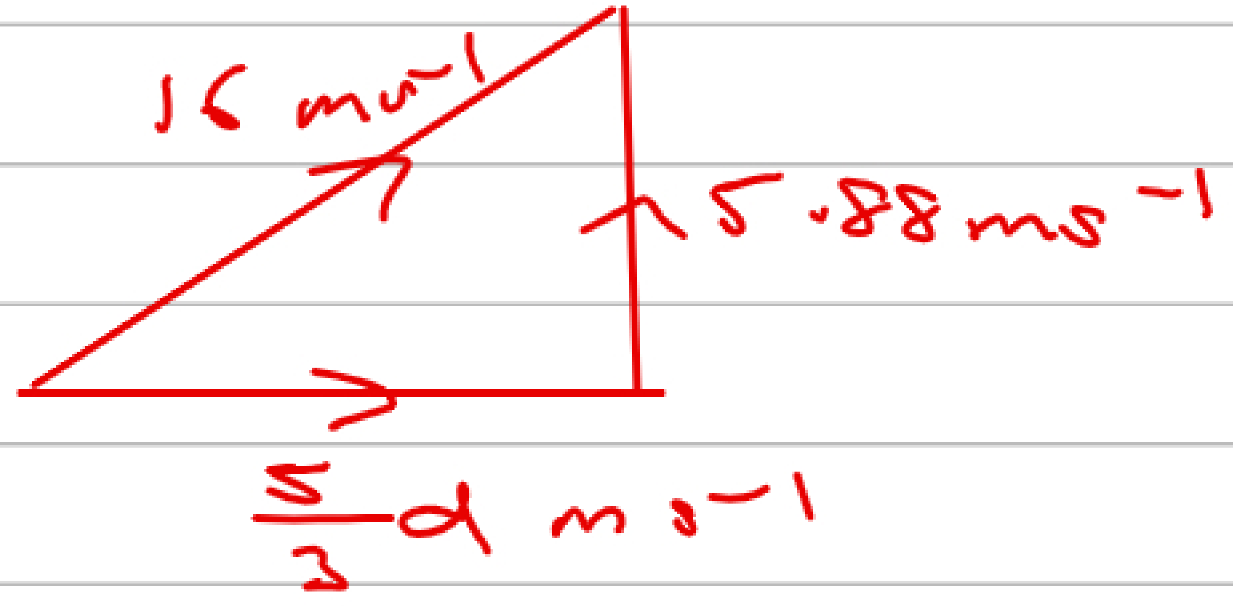
$\therefore t = \frac{3}{5}$

$s = ut + \frac{1}{2} a t^2$

$s = 11.76 \times \frac{3}{5} - \frac{1}{2} \times -9.8 \times \left(\frac{3}{5}\right)^2$

$s = 5.292\text{ m}$

d)



at $t = \frac{3}{5}$

$v = u + at$

$v = 11.76 + -9.8 \times \frac{3}{5}$

$v = 5.88\text{ ms}^{-1}$

$$\therefore \quad 16^2 = 5.88^2 + \left(\frac{5}{3} d\right)^2$$

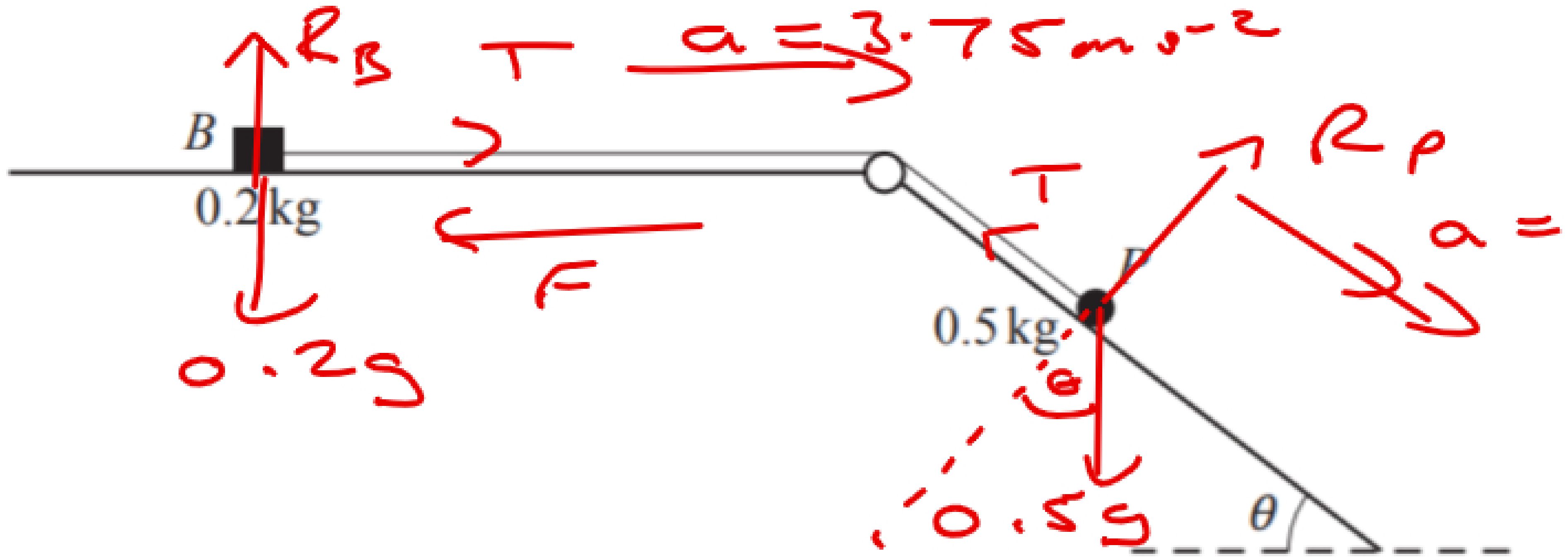
$$256 = 34.5744 + \frac{25}{9} d^2$$

$$d = \sqrt{\frac{256 - 34.5744}{\frac{25}{9}}}$$

$$d = 8.928225$$

$$d = 8.93\ \text{m} \quad (3\text{sf})$$

9



The diagram shows a small block B, of mass 0.2 kg, and a particle P, of mass 0.5 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane.

The block can move on the horizontal surface, which is rough. The particle can move on the inclined plane, which is smooth and which makes an angle of θ with the horizontal where $\tan\theta = \frac{3}{4}$.

The system is released from rest. In the first 0.4 seconds of the motion P moves 0.3 m down the plane and B does not reach the pulley.

(a) Find the tension in the string during the first 0.4 seconds of the motion. **[4]**

(b) Calculate the coefficient of friction between B and the horizontal surface. **[5]**

$s = 0.3$
$u = 0$
$v = ?$
$a =$
$t = 0.4$

$s = ut + \frac{1}{2}at^2$

$0.3 = \frac{1}{2} \times a \times 0.4^2$

$a = \frac{0.3 \times 2}{0.4^2} = 3.75\ ms^{-2}$

Equation of motion for P ($\rightarrow$)

$0.5 \times 3.75 = 0.5g\sin\theta - T$

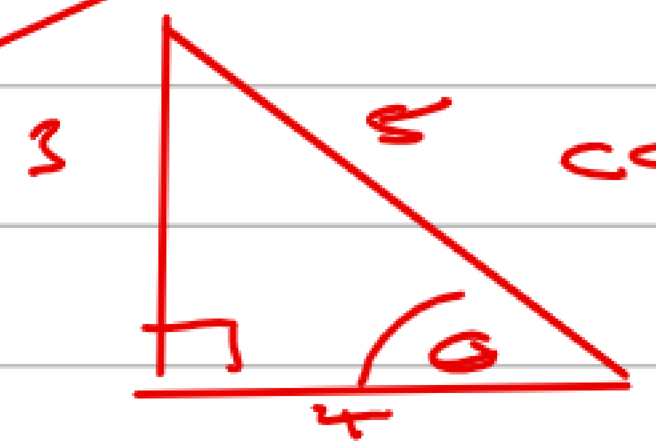
$0.5 \times 3.75 \quad 0.5 \times 9.8 \times \frac{3}{5} - T$

$T = 0.5 \times 9.8 \times \frac{3}{5} - 0.5 \times 3.75$

$T = 1.065\ N$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$



Equation of motion for B

$$0.2 \times 3.75 = T - F$$

$$F = T - 0.2 \times 3.75$$

$$F = 1.065 - 0.2 \times 3.75$$

$$F = 0.315 \text{ N}$$

Limiting friction

$$F = \mu R_B \quad (1)$$

R (↑) for B only

$$R_B = 0.2g \quad (2)$$

in (1) $\quad 0.315 = \mu \times 0.2g$

$$\mu = \frac{0.315}{0.2 \times 9.8} \Rightarrow \mu = \frac{9}{56} = 0.161$$

(3 sf)

10 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle R of mass 2 kg is moving on a smooth horizontal surface under the action of a single horizontal force **F** N. At time t seconds, the velocity $\mathbf{v}$ m s^{-1} of R, relative to a fixed origin O, is given by $\mathbf{v} = (pt^2 - 3t)\mathbf{i} + (8t + q)\mathbf{j}$, where p and q are constants and $p < 0$.

(a) Given that when $t = 0.5$ the magnitude of **F** is 20, find the value of p. **[6]**

When $t = 0$, R is at the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m.

(b) Find, in terms of q, an expression for the displacement vector of R at time t. **[4]**

When $t = 1$, R is at a point on the line L, where L passes through O and the point with position vector $2\mathbf{i} - 8\mathbf{j}$.

(c) Find the value of q. **[3]**

a) $\underset{\sim}{F} = m \times \underset{\sim}{a}$

$\underset{\sim}{v} = \begin{pmatrix} pt^2 - 3t \\ 8t + q \end{pmatrix}$

differentiating

$\underset{\sim}{a} = \begin{pmatrix} 2pt - 3 \\ 8 \end{pmatrix}$

$m \times \underset{\sim}{a}$

$\underset{\sim}{F} = 2 \begin{pmatrix} 2pt - 3 \\ 8 \end{pmatrix}$

$\underset{\sim}{F} = \begin{pmatrix} 4pt - 6 \\ 16 \end{pmatrix}$

$\therefore \sqrt{(4pt - 6)^2 + (16)^2} = 20$

$\therefore \sqrt{(4pt-6)^2+(16)^2} = 20$

$t = 0.5$

Squaring

$(4pt-6)^2 + 16^2 = 400$

$(2p-6)^2 + 16^2 = 400$

$4p^2 - 24p + 36 + 256 - 400 = 0$

$4p^2 - 24p - 108$

$p = 9$ or $\underline{\underline{p = -3}}$

not possible as $p < 0$

When $t = 0$, R is at the point with position vector $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m}$.

(b) Find, in terms of q, an expression for the displacement vector of R at time t. **[4]**

$$\underset{\sim}{v} = \begin{pmatrix} -3t^2 - 3t \\ 8t + q \end{pmatrix}$$

Integrating

$$\underset{\sim}{R} = \begin{pmatrix} -t^3 - \frac{3}{2}t^2 \\ 4t^2 + qt \end{pmatrix} + \underset{\sim}{c}$$

at $t = 0$ $\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \underset{\sim}{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \Rightarrow \underset{\sim}{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

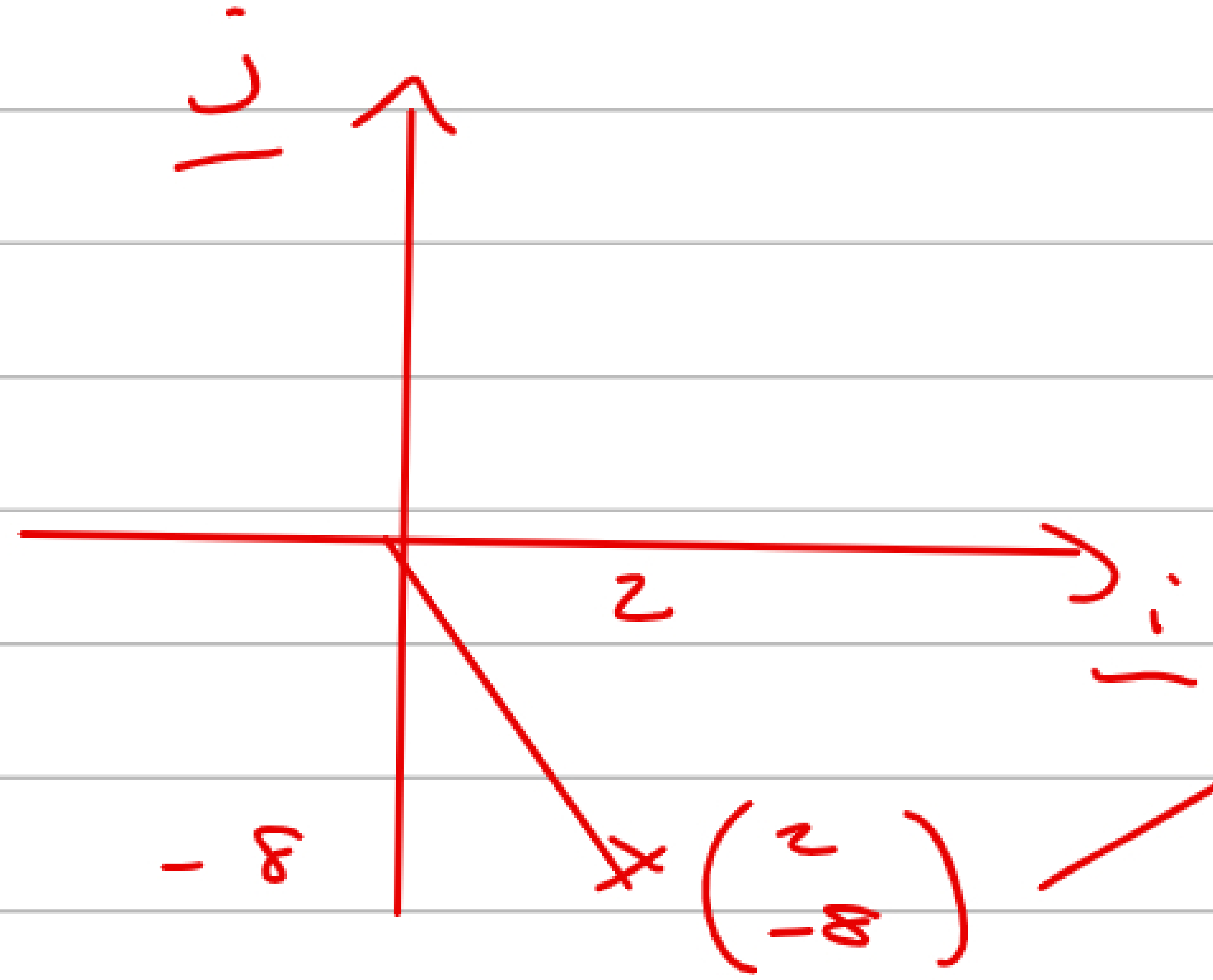
$$\underset{\sim}{R} = \begin{pmatrix} -t^3 - \frac{3}{2}t^2 + 2 \\ 4t^2 + qt - 3 \end{pmatrix}$$

When $t = 1$, R is at a point on the line L, where L passes through O and the point with position vector $2\mathbf{i} - 8\mathbf{j}$.

(c) Find the value of q. **[3]**

$$\underset{\sim}{R} = \begin{pmatrix} -t^3 - \frac{3}{2}t^2 + 2 \\ 4t^2 + qt - 3 \end{pmatrix}$$

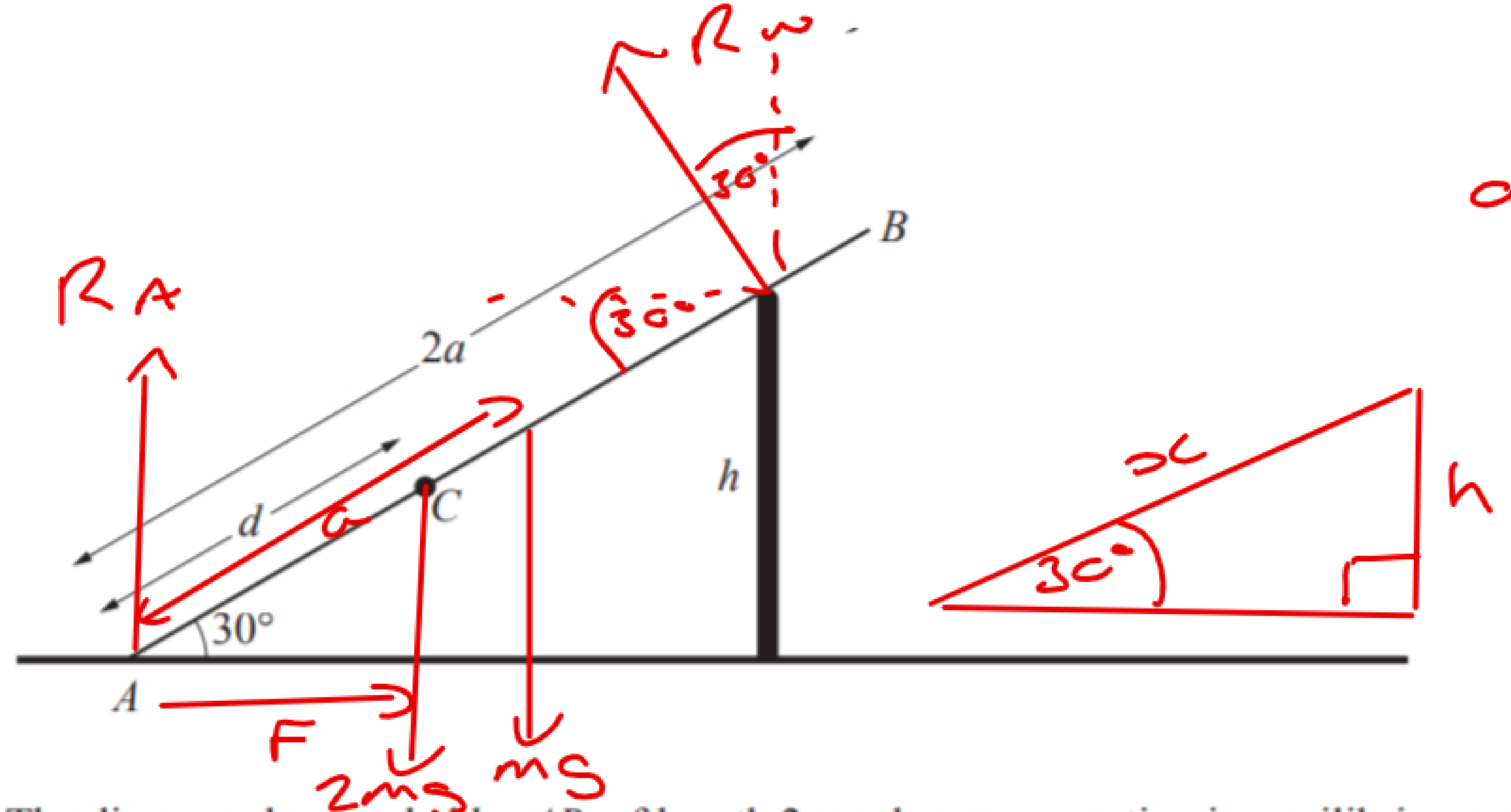
at $t = 1$ $\quad \underset{\sim}{R} = \begin{pmatrix} -1 - \frac{3}{2} + 2 \\ 4 + q - 3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 1 + q \end{pmatrix}$



on line L, $\underset{\sim}{j}$ component is $-4 \times \underset{\sim}{i}$ component

$$\therefore \quad 1 + q = -4\left(-\frac{1}{2}\right)$$
$$1 + q = 2$$
$$q = 1$$

11



The diagram shows a ladder AB, of length $2a$ and mass m, resting in equilibrium on a vertical wall of height h. The ladder is inclined at an angle of 30° to the horizontal. The end A is in contact with horizontal ground. An object of mass $2m$ is placed on the ladder at a point C where $AC = d$.

The ladder is modelled as uniform, the ground is modelled as being rough, and the vertical wall is modelled as being smooth.

(a) Show that the normal contact force between the ladder and the wall is $\dfrac{mg(a+2d)\sqrt{3}}{4h}$. **[4]**

a) m (A)

$x \times R_w = a\cos 30° \times mg$
$+ d\cos 30° \times 2mg$ (1)

$\sin 30° = \dfrac{h}{x}$

$x = \dfrac{h}{\sin 30°} \Rightarrow x = 2h$

in (1)

$2h R_w = a \times \dfrac{\sqrt{3}}{2} mg + d \dfrac{\sqrt{3}}{\cancel{2}} \cancel{2} mg$

$R_w = \dfrac{\frac{a}{2}\sqrt{3}mg + d\sqrt{3}mg}{2h}$

$$R_W = \frac{\frac{a}{2}\sqrt{3}mg + \frac{d\sqrt{3}mg}{1}}{2h} = \frac{a\sqrt{3}mg}{4h} + \frac{d\sqrt{3}mg}{2h}$$

$$= \frac{a\sqrt{3}mg}{4h} + \frac{2d\sqrt{3}mg}{4h}$$

$$= \frac{\sqrt{3}mg(a+2d)}{4h}$$

(as required)

It is given that the equilibrium is limiting and the coefficient of friction between the ladder and the ground is $\frac{1}{8}\sqrt{3}$.

(b) Show that $h = k(a+2d)$, where k is a constant to be determined. **[7]**

$F = \mu R_A$

$F = \frac{\sqrt{3}}{8} R_A$

$R_A = \frac{8F}{\sqrt{3}}$ ②

Resolving (↑) whole system

$R_A + R_W \cos 30° = 3mg$ ③

Resolving (→) $F = R_W \sin 30°$ ④

$$R_A + R_w \cos 30^\circ = 3mg \quad (3)$$

$$\frac{8F}{\sqrt{3}} + R_w \cos 30^\circ = 3mg$$

$$\frac{8 \times R_w \sin 30^\circ}{\sqrt{3}} + R_w \cos 30^\circ = 3mg$$

$$R_w \left(\frac{4}{\sqrt{3}} + \frac{\sqrt{3}}{2} \right) = 3mg$$

$$\frac{11\sqrt{3}}{6} R_w = 3mg$$

$$R_w = \frac{6\sqrt{3}}{11} mg$$

In equation for R_w

$$\frac{\sqrt{3}mg(a+2d)}{4h} = \frac{6\sqrt{3}}{11}mg$$

make h the subject

$$\frac{\cancel{\sqrt{3}}\,\cancel{mg}(a+2d)\times 11}{6\cancel{\sqrt{3}}\times 4\times \cancel{mg}} = h$$

$$\frac{11}{24}(a+2d) = h$$

$$\therefore\ k = \frac{11}{24}$$

c) h cannot exceed $2a\sin 30^\circ = a$ (as then the ladder would not reach top of wall)

$$\frac{11}{24}(a+2d) \le a$$

$$11a + 22d \le 24a \qquad d \le \frac{13}{22}a$$

$$22d \le 13a$$

so greatest d is $\frac{13}{22}a$

(d) State one improvement that could be made to the model. **[1]**

consider ladder as non-uniform

include friction on wall

consider thickness of ladder

consider ladder may bend