AQA

Please write clearly in block capitals.

Centre number	
Surname	
Forename(s)	
Candidate signature	

I declare this is my own work.

YORKSHIRE MATHS TUTOR

A-level
MATHEMATICS
Paper 3

Friday 12 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
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8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
TOTAL	

J U N 2 0 7 3 5 7 3 0 1

PB/Jun20/E6

7357/3

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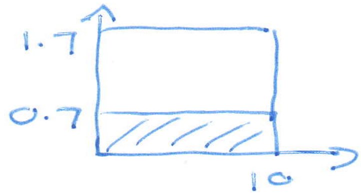
Section A

Answer **all** questions in the spaces provided.

1 Given that

$$\int_0^{10} f(x)\,dx = 7$$



deduce the value of

$$\int_0^{10} \left(f(x) + 1 \right) dx$$

up 1 unit

$1.7 \times 10 = 17$

Circle your answer.

[1 mark]

-3 7 8 17

2 Given that

$$6\cos\theta + 8\sin\theta \equiv R\cos(\theta + \alpha)$$

find the value of R.

Circle your answer.

[1 mark]

6 8 10 14

$R = \sqrt{6^2 + 8^2} = 10$

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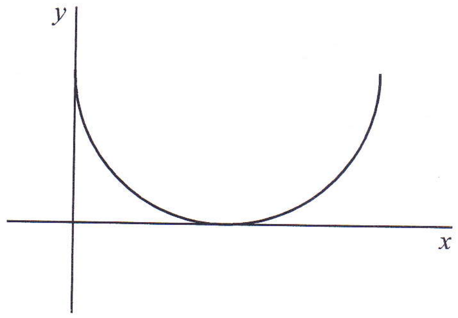
3 Determine which one of these graphs does **not** represent y as a function of x.

Tick (✓) **one** box.

[1 mark]



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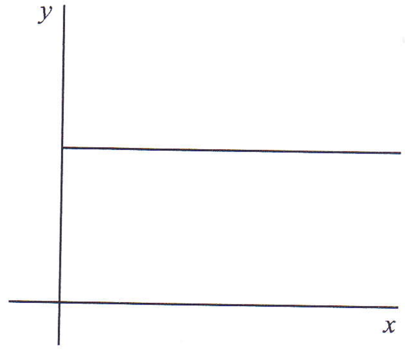

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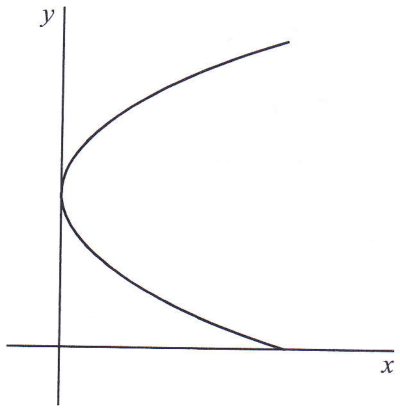
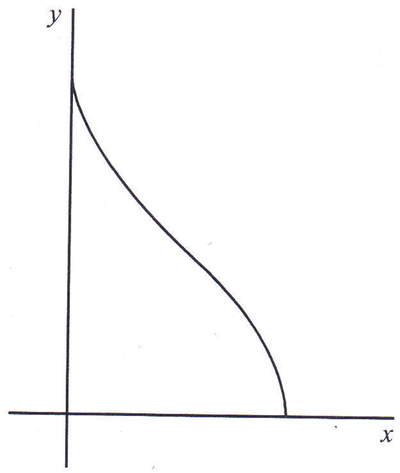


$\leftarrow f(x) = y^2 \ldots$

☑



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4 $p(x) = 4x^3 - 15x^2 - 48x - 36$

4 (a) Use the factor theorem to prove that $x - 6$ is a factor of $p(x)$.

[2 marks]

$p(6) = 4 \times 6^3 - 15 \times 6^2 - 48 \times 6 - 36 = 0$

as $p(6) = 0$

$x - 6$ is a factor of $p(x)$

4 (b) (i) Prove that the graph of $y = p(x)$ intersects the x-axis at exactly one point.

[4 marks]

$4x^2 + 9x + 6$

$x - 6 \,|\, 4x^3 - 15x^2 - 48x - 36$

$- \; 4x^3 - 24x^2$

$9x^2 - 48x$

$- \; 9x^2 - 54x$

$6x - 36$

$6x - 36$

0

$4x^2 + 9x + 6$

$b^2 - 4ac = 81 - 4 \times 4 \times 6 = -15$

as $b^2 - 4ac < 0$ has no real roots

$\therefore$ Curve only intersects x-axis at $(6, 0)$

Do not write outside the box

4 (b) (ii) State the coordinates of this point of intersection.

[1 mark]

(6, 0)

Turn over for the next question

Do not write outside the box

5 The number of radioactive atoms, N, in a sample of a sodium isotope after time t hours can be modelled by

$$N = N_0 e^{-kt}$$

where N_0 is the initial number of radioactive atoms in the sample and k is a positive constant.

The model remains valid for large numbers of atoms.

5 (a) It takes 15.9 hours for half of the sodium atoms to decay.

Determine the number of days required for at least 90% of the number of atoms in the original sample to decay.

[5 marks]

a) $N = N_o e^{-kt}$

at $t = 0$, $N = N_o$

at $t = 15.9$ $N = \frac{N_o}{2}$

$\frac{N_o}{2} = N_o e^{-k \times 15.9}$

$\frac{1}{2} = e^{-15.9k}$

$\ln\left(\frac{1}{2}\right) = -15.9\,k$

$k = \frac{\ln\left(\frac{1}{2}\right)}{-15.9}$

at 90% decay $N = \frac{N_o}{10}$

$\frac{\cancel{N_o}}{10} = \cancel{N_o} e^{-\frac{\ln\left(\frac{1}{2}\right)}{-15.9} \times t}$

$\frac{1}{10} = e^{\frac{\ln\left(\frac{1}{2}\right)}{15.9} t}$

$\ln\left(\frac{1}{10}\right) = \frac{\ln\left(\frac{1}{2}\right)}{15.9} t$

$\frac{15.9 \times \ln\left(\frac{1}{10}\right)}{\ln\left(\frac{1}{2}\right)} = t$

$t = 52.81865$ hours

$(\div 24)$

$t = 2.2$ days (1dp)

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5 (b) Find the percentage of the atoms remaining after the first week. $t = 168$ hours

Give your answer to two significant figures.

[2 marks]

$N = N_0 \times e^{-\frac{\ln(\frac{1}{2})}{-15.9} \times 168} = N_0 \times 0.0006596$

$\times 100$

0.066% (2sf)

5 (c) Explain why the model can only provide an estimate for the number of remaining atoms.

[1 mark]

The model is continuous but the number of atoms is discrete

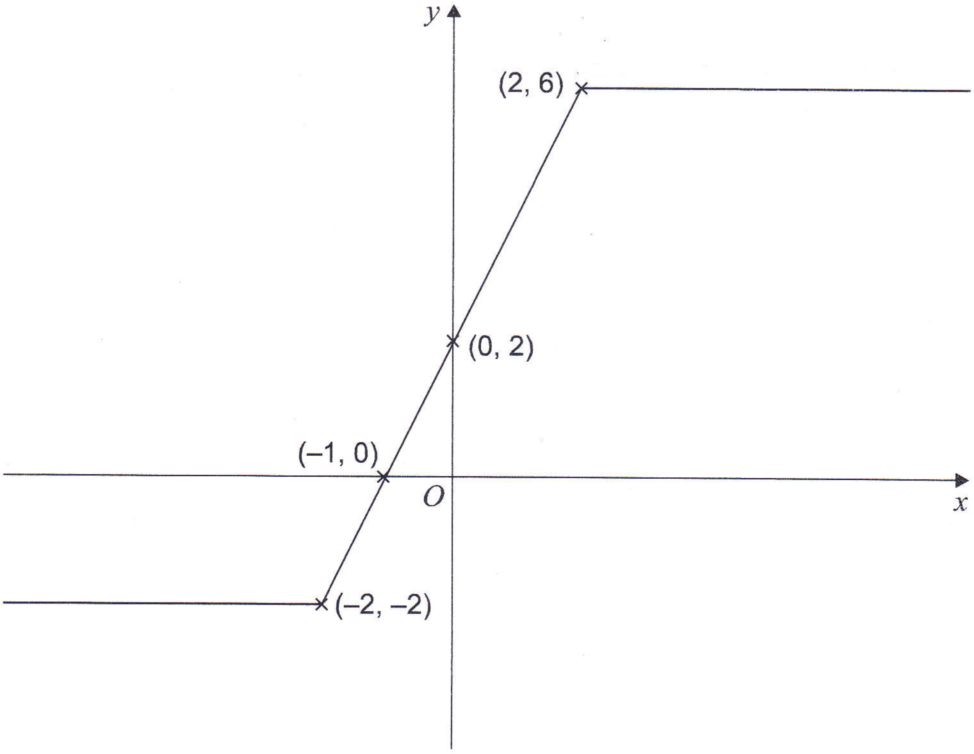
5 (d) Explain why the model is invalid in the long run.

[1 mark]

As t gets larger e^{-kt} tends to zero, but never actually reaches zero. So the model will eventually predict less than one atom, but never zero.

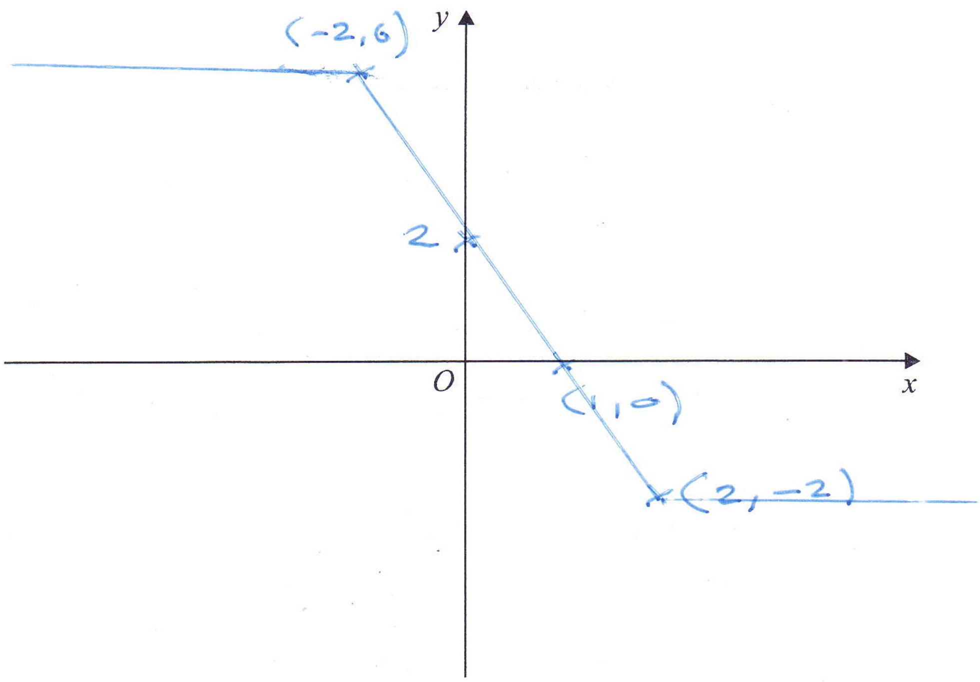
Turn over for the next question

6 The graph of $y = \mathrm{f}(x)$ is shown below.



6 (a) Sketch the graph of $y = \mathrm{f}(-x)$ reflection in y-axis

[2 marks]

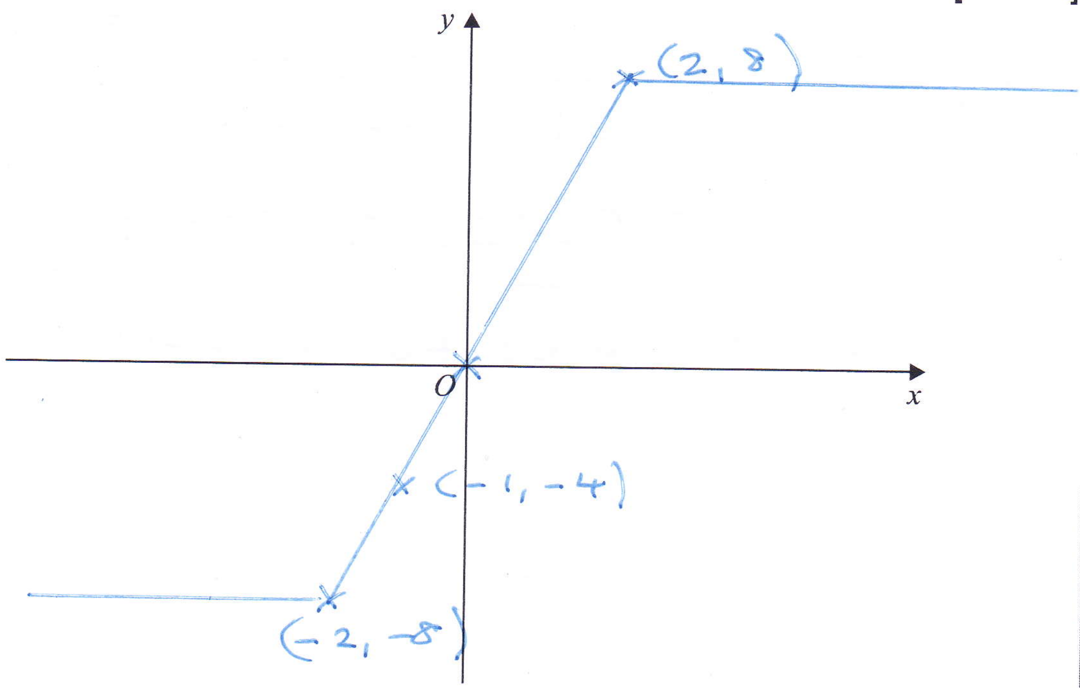


y coord $\times 2$
then down 4

Do not write outside the box

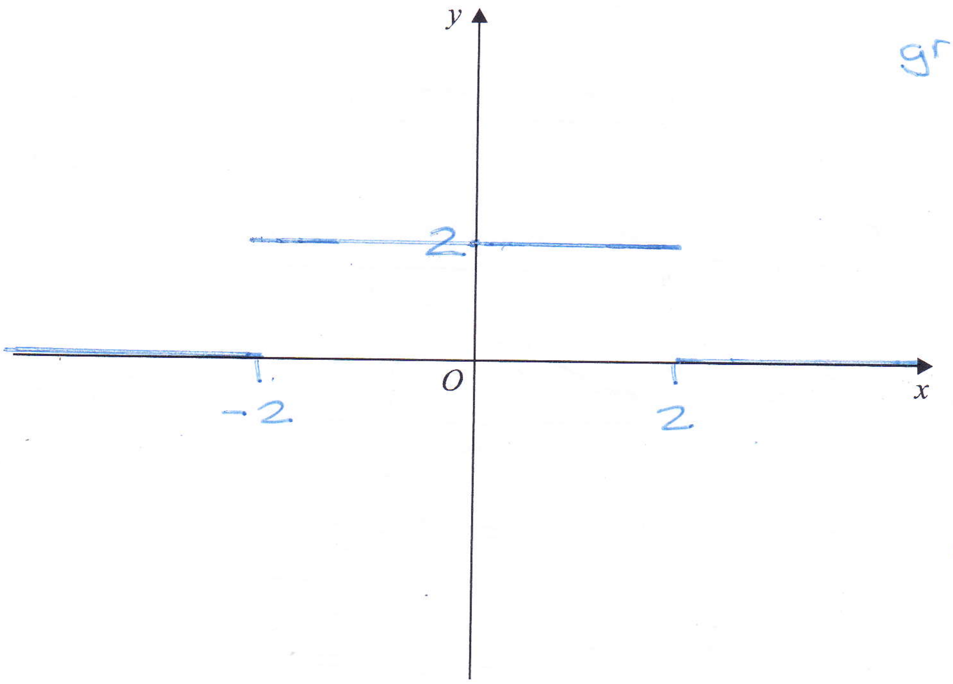
6 (b) Sketch the graph of $y = 2\mathrm{f}(x) - 4$

[2 marks]



6 (c) Sketch the graph of $y = \mathrm{f}'(x)$

[3 marks]



gradient between
(2, 6)
(-2, -2)

$= \frac{6 - -2}{2 - -2}$

$= \frac{8}{4} = 2$

gradient 0
for
$x < -2$
and
$x > 2$

Turn over for the next question

$2! = 2 \times 1$
$3! = 3 \times 2 \times 1$
$n! = n(n-1)\dots 1$
$(n-2)! = (n-2)\dots 1$

Do not write outside the box

7 (a) Using ${}^nC_r = \frac{n!}{r!(n-r)!}$ show that ${}^nC_2 = \frac{n(n-1)}{2}$

[2 marks]

$r = 2,\quad {}^nC_2 = \frac{n(n-1)(n-2)\dots 1}{2! \times (n-2)\dots 1}$

$= \frac{n(n-1)}{2} \frac{\times (n-2)\dots 1}{\times (n-2)\dots 1}$

which cancels to $\frac{n(n-1)}{2}$

7 (b) (i) Show that the equation

$$2 \times {}^nC_4 = 51 \times {}^nC_2$$

simplifies to

$$n^2 - 5n - 300 = 0$$

[3 marks]

${}^nC_2 = \frac{n(n-1)}{2}$

$C_4 = \frac{n(n-1)(n-2)(n-3) \times (n-4)\dots 1}{4! \quad (n-4)\dots 1}$

${}^nC_4 = \frac{n(n-1)(n-2)(n-3)}{24}$

$\therefore \cancel{2} \times \frac{\cancel{n}\cancel{(n-1)}(n-2)(n-3)}{\cancel{24}\,12} = 51\frac{\cancel{n}\cancel{(n-1)}}{2}$

$(n-2)(n-3) = \frac{12 \times 51}{2}$

$n^2 - 3n - 2n + 6 = 306$

$n^2 - 5n - 300 = 0$

as required

Do not write outside the box

7 (b) (ii) Hence, solve the equation

$$2 \times {}^{n}C_4 = 51 \times {}^{n}C_2$$

[2 marks]

$n^2 - 5n - 300 = 0$

$(n + 15)(n - 20) = 0$

$n = -15$ or $n = 20$

as $n > 0$

$n = 20$

Turn over for the next question

Do not write outside the box

8 The sum to infinity of a geometric series is 96 $S_\infty = \frac{a}{1-r}$

The first term of the series is less than 30

The second term of the series is 18 $ar = 18$

8 (a) Find the first term and common ratio of the series.

[5 marks]

$S_\infty = \frac{a}{1-r} = 96$ (1)

$ar = 18$

$r = \frac{18}{a}$

in (1) $\frac{a}{1 - \frac{18}{a}} = 96$

$\frac{a}{\frac{a-18}{a}} = 96$

$\frac{a^2}{a-18} = 96$

$a^2 = 96(a-18)$

$a^2 - 96a + 1728 = 0$

$a = 72$ or $a = 24$ (polynomial on calculator

as first term < 30

$a = 24$

$r = \frac{18}{24} = \frac{3}{4}$

8 (b) (i) Show that the nth term of the series, u_n, can be written as

$$u_n = \frac{3^n}{2^{2n-5}}$$

[4 marks]

$a \quad ar \quad ar^2$

$24 \quad 18 \quad 13.5$

$\times\frac{3}{4} \quad \times\frac{3}{4}$

$u_1 = a = (3^1 \times 2^3) = 3^1 \times 2^3$

$u_2 = ar = (3^1 \times 2^3) \times \frac{3^1}{2^2} = 3^2 \times 2^1$

$u_3 = ar^2 = (3^1 \times 2^3) \times \frac{3^2}{2^4} = 3^3 \times 2^{-1}$

Do not write outside the box

$3^1 \quad 3^2 \quad 3^3$ — $1 \quad 2 \quad 3 \quad \therefore$ sequence for powers is n

$2^3 \quad 2^1 \quad 2^{-1}$ — $\underbrace{3 \quad 1}_{-2} \underbrace{\quad -1}_{-2}$ — $-2n+5$

$\therefore u_n = 3^n \times 2^{(-2n+5)}$

$= \dfrac{3^n}{2^{2n-5}}$

$2^{-2n+5} = \dfrac{1}{2^{2n-5}}$

8 (b) (ii) Hence show that

$$\log_3 u_n = n(1 - 2\log_3 2) + 5\log_3 2$$

[3 marks]

$u_n = \dfrac{3^n}{2^{2n-5}}$

$\log_3 u_n = \log_3 \left(\dfrac{3^n}{2^{2n-5}}\right)$

$\log_3 u_n = \log_3 3^n - \log_3 (2^{2n-5})$

$\log_3 u_n = n - (2n-5)\log_3 2$

$\log_3 u_n = n - 2n\log_3 2 + 5\log_3 2$

$\log_3 u_n = n(1 - 2\log_3 2) + 5\log_3 2$

as required

Turn over ▶

Do not write outside the box

9 (a) For $\cos\theta \neq 0$, prove that

Identities

$\cosec 2\theta = \frac{1}{\sin 2\theta}$

$$\operatorname{cosec} 2\theta + \cot 2\theta = \cot\theta$$

[4 marks]

$\cot 2\theta = \frac{\cos 2\theta}{\sin 2\theta}$ $\cos 2\theta = 2\cos^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta} = \frac{1+\cos 2\theta}{\sin 2\theta}$

$= \frac{1 + 2\cos^2\theta - 1}{2\sin\theta\cos\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \cot\theta$

as required

9 (b) Explain why

$$\cot\theta \neq \operatorname{cosec} 2\theta + \cot 2\theta$$

when $\cos\theta = 0$

[1 mark]

when $\cos\theta = 0$

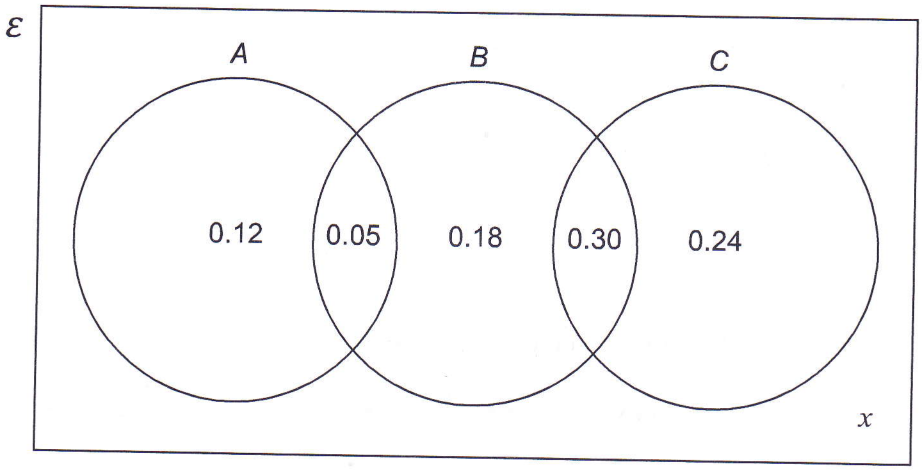
$\cosec 2\theta = \frac{1}{\sin 2\theta} = \frac{1}{2\sin\theta\cos\theta}$

This is undefined as we would be dividing by zero

Section B

Answer **all** questions in the spaces provided.

10 The probabilities of events A, B and C are related, as shown in the Venn diagram below.



Find the value of x.

Circle your answer.

add to 1

[1 mark]

0.11 0.46 0.54 0.89

11 The table below shows the temperature on Mount Everest on the first day of each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temperature (°C)	−17	−16	−14	−9	−2	2	6	5	−3	−4	−11	−18

Menu 6

1: 1 variable

enter values in x column

Calculate the standard deviation of these temperatures.

Circle your answer.

[1 mark]

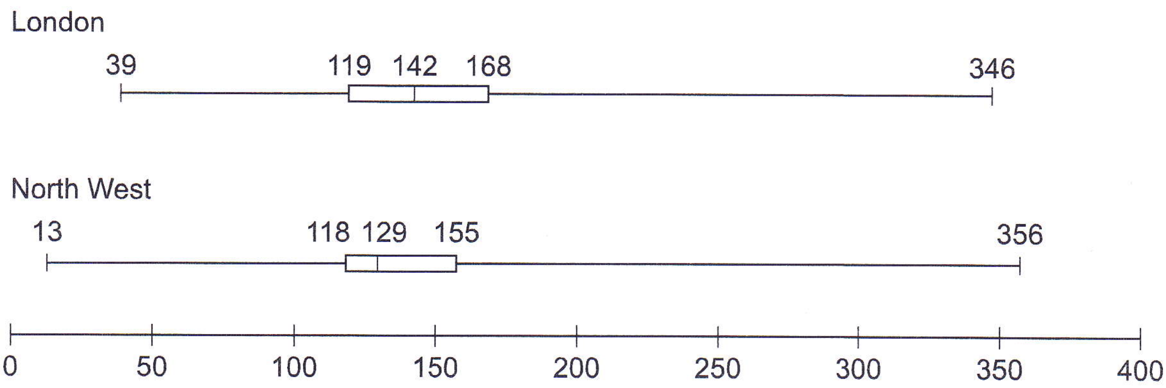
−6.75 5.82 8.24 67.85

Option 3: 1 variable calc

$\sigma x = 8.24$

standard deviation

12 The box plot below summarises the CO_2 emissions, in g/km, for cars in the Large Data Set from the London and North West regions.



12 (a) Using the box plot, give **one** comparison of central tendency and **one** comparison of spread for the two regions.

[2 marks]

Comparison of central tendency

Median in London is greater than median in North West

Comparison of spread

The IQR for London is greater than the IQR for North West

Do not write outside the box

12 (b) Jaspal, an environmental researcher, used all of the data in the Large Data Set to produce a statistical comparison of the CO_2 and CO emissions in regions of England.

Using your knowledge of the Large Data Set, give **two** reasons why his conclusions may be invalid.

[2 marks]

Not all makes of car are included in the database

Not all English regions are included

Turn over for the next question

Do not write outside the box

13 Diedre is a head teacher in a school which provides primary, secondary and sixth-form education.

There are 200 teachers in her school.

The number of teachers in each level of education along with their gender is shown in the table below.

	Primary	**Secondary**	**Sixth-form**
Male	9	24	23
Female	35	85	24

13 (a) A teacher is selected at random. Find the probability that:

13 (a) (i) the teacher is female

[1 mark]

$$\frac{35+85+24}{200} = \frac{18}{25}$$

13 (a) (ii) the teacher is **not** a sixth-form teacher.

[1 mark]

$$\frac{9+24+35+85}{200} = \frac{153}{200}$$

13 (b) Given that a randomly chosen teacher is male, find the probability that this teacher is **not** a primary teacher.

[2 marks]

$$\frac{24+23}{9+24+23} = \frac{47}{56}$$

male

Do not write outside the box

13 (c) Diedre wants to select three different teachers at random to be part of a school project.

Calculate the probability that all three chosen are secondary teachers.

[2 marks]

$24 + 85 = 109$ secondary

$\frac{109}{200} \times \frac{108}{199} \times \frac{107}{198} = 0.15984...$

$= 0.16$ (2 sf)

Turn over for the next question

Do not write outside the box

14 It is known that a hospital has a mean waiting time of 4 hours for its Accident and Emergency (A&E) patients.

After some new initiatives were introduced, a random sample of 12 patients from the hospital's A&E Department had the following waiting times, in hours.

4.25	3.90	4.15	3.95	4.20	4.15
5.00	3.85	4.25	4.05	3.80	3.95

Carry out a hypothesis test at the 10% significance level to investigate whether the mean waiting time at this hospital's A&E department has changed. 2 tail test

You may assume that the waiting times are normally distributed with standard deviation 0.8 hours.

[7 marks]

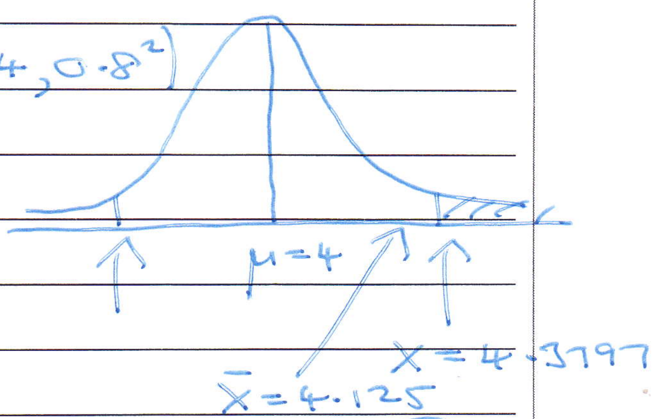
$\bar{x}$ = Sample mean = (4.25 + 3.9 + 4.15 + 3.95 + 4.2 + 4.15 + 5 + 3.85 + 4.25 + 4.05 + 3.8 + 3.95) ÷ 12 = 4.125

$\mu = 4$

$\sigma = 0.8$ $\quad X \sim N(4, 0.8^2)$

$H_0: \mu = 4$

$H_1: \mu \neq 4$



5% lower tail

5% upper tail

Used a sample mean $\therefore \sigma = \frac{\sigma}{\sqrt{n}}$

$= \frac{0.8}{\sqrt{12}} = 0.2309$ (4dp)

Inverse normal

Area 0.95 (95%)

$\sigma = 0.2309$

$\mu = 4$

$x = 4.3797$

$\bar{x} < 4.3797$ so we accept H_0

There is insufficient evidence to suggest that the mean waiting time has changed

Turn over for the next question

Turn over ▶

Do not write outside the box

15 A political party is holding an election to choose a new leader.

A statistician within the party decides to sample 70 party members to find their opinions of the leadership candidates.

There are 4735 members under 30 years old and 8565 members 30 years old and over.

The statistician wants to use a sample of 70 party members in the survey.

He decides to use a random stratified sample.

15 (a) Calculate how many of each age group should be included in his sample.

[2 marks]

Under 30 4735

30 + 8565

13300

Under 30 $= \frac{4735}{13300} \times 70 = 24.9 = 25$

30 + $= \frac{8565}{13300} \times 70 = 45.07 = 45$

15 (b) Explain how he could collect the random sample of members under 30 years old.

[3 marks]

Give each member under 30 a number between 1 and 4735

Generate random 4 digit numbers using calculator or software

Keep generating until 25 different numbers in range 1 to 4735 have been picked

Do not write outside the box

16 An educational expert found that the correlation coefficient between the hours of revision and the scores achieved by 25 students in their A-level exams was 0.379

Her data came from a bivariate normal distribution.

Carry out a hypothesis test at the 1% significance level to determine if there is a positive correlation between the hours of revision and the scores achieved by students in their A-level exams.

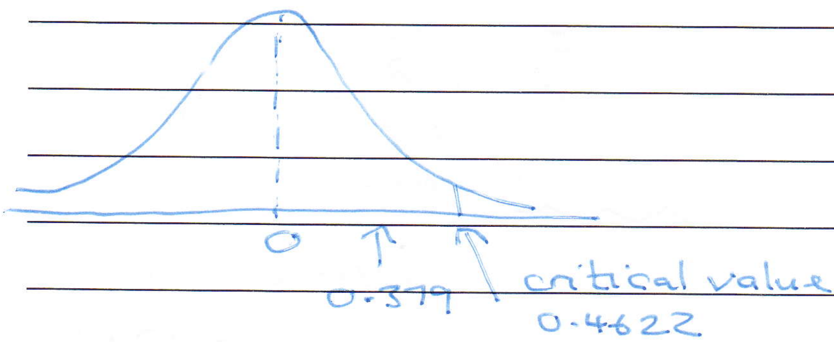
The critical value of the correlation coefficient is 0.4622

[4 marks]

$r = 0.379$

$H_0 : \rho = 0$ (no correlation)

$H_1 : \rho > 0$ (positive correlation)



As the test statistic $r = 0.379$

< 0.4622 (not in tail)

accept H_0

There is insufficient evidence to suggest there is a positive correlation between the hours of revision and the scores achieved in their A level exams

Turn over for the next question

17 The lifetime of Zaple smartphone batteries, X hours, is normally distributed with mean 8 hours and standard deviation 1.5 hours.

$X \sim N(8, 1.5^2)$

17 (a) (i) Find $P(X \neq 8)$

[1 mark]

$p = 1$

17 (a) (ii) Find $P(6 < X < 10)$

[1 mark]

Normal CD

Lower 6

Upper 10

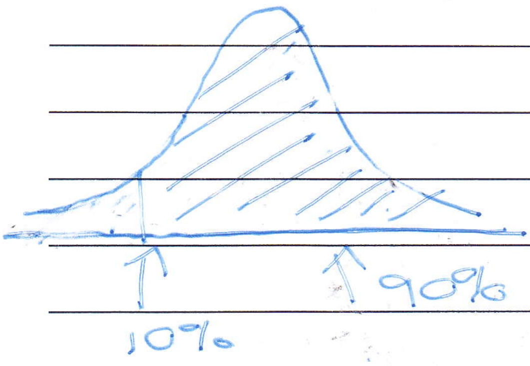
σ 1.5

μ 8

$p = 0.818$ (3 sf)

17 (b) Determine the lifetime exceeded by 90% of Zaple smartphone batteries.

[2 marks]



Inverse normal

area = 0.1

$\sigma = 1.5$

$\mu = 8$

$X = 6.08$ (3sf)

Do not write outside the box

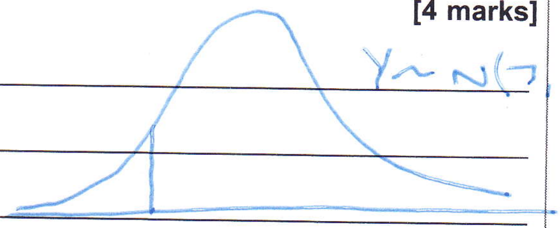
17 (c) A different smartphone, Kaphone, has its battery's lifetime, Y hours, modelled by a normal distribution with mean 7 hours and standard deviation σ.

25% of randomly selected Kaphone batteries last less than 5 hours.

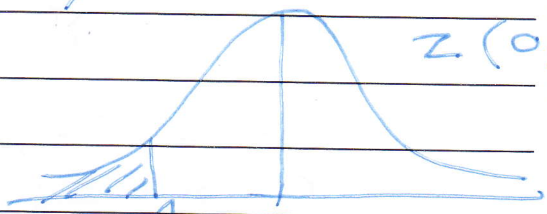
Find the value of σ, correct to three significant figures.

[4 marks]

$Y \sim N(7, \sigma^2)$



$Z = \dfrac{Y - \mu}{\sigma}$



$-0.674489 = \dfrac{5-7}{\sigma}$

$\sigma = \dfrac{5-7}{-0.674489}$

$\sigma = 2.9652077$

$\sigma = 2.97$ (3sf)

Inverse normal
area $= 0.25$
$\sigma = 1$
$\mu = 0$
$Z = -0.674489$

Turn over for the next question

Do not write outside the box

18 Tiana is a quality controller in a clothes factory. She checks for four possible types of defects in shirts.

Of the shirts with defects, the proportion of each type of defect is as shown in the table below.

Type of defect	Colour	Fabric	Sewing	Sizing
Probability	0.25	0.30	0.40	0.05

Shirts with defects are packed in boxes of 30 at random.

18 (a) Find the probability that:

18 (a) (i) a box contains exactly 5 shirts with a colour defect

[2 marks]

$X \sim B(30, 0.25)$

$P(X = 5)$

${}^{30}C_5 \times 0.25^5 \times 0.75^{25}$

$= 0.1047$

18 (a) (ii) a box contains fewer than 15 shirts with a sewing defect

[2 marks]

$X \sim B(30, 0.4)$

$P(X < 15) = P(X \leq 14)$

Binomial CD

$x = 14$

$N = 30$ $\qquad p = 0.8246$

$p = 0.4$

Do not write outside the box

18 (a) (iii) a box contains at least 20 shirts which do **not** have a fabric defect.

[3 marks]

0·7

$X \sim B(30, 0.7)$

$P(X \geq 20) = 1 - P(X \leq 19)$

18 19 20 21 ... 30

$= 1 - 0.26962$

$= 0.73037$

Binomial CD

$x = 19$

$n = 30$

$p = 0.7$

$= 0.26962$

Question 18 continues on the next page

18 (b) Tiana wants to investigate the proportion, p, of defective shirts with a fabric defect.

She wishes to test the hypotheses

$$H_0 : p = 0.3$$

$$H_1 : p < 0.3$$

She takes a random sample of 60 shirts with a defect and finds that x of them have a fabric defect.

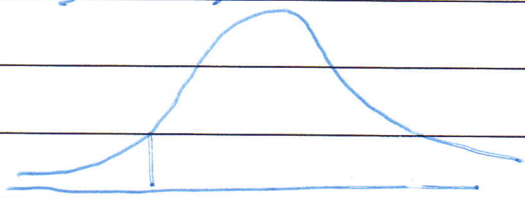
18 (b) (i) Using a 5% level of significance, find the critical region for x.

[5 marks]

$n = 60$ $p = 0.3$

$X \sim B(60, 0.3)$

Binomial CD



$P(X \leq 5) = 5 \times 10^{-5}$ 5%

$P(X \leq 10) = 0.01387$ < 0.05

$P(X \leq 11) = 0.0294$ < 0.05

$P(X \leq 12) = 0.05677$ > 0.05

Critical region is $X \leq 11$

Do not write outside the box

18 (b) (ii) In her sample she finds 13 shirts with a fabric defect.

Complete the test stating her conclusion in context.

[2 marks]

$13 > 11$ so not in the critical region, accept H_0

There is insufficient evidence to suggest the proportion of shirts with a fabric defect has decreased.

END OF QUESTIONS