

2.

A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \quad x > 0$$

(a) Find (i) $\frac{dy}{dx}$ (ii) $\frac{d^2y}{dx^2}$

(3)

$$a) (i) \quad y = x^2 - 2x - 24x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$$

$$(ii) \quad \frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$$

b) $\frac{dy}{dx} = 0$ at stationary point

$$\text{at } x=4, \quad \frac{dy}{dx} = 2 \times 4 - 2 - 12 \times \frac{1}{4^{\frac{1}{2}}}$$

$$= 8 - 2 - 6 = 0$$

as $\frac{dy}{dx} = 0$, stationary point at $x=4$

$$c) \quad \text{at } x=4, \quad \frac{d^2y}{dx^2} = 2 + 6 \times 4^{-\frac{3}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}$$

as $\frac{d^2y}{dx^2} > 0$ this is a minimum point

3.

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} = A + \frac{B}{x-3} + \frac{C}{1-2x}$$

(a) Find the values of the constants A , B and C .

(4)

$$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)} \quad x > 3$$

(b) Prove that $f(x)$ is a decreasing function.

(3)

$$\begin{aligned} \text{a)} \quad (x-3)(1-2x) &= x - 2x^2 - 3 + 6x \\ &= -2x^2 + 7x - 3 \end{aligned}$$

$$\begin{array}{r} -2x^2 + 7x - 3 \quad | \quad -6x^2 + 11x + 1 \\ \underline{-6x^2 + 21x - 9} \\ -10x + 10 \end{array}$$

$$\therefore A = 3 \quad \text{and}$$

$$\frac{-10x+10}{(x-3)(1-2x)} = \frac{B}{x-3} + \frac{C}{1-2x}$$

$$-10x+10 = B(1-2x) + C(x-3)$$

$$\begin{aligned} \text{Let } x=3, \quad -30+10 &= B(1-6) \\ -20 &= -5B \\ B &= 4 \end{aligned}$$

$$\begin{aligned} \text{Let } x = \frac{1}{2}, \quad -10 \times \frac{1}{2} + 10 &= C\left(\frac{1}{2} - 3\right) \\ 5 &= -2.5C \\ C &= -2 \end{aligned}$$

$$A = 3, \quad B = 4, \quad C = -2$$

$$b) f(x) = 3 + 4(x-3)^{-1} - 2(1-2x)^{-1}$$

$$f'(x) = -4(x-3)^{-2} + 2 \times 2(1-2x)^{-2}$$

$$f'(x) = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2}$$

as $x > 3$
always
+ve

squared
means
always +ve

$\therefore -\frac{4}{(x-3)^2}$ is always
negative

and $-\frac{4}{(1-2x)^2}$ is always
negative

so $f'(x)$ is always
negative for $x > 3$

so $f(x)$ is a decreasing
function

5.

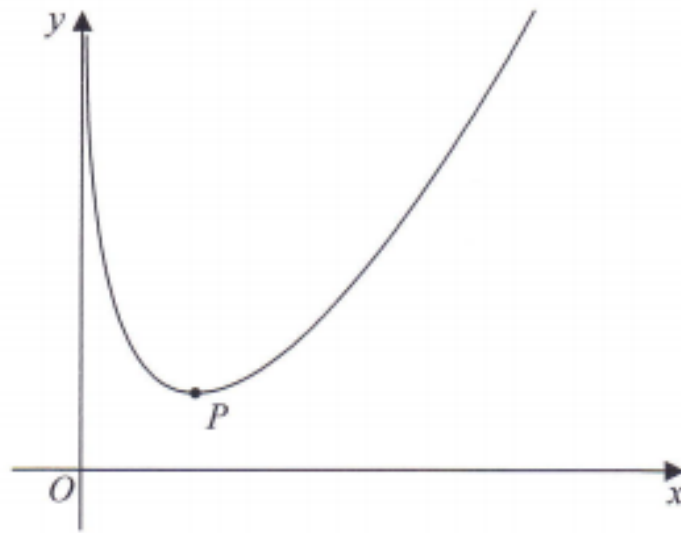


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4 \ln x \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} \quad (4)$$

The point P , shown in Figure 1, is the minimum turning point on C .

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}} \quad (3)$$

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

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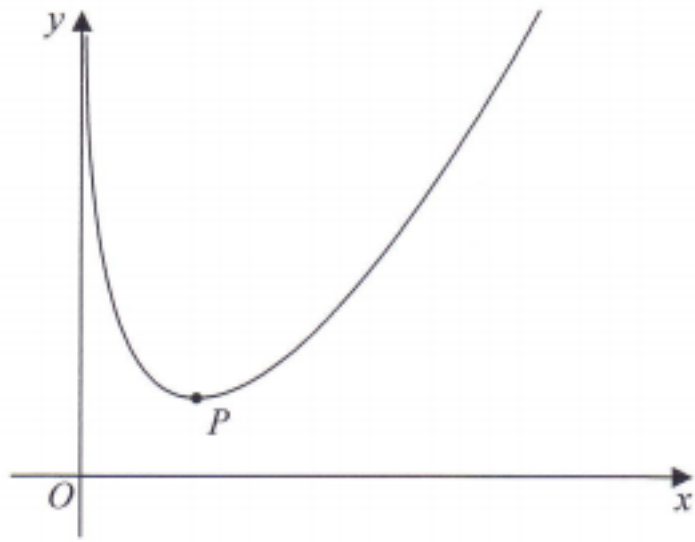


Figure 1

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