YORKSHIRE MATHS TUTOR

1. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

$2\log(4-x) - \log(x+8) = 0$

$\log(4-x)^2 - \log(x+8) = 0$

$\log\frac{(4-x)^2}{x+8} = 0$

$\frac{(4-x)^2}{x+8} = 10^0$

$(4-x)^2 = x+8$

$16 - 8x + x^2 = x + 8$

$x^2 - 9x + 8 = 0$ (as required)

b) (i) $(x-8)(x-1) = 0$

$x = 8$, $x = 1$

(ii) $x = 8$ is not a solution as $\log(4-8)$ gives a math error

2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

$$\ln 4^{3p-1} = \ln 5^{210}$$

$$(3p-1)\ln 4 = 210 \ln 5$$

$$3p\ln 4 - \ln 4 = 210 \ln 5$$

$$3p \ln 4 = 210 \ln 5 + \ln 4$$

$$p = \frac{210 \ln 5 + \ln 4}{3 \ln 4}$$

$$p = 81.6008$$

$$p = 81.6 \quad (1dp)$$

3\. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of k,

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to k.

(3)

a) $\left[\frac{2}{3}\ln(3x-k)\right]_k^{3k}$

$= \left(\frac{2}{3}\ln(9k-k)\right) - \left(\frac{2}{3}\ln(3k-k)\right)$

$= \frac{2}{3}\ln(8k) - \frac{2}{3}\ln 2k$

$= \frac{2}{3}\ln\frac{8k}{2k}$

$= \frac{2}{3}\ln 4$

b) $\int_k^{2k} 2(2x-k)^{-2}\,dx$

$= \left[\frac{2}{-1\times 2}(2x-k)^{-1}\right]_k^{2k}$

$= \left[-\frac{1}{2x-k}\right]_k^{2k}$

$= \left(-\frac{1}{4k-k}\right) - \left(-\frac{1}{2k-k}\right)$

$= -\frac{1}{3k} + \frac{1}{k}$

$= \frac{-1+3}{3k} = \frac{2}{3k}$

$\therefore \propto \frac{1}{k}$

inversely proportional to k

P 5 8 3 4 8 A 0 1 6 4 4

4. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V\,\text{km}\,\text{h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

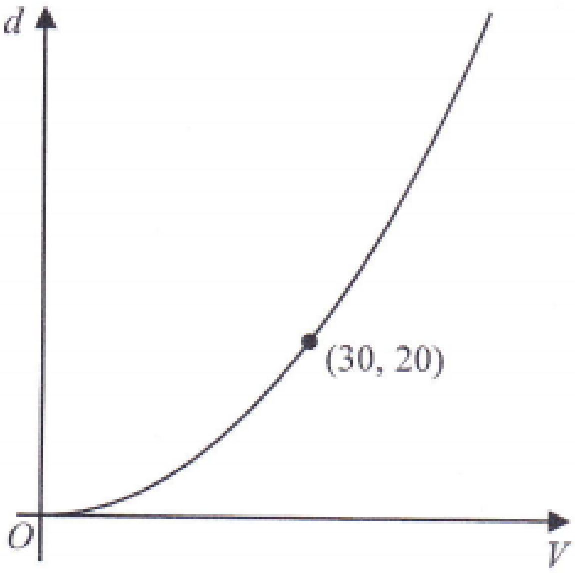


Figure 5

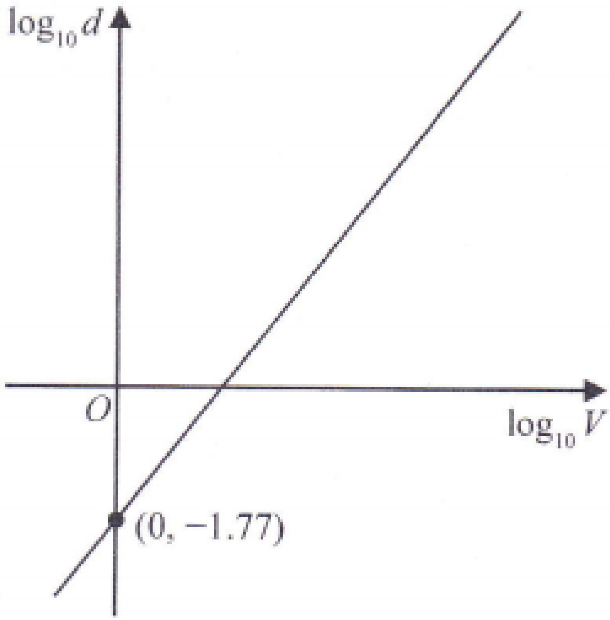


Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at $60\,\text{km}\,\text{h}^{-1}$ in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

(3)

a) As the gradient is a constant (straight line) in figure 6. Figure 6 has the data coded in form where $Y = \log_{10} d$ and $X = \log_{10} V$. If data is coded in this form and produces a straight line, it can be modelled by an exponential relationship

in form $d = kV^n$

b) $d = kV^n$

coded $Y = \log_{10} d$
$X = \log_{10} V$

$\log_{10} d = \log_{10} k + n\log_{10} V$

$\log_{10} k = -1.77$ from figure 6

$k = 10^{-1.77} \approx 0.017$

(Given $k \simeq 0.017$)

using $d = kV^n$

fig 5
$d = 20$
$V = 30$

$20 = 0.017 \times 30^n$

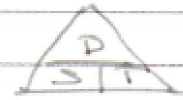
$\frac{20}{0.017} = 30^n$

$\ln\left(\frac{20}{0.017}\right) = \ln 30^n$

$\ln\left(\frac{20}{0.017}\right) = n \ln 30$

$\Rightarrow n = \frac{\ln\left(\frac{20}{0.017}\right)}{\ln 30} = 2.08$ (3sf)

c) $V = 60\,km\,h^{-1}$



Thinking distance $= \dfrac{60 \times 0\cdot 8}{3600} = \dfrac{1}{75}$ km

km h^{-1} hours

$\dfrac{1}{75}$ km $= \dfrac{1000}{75} = \dfrac{40}{3}$ m

$d = kV^n$

$d = 0\cdot017 \times 60^{2\cdot08}$

$= 84\cdot9187\ldots$ m

Total stopping distance

$= \dfrac{40}{3} + 84\cdot9187 = 98\cdot25$ m (2dp)

Yes, he will stop before the puddle

8. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

a) $V = ae^{-kt}$ (depreciating)

when $t = 0$ $20000 = a \times b^0$

$\Rightarrow a = 20000$

at $t = 1$, $V = 16000$

$16000 = 20000e^{-k}$

$\frac{16000}{20000} = e^{-k} \Rightarrow \ln\frac{4}{5} = -k$

$\Rightarrow k = -0.2231$

$V = 20000e^{-0.223t}$ (use 0.2231

b) at $t = 10$

$V = 20000e^{-0.2231435 \times 10} = £2147.4$

This is fairly close to £2000 so reliable

c) Increase the value of constant ~~k~~ from -0.2231435

9. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

$$3 \times 2^x = 15 - 2^{x+1}$$

$$3 \times 2^x = 15 - 2^x \times 2^1$$

$$3 \times 2^x + 2 \times 2^x = 15$$

$$5 \times 2^x = 15$$

$$2^x = \frac{15}{5}$$

$$2^x = 3$$

$$\log_2 2^x = \log_2 3$$

$$x = \log_2 3$$

10. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

a) $\theta = A - Be^{-0.07t}$

at $t = 0$, $\theta = 18$

$18 = A - Be^{0}$

$18 = A - B$ (1)

at $t = 10$, $\theta = 44$

$44 = A - Be^{-0.07\times10}$ (2)

(2) − (1) gives

$26 = -Be^{-0.7} + B$

$26 = B(1 - e^{-0.7})$

$\frac{26}{1-e^{-0.7}} = B$

$B = 51.6$ (3 sf)

in (1) $A = 18 + 51.64728$

$A = 69.6$ (3 sf)

b) The maximum temperature is 69.6°C from model.

The model is not appropriate as ethanol has a boiling point of 78°C

11. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

a) $\log a - \log b = \log(a-b)$

$\log \frac{a}{b} = \log(a-b)$

$\frac{a}{b} = a - b$

$a = b(a-b)$

$a = ab - b^2$

$b^2 = ab - a$

$b^2 = a(b-1)$

$\frac{b^2}{b-1} = a$

b) $b \neq 1$

as cannot divide by zero

12. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(You do not need to evaluate any unknown constants in your equation.)

(2)

$$n = Ae^{kt}$$

where A and k are positive constants