

4. The depth of water,  $D$  metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \quad 0 \leq t < 24$$

where  $t$  is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.

a)  $D = 5 + 2\sin(30t)$

enters harbour at 6:30 am  
 $t = 6.5$  hours

$$\begin{aligned} D &= 5 + 2\sin(30 \times 6.5) \\ &= 4.48236 \\ &= 4.48 \text{ m (3sf)} \end{aligned}$$

b) 2 hours to load cargo

at 8:30 am  $t = 8.5$

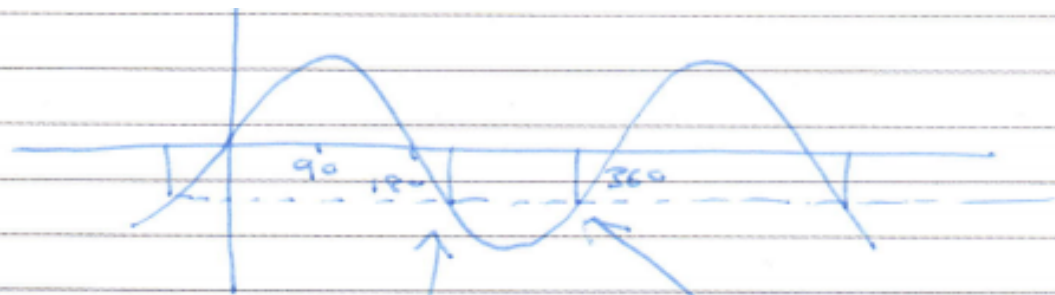
$$\begin{aligned} D &= 5 + 2\sin(30 \times 8.5) \\ &= 3.068 \text{ m (to 3sf)} \end{aligned}$$

need  $D = 3.8$

$$\begin{aligned} 3.8 &= 5 + 2\sin(30t) \\ \frac{3.8 - 5}{2} &= \sin(30t) \end{aligned}$$

$$-\frac{3}{5} = \sin(30t)$$

$$\sin^{-1}\left(-\frac{3}{4}\right) = -36.86989^\circ$$



$$180 + 36.869 \\ = 216.869$$

$$360 - 36.869 \\ = 323.130$$

$$30t = 216.869$$

$$t = 7.2289 \text{ hrs}$$

before 8:30am

$$30t = 323.130$$

$$t = 10.771 \text{ hrs}$$

$$10.771 \text{ hrs} \\ = 10 \text{ hrs } 46 \text{ mins}$$

Boat can leave at 10:46 am

5. (a) Solve, for  $-180^\circ \leq \theta \leq 180^\circ$ , the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(6)

(b) Deduce the smallest positive solution to the equation

$$5 \sin(2x - 50^\circ) = 9 \tan(x - 25^\circ)$$

(2)

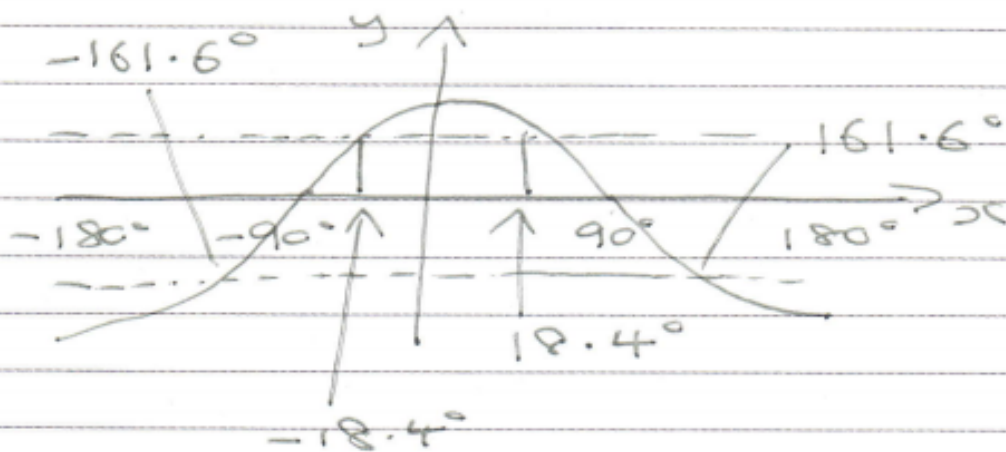
$$5 \sin 2\theta = 9 \tan \theta$$

$$5 \times 2 \sin \theta \cos \theta = 9 \frac{\sin \theta}{\cos \theta}$$

$$\cos^2 \theta = \frac{9}{10}$$

$$\cos \theta = +\sqrt{0.9} \quad \cos \theta = -\sqrt{0.9}$$

$$\theta = 18.434^\circ \quad \theta = 161.565^\circ$$



$$\theta = -161.6^\circ, -18.4^\circ, 18.4^\circ, 161.6^\circ$$

b)  $5 \sin(2x - 50^\circ) = 9 \tan(x - 25^\circ)$   
 $5 \sin 2(x - 25^\circ) = 9 \tan(x - 25^\circ)$   
 $\therefore \theta = x - 25^\circ$

$$\therefore -161.6 = x - 25 \Rightarrow x = -136.6^\circ$$

$$-18.4 = x - 25 \Rightarrow x = 6.6^\circ$$

$$18.4 = x - 25 \Rightarrow x = 43.4^\circ$$

$$161.6 = x - 25 \Rightarrow x = 186.6$$

Smallest solution is  $x = 6.6^\circ$

7. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta \quad \theta \neq (180n)^\circ \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence, or otherwise, solve for  $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ)$$























































