

3.

In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of  $x^4$  is 15120Find the value of  $a$ .

(3)

$$\begin{aligned} & {}^7C_0 a^7 + {}^7C_1 a^6 \times 2x + {}^7C_2 a^5 (2x)^2 \\ & + {}^7C_3 a^4 (2x)^3 + {}^7C_4 a^3 (2x)^4 \\ & + \dots \end{aligned}$$

$${}^7C_4 \times a^3 \times 2^4 = 15120$$

$$35 \times 16 \times a^3 = 15120$$

$$a^3 = \frac{15120}{35 \times 16}$$

$$a = \sqrt[3]{\frac{15120}{35 \times 16}}$$

$$a = 3$$

4. A car has six forward gears.

The fastest speed of the car

- in 1<sup>st</sup> gear is  $28 \text{ km h}^{-1}$
- in 6<sup>th</sup> gear is  $115 \text{ km h}^{-1}$

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3<sup>rd</sup> gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5<sup>th</sup> gear.

(3)

1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>
28		$a+2d$			115

$$a = 28$$

$$a + 5d = 115$$

$$\therefore 28 + 5d = 115$$

$$5d = 115 - 28$$

$$d = \frac{115 - 28}{5} = 17.4$$

$$3^{\text{rd}} = a + 2d = 28 + 2 \times 17.4 = 62.8 \text{ km/h}$$

$$b) \quad 1^{\text{st}} = a = 28$$

$$6^{\text{th}} = ar^5 = 115$$

$$r^5 = \frac{115}{28}$$

$$r = \sqrt[5]{\frac{115}{28}} = 1.326502161$$

$$\begin{aligned} 5^{\text{th}} &= ar^4 = 28 \times 1.326502161^4 \\ &= 86.6941671 \\ &= 86.7 \text{ km/h}^{-1} \text{ (1dp)} \end{aligned}$$

5. The value,  $\pounds V$ , of a vintage car  $t$  years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \quad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was  $\pounds 32\,000$  on 1st January 2005 and  $\pounds 50\,000$  on 1st January 2012

- (a) (i) find  $p$  to 4 decimal places,

$$t = 4$$

$$t = 11$$

- (ii) show that  $A$  is approximately 24 800

(4)

- (b) With reference to the model, interpret

- (i) the value of the constant  $A$ ,

- (ii) the value of the constant  $p$ .

(2)

Using the model,

- (c) find the year during which the value of the car first exceeds  $\pounds 100\,000$

(4)

(4)

a) (i)  $v = Ap^t$

$$32000 = Ap^4 \quad (1)$$

$$50000 = Ap^{11} \quad (2)$$

$$(2) \div (1) \quad \frac{50000}{32000} = p^7$$

$$p = \sqrt[7]{\frac{50000}{32000}} = 1.0658 \quad (4dp)$$

(ii) (1) gives  $A = \frac{32000}{1.0658^4}$

$$A = 24796.8022$$

$$A = 24800 \quad (3sf)$$

b) (i)  $A$  ( $\pounds 24800$ ) is the value of the car on 1st January 2001

(ii)  $p$  is the multiplier indicating the percentage rise per year  
 $p = 1.0658$   
is a rise of  $6.58\%$  per year

$$c) \quad Ap^t > 100000$$

$$24800 \times 1.0658^t > 100000$$
$$1.0658^t > \frac{100000}{24800}$$

$$\ln 1.0658^t > \ln \left( \frac{100000}{24800} \right)$$

$$t \ln 1.0658 > \frac{\ln 100000}{24800}$$

$$t > \frac{\ln \left( \frac{100000}{24800} \right)}{\ln(1.0658)}$$

$$\ln(1.0658)$$

$$t > 21.8$$

$$2001 + 21.8 = 2022.8$$

Year it first exceeds  
 $\pounds 100000$  is 2022

7. (i) Show that  $\sum_{r=1}^{16} (3 + 5^r + 2^r) = 131\,798$  (4)

(ii) A sequence  $u_1, u_2, u_3, \dots$  is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of  $\sum_{r=1}^{100} u_r$  (3)























































