

1.

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that  $(x + 3)$  is a factor of  $f(x)$ , find the value of the constant  $a$ .

(3)

$$f(-3) = 0 \text{ if } (x+3) \text{ is a factor}$$

$$f(-3) = 3 \times (-3)^3 + 2a \times (-3)^2 - 4(-3) + 5a$$

$$0 = -81 + 18a + 12 + 5a$$

$$81 - 12 = 23a$$

$$69 = 23a$$

$$a = 3$$

4.

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express  $y$  as a function of  $x$ .

(3)

$$2^x \times (2^2)^y = \frac{1}{2\sqrt{2}}$$

$$2^x \times 2^{2y} = \frac{1}{2\sqrt{2}}$$

$$2^{2y} = \frac{1}{2 \times 2^x \times 2^{\frac{1}{2}}}$$

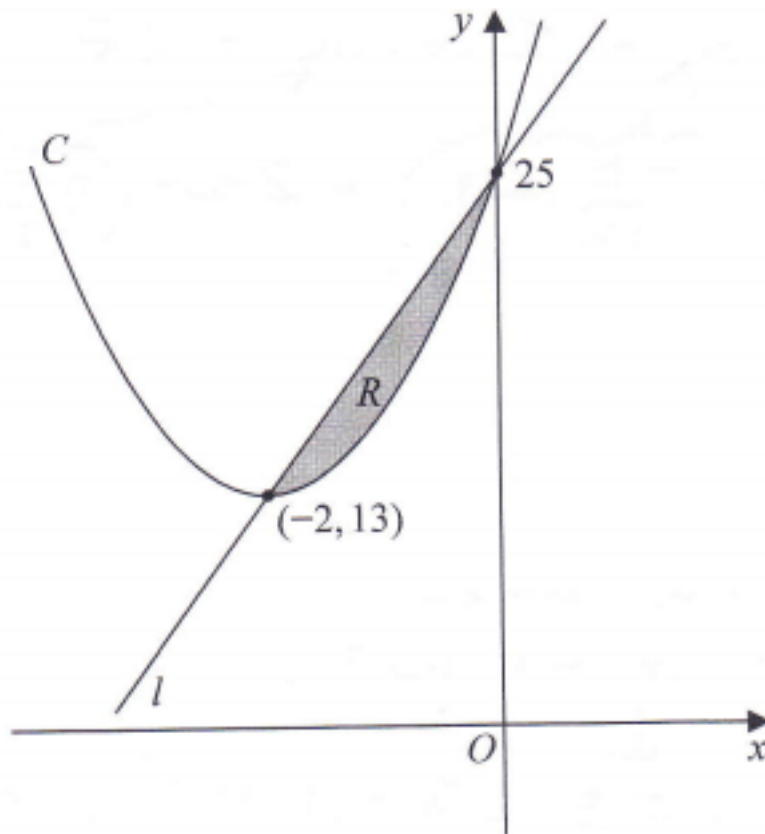
$$2^{2y} = \frac{1}{2^{\frac{3}{2} + x}}$$

$$2^{2y} = 2^{-\frac{3}{2} - x}$$

$$\therefore 2y = -\frac{3}{2} - x$$

$$y = -\frac{3}{4} - \frac{1}{2}x$$

8.



**Figure 1**

Figure 1 shows a sketch of a curve  $C$  with equation  $y = f(x)$  and a straight line  $l$ .

The curve  $C$  meets  $l$  at the points  $(-2, 13)$  and  $(0, 25)$  as shown.

The shaded region  $R$  is bounded by  $C$  and  $l$  as shown in Figure 1.

Given that

- $f(x)$  is a quadratic function in  $x$
- $(-2, 13)$  is the minimum turning point of  $y = f(x)$

use inequalities to define  $R$ .

line  $l$   $(0, 25)$   
 $(-2, 13)$   $m = \frac{25-13}{0-(-2)} = \frac{12}{2} = 6$  (5)

$l$  is  $y = 6x + 25$

Curve, Using minimum pt  $\leftarrow$  minimum pt of curve is  $(-2, 13)$

$y = a(x+2)^2 + 13$

format for completing the square

$\therefore y = a(x^2 + 4x + 4) + 13$   
 $= ax^2 + 4ax + 4a + 13$

when  $x = 0, y = 25$

$\therefore 4a + 13 = 25, a = 3$

Inequality for R is

$$3(x+2)^2 + 13 < y < 6x + 25$$

9.

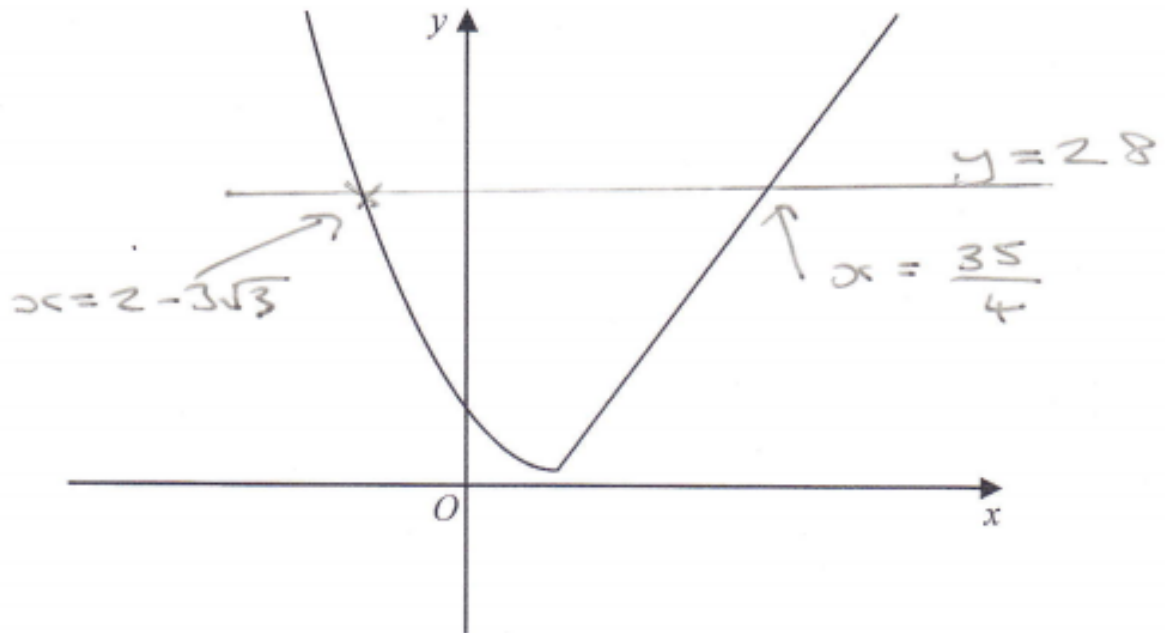


Figure 4

Figure 4 shows a sketch of the graph of  $y = g(x)$ , where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leq 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of  $gg(0)$ .

(2)

(b) Find all values of  $x$  for which

$$g(x) > 28$$

(4)

The function  $h$  is defined by

$$h(x) = (x-2)^2 + 1 \quad x \leq 2$$

(c) Explain why  $h$  has an inverse but  $g$  does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

$$\begin{aligned} \text{a) } g(0) &= (0-2)^2 + 1 = 5 \\ g(5) &= 4 \times 5 - 7 = 13 \end{aligned}$$

$$\begin{aligned} \text{b) } (x-2)^2 + 1 &= 28 \\ x^2 - 4x + 4 + 1 &= 28 \\ x^2 - 4x - 23 &= 0 \end{aligned}$$

used

using quadratic formula

$$x = 2 + 3\sqrt{3} \quad \text{or} \quad x = 2 - 3\sqrt{3}$$

from diagram

$$4x - 7 = 28$$

$$4x = 35$$

$$x = \frac{35}{4}$$

$g(x) > 28$  for

$$x < 2 - 3\sqrt{3} \quad \text{or} \quad x > \frac{35}{4}$$

c)  $h$  is a one to one function  
 $g$  is a many to one function

d)  $h(x) = (x-2)^2 + 1$

$$y = (x-2)^2 + 1$$

$$y-1 = (x-2)^2$$

$$\sqrt{y-1} = x-2$$

$$\sqrt{y-1} + 2 = x$$

$$h^{-1}(x) = \sqrt{x-1} + 2$$

$$\sqrt{x-1} + 2 = -\frac{1}{2}$$

$$\sqrt{x-1} = \left(-\frac{5}{2}\right)$$

$$x-1 = \frac{25}{4}$$

$$x = 1 + \frac{25}{4} = \frac{29}{4}$$





















































