**1.**

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant $a$.

**(3)**

$f(-3) = 0$ if $(x+3)$ is a factor

$f(-3) = 3 \times (-3)^3 + 2a \times (-3)^2 - 4(-3)$
$+ 5a$

$0 = -81 + 18a + 12 + 5a$

$81 - 12 = 23a$

$69 = 23a$

$a = 3$

4. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express $y$ as a function of $x$.

**(3)**

$$2^x \times (2^2)^y = \frac{1}{2\sqrt{2}}$$

$$2^x \times 2^{2y} = \frac{1}{2\sqrt{2}}$$

$$2^{2y} = \frac{1}{2^1 \times 2^x \times 2^{\frac{1}{2}}}$$

$$2^{2y} = \frac{1}{2^{\frac{3}{2}+x}}$$

$$2^{2y} = 2^{-\frac{3}{2}-x}$$

$$\therefore \quad 2y = -\frac{3}{2} - x$$

$$y = -\frac{3}{4} - \frac{1}{2}x$$

8.

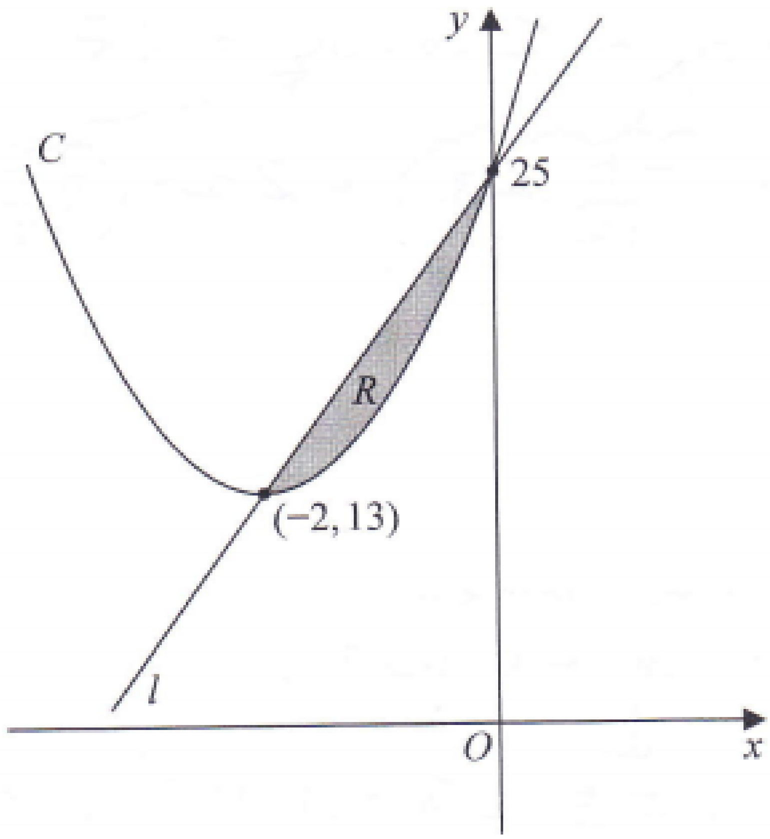


**Figure 1**

Figure 1 shows a sketch of a curve $C$ with equation $y = f(x)$ and a straight line $l$.

The curve $C$ meets $l$ at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region $R$ is bounded by $C$ and $l$ as shown in Figure 1.

Given that

- $f(x)$ is a quadratic function in $x$
- $(-2, 13)$ is the minimum turning point of $y = f(x)$

use inequalities to define $R$.

**(5)**

line L $(0, 25)$ $(-2, 13)$ $m = \frac{25-13}{0--2} = \frac{12}{2} = 6$

L is $y = 6x + 25$

Curve, Using minimum pt
$y = a(x+2)^2 + 13$

minimum pt of curve is $(-2, 13)$

format for completing the square

$\therefore\ y = a(x^2 + 4x + 4) + 13$
$= ax^2 + 4ax + 4a + 13$

when $x = 0$, $y = 25$

$\therefore\ 4a + 13 = 25$, $a = 3$

Inequality for R is

$$3(x+2)^2 + 13 < y < 6x + 25$$

9.

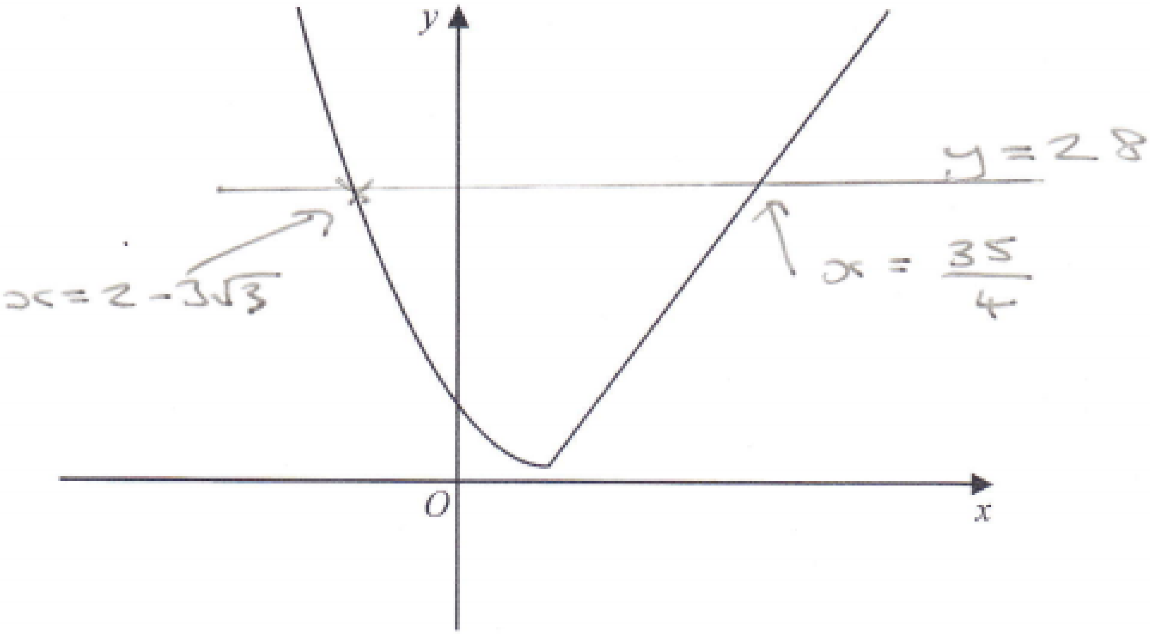


**Figure 4**

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $\mathrm{gg}(0)$.

**(2)**

(b) Find all values of $x$ for which

$$\mathrm{g}(x) > 28$$

**(4)**

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

**(1)**

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

**(3)**

a) $g(0) = (0-2)^2 + 1 = 5$

$g(5) = 4 \times 5 - 7 = 13$

b) $(x-2)^2 + 1 = 28$

$x^2 - 4x + 4 + 1 = 28$

$x^2 - 4x - 23 = 0$

using quadratic formula

ued

$x = 2 + 3\sqrt{3}$ or $x = 2 - 3\sqrt{3}$

from diagram

$4x - 7 = 28$

$4x = 35$

$x = \frac{35}{4}$

$g(x) > 28$ for

$x < 2 - 3\sqrt{3}$ or $x > \frac{35}{4}$

c) h is a one to one function

g is a many to one function

d) $h(x) = (x-2)^2 + 1$

$y = (x-2)^2 + 1$

$y - 1 = (x-2)^2$

$\sqrt{y-1} = x - 2$

$\sqrt{y-1} + 2 = x$

$h^{-1}(x) = \sqrt{x-1} + 2$

$\sqrt{x-1} + 2 = -\frac{1}{2}$

$\sqrt{x-1} = \left(-\frac{5}{2}\right)$

$x - 1 = \frac{25}{4}$

$x = 1 + \frac{25}{4} = \frac{29}{4}$

**11.**

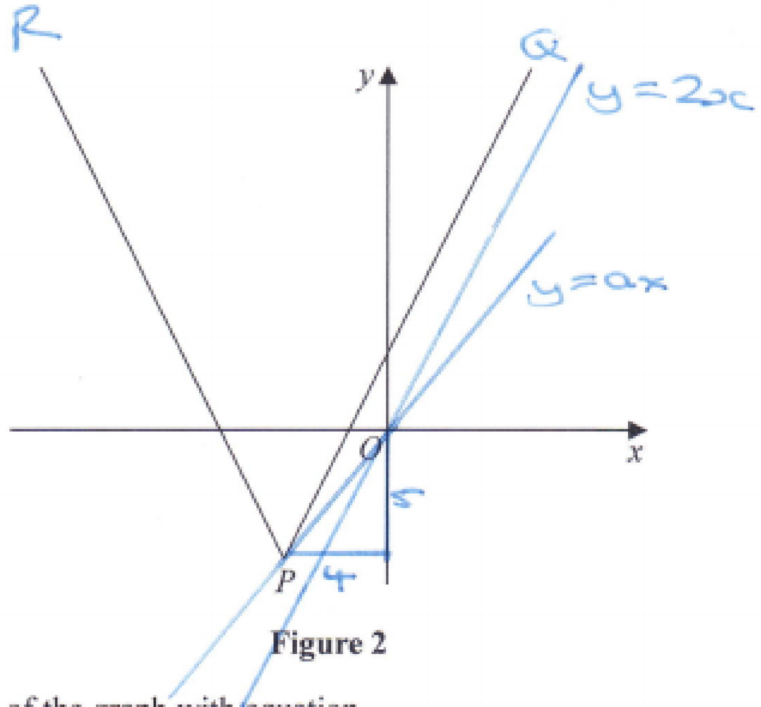


**Figure 2**

Figure 2 shows a sketch of the graph with equation

$$y = 2|x+4| - 5$$

The vertex of the graph is at the point $P$, shown in Figure 2.

(a) Find the coordinates of $P$.

**(2)**

(b) Solve the equation

$$3x + 40 = 2|x+4| - 5$$

**(2)**

A line $l$ has equation $y = ax$, where $a$ is a constant.

Given that $l$ intersects $y = 2|x+4| - 5$ at least once,

(c) find the range of possible values of $a$, writing your answer in set notation.

**(3)**

a) $-2(x+4)-5 = 2(x+4)-5$

$-2x-8-\cancel{5} = 2x+8-\cancel{5}$

$-16 = 4x$

$x = -4$

$y = 2(-4+4)-5 = -5$

$P(-4,-5)$

b) $3x + 40 = 2(x+4) - 5$

$3x + 40 = 2x + 8 - 5$

$x = 8 - 5 - 40$

$x = -37$ (not possible as minimum pt is $P(-4, -5)$)

$3x + 40 = -2(x+4) - 5$

$3x + 40 = -2x - 8 - 5$

$5x = -53$

$x = -10.6$

c) $y = ax$ must go through origin

Gradient $PO = \frac{5}{4} = 1.25$

This line would have 1 solution.
Any gradient less than or equal to $\frac{5}{4}$ has 1 or 2 solutions

Gradient of $PQ$ is 2

Line $y = 2x$ is parallel to $PQ$.
Any line with a gradient $> 2$ would intersect with $PQ$ or $PR$

$\therefore$ $a \leq 1.25$ or $a > 2$

in set notation

$\{a : a \leq 1.25\} \cup \{a : a > 2\}$