YORKSHIRE MATHS TUTOR

1. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

a) $A\begin{pmatrix}2\\3\\-4\end{pmatrix}$ $B\begin{pmatrix}4\\-2\\3\end{pmatrix}$

$C\begin{pmatrix}a\\5\\-2\end{pmatrix}$

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix}4-2\\-2-3\\3--4\end{pmatrix} = \begin{pmatrix}2\\-5\\7\end{pmatrix}$

$D = \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}2\\-5\\7\end{pmatrix} = \begin{pmatrix}6\\-7\\10\end{pmatrix}$

b) $4 = \sqrt{(a-2)^2 + (5-3)^2 + (-2--4)^2}$

$4 = \sqrt{a^2 - 4a + 4 + 4 + 4}$

$4 = \sqrt{a^2 - 4a + 12}$

$16 = a^2 - 4a + 12$

$0 = a^2 - 4a - 4$

$a = \dfrac{--4 \pm \sqrt{(-4)^2 - 4 \times 1 \times -4}}{2}$

$a = 2 + 2\sqrt{2}$ or $\underline{\underline{a = 2 - 2\sqrt{2}}}$

not possible as $a < 0$

2. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$ $\underset{\sim}{a}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ $\underset{\sim}{b}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$ $\underset{\sim}{c}$

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

a) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = (3\underset{\sim}{i} - 3\underset{\sim}{j} - 4\underset{\sim}{k}) - (2\underset{\sim}{i} + 5\underset{\sim}{j} - 6\underset{\sim}{k})$

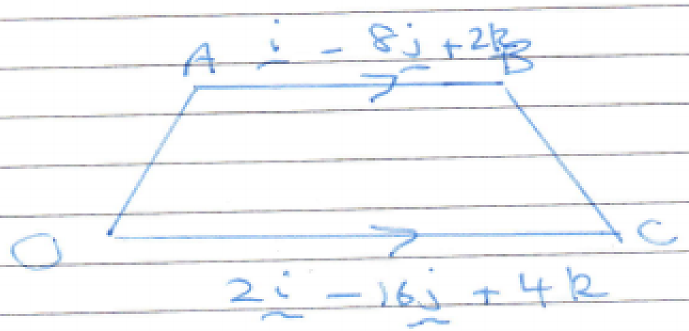
$= \underset{\sim}{i} - 8\underset{\sim}{j} + 2\underset{\sim}{k}$

b) $\overrightarrow{BC} = \underset{\sim}{c} - \underset{\sim}{b} = (2\underset{\sim}{i} - 16\underset{\sim}{j} + 4\underset{\sim}{k}) - (3\underset{\sim}{i} - 3\underset{\sim}{j} - 4\underset{\sim}{k})$

$= -\underset{\sim}{i} - 13\underset{\sim}{j} + 8\underset{\sim}{k}$

$\overrightarrow{OC} = 2\underset{\sim}{i} - 16\underset{\sim}{j} + 4\underset{\sim}{k}$

this is an exact multiple of $\overrightarrow{AB}$ $\therefore$ $\overrightarrow{OC}$ and $\overrightarrow{AB}$ are parallel



but $|\overrightarrow{OC}| = \sqrt{2^2 + 16^2 + 4^2} = 2\sqrt{69}$

$|\overrightarrow{AB}| = \sqrt{1^2 + 8^2 + 2^2} = \sqrt{69}$

as parallel but $|\overrightarrow{OC}|$ is double size of $|\overrightarrow{AB}|$ they are parallel but not same size, so it is a trapezium

4. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

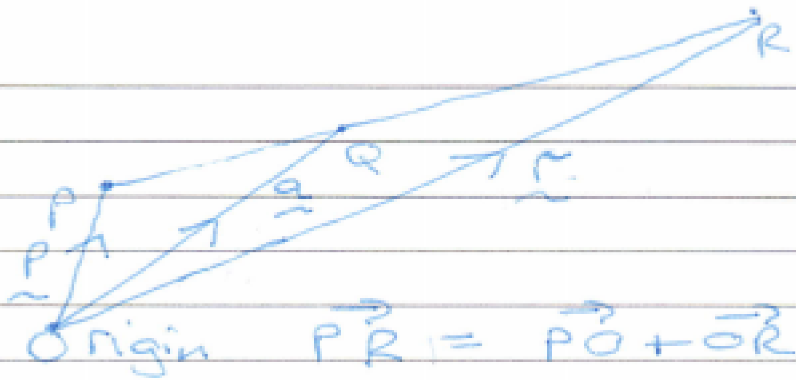
Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)



$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\underset{\sim}{p} + \underset{\sim}{r}$

$\overrightarrow{PQ} = \frac{1}{3}\overrightarrow{PR} = \frac{1}{3}(-\underset{\sim}{p} + \underset{\sim}{r})$

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$\frac{1}{3}(-\underset{\sim}{p} + \underset{\sim}{r}) = -p + q$

$-\frac{1}{3}\underset{\sim}{p} + \underset{\sim}{p} + \frac{1}{3}\underset{\sim}{r} = \underset{\sim}{q}$

$\frac{2}{3}\underset{\sim}{p} + \frac{1}{3}\underset{\sim}{r} = \underset{\sim}{q}$

$\frac{1}{3}(2\underset{\sim}{p} + \underset{\sim}{r}) = \underset{\sim}{q}$

as required

7.

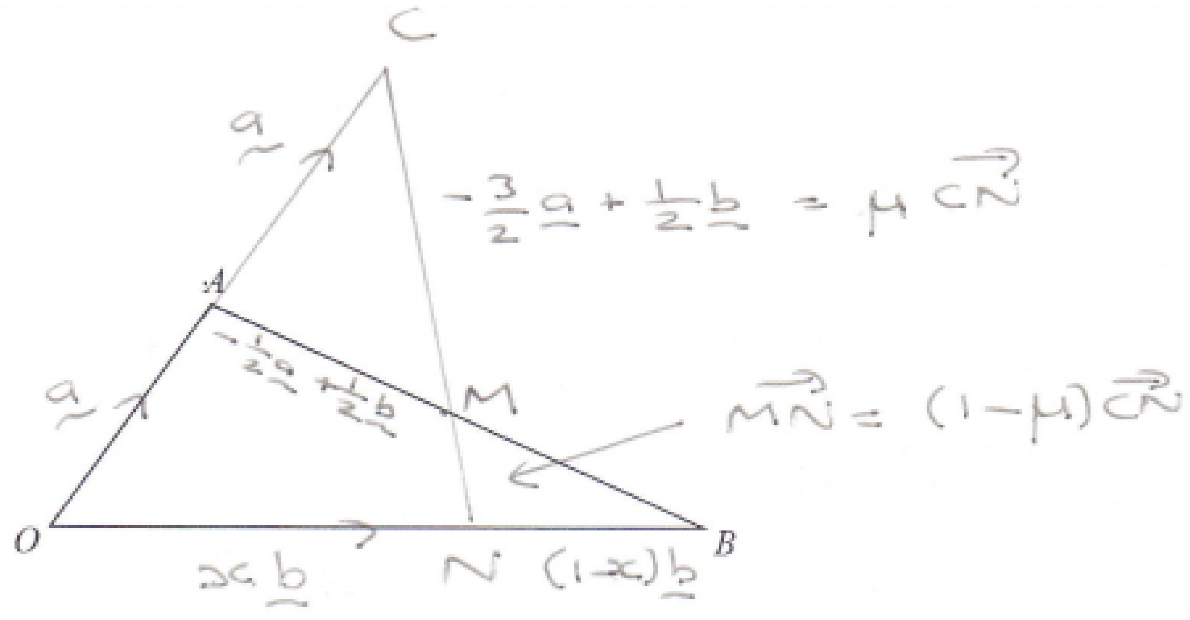


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON:NB = 2:1$

(2)

a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b$

$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB} = -\frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = -a - \frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{CM} = -\frac{3}{2}a + \frac{1}{2}b$

b) let $\overrightarrow{CM} = \mu\overrightarrow{CN}$ $\therefore$ $\overrightarrow{MN} = (1-\mu)\overrightarrow{CN}$

let $\overrightarrow{ON} = \lambda b$ $\therefore$ $\overrightarrow{NB} = (1-\lambda)b$

$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} = 2a + \frac{1}{\mu}\overrightarrow{CM}$

$= 2a + \frac{1}{\mu}\left(-\frac{3}{2}a + \frac{1}{2}b\right)$ 1

Equating coefficients of $\underset{\sim}{a}$
using ①

$\overrightarrow{ON} \quad \left(2 - \frac{3}{2\mu}\right)\underset{\sim}{a} + \frac{1}{\mu} \times \frac{1}{2}\underset{\sim}{b}$ ②

as $\underset{\sim}{a}$ component $= 0$

$2 - \frac{3}{2\mu} = 0$

$2 = \frac{3}{2\mu}$

$4\mu = 3$

$\mu = \frac{3}{4}$

$\therefore$ in ② gives

$\left(2 - \frac{3}{2\mu}\right)\underset{\sim}{a} + \frac{1}{\mu} \times \frac{1}{2}\underset{\sim}{b}$

let $\lambda = \frac{1}{\mu} = \frac{1}{3/4} = \frac{4}{3}$

$\therefore \lambda = \frac{4}{3}$

c) let $\overrightarrow{ON} = x \times \overrightarrow{OB}$

$NB = (1-x)\overrightarrow{OB}$

Equating coefficients of $\underset{\sim}{b}$ for $\overrightarrow{ON}$

$\frac{4}{3} \times \frac{1}{2} = x$

$x = \frac{4}{6} = \frac{2}{3}$

$\therefore ON = \frac{2}{3}\underset{\sim}{b} \quad \overrightarrow{NB} = \frac{1}{3}\underset{\sim}{b}$

$\therefore ON : NB = 2 : 1$