

2. "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

$$m = \sqrt{3} \quad n = \sqrt{12}$$

$$mn = \sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$$

where 6 is rational

5. Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

Assume that for all values of n , where n is an integer, $n^2 + 2 = 4n$ (ie $n^2 + 2$ is divisible by 4)

$$n^2 - 4n + 2 = 0$$

using quadratic formula

$$n = 2 + \sqrt{2} \quad \text{or} \quad n = 2 - \sqrt{2}$$

\therefore as n is not an integer, so this contradicts the original assumption that $n^2 + 2$ is divisible by 4 for all n

\therefore for all n , $n^2 + 2$ is not divisible by 4

6. A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

a) ① $S_n = a + ar + ar^2 + \dots + ar^{n-1}$

② $r S_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$

① - ② gives

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

8. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

Factorising, then assume that there are positive integers p and q such that $(2p+q)(2p-q) = 25$

Factors of 25 are 1 and 25
5 and 5

$$\therefore 2p+q = 25 \quad (1) \text{ First pair}$$

$$2p-q = 1 \quad (2)$$

$$(1) + (2) \quad 4p = 26$$

$$p = 6.5 \text{ in (1) gives } q = 12$$

or

$$2p+q = 1 \quad (1)$$

Use algebra to prove that the square of any natural number is **either** a multiple of 3 or one more than a multiple of 3

(4)

$$p = 6.5, q = 12$$

or

$$2p+q = 5 \quad (1)$$

$$2p-q = 5 \quad (2)$$

$$(1) + (2) \quad 4p = 10$$

$$p = 2.5, q = 0$$

This is a contradiction as there are no integer solutions for any of the combinations

Hence there are no positive integers p and q such that $4p^2 - q^2 = 25$

9. Use algebra to prove that the square of any natural number is **either** a multiple of 3 or one more than a multiple of 3

(4)

Natural numbers are

$$3k, 3k+1, 3k+2$$

$$(3k)^2 = 9k^2 = 3 \times 3k^2$$

a multiple of 3

$$(3k+1)^2 = 9k^2 + 6k + 1$$
$$= 3(3k^2 + 2k) + 1$$

one more than a
multiple of 3

$$(3k+2)^2 = 9k^2 + 12k + 4$$
$$= 9k^2 + 12k + 3 + 1$$
$$= 3(3k^2 + 4k + 1) + 1$$

one more than a
multiple of 3

Each of the 3 cases above is
either a multiple of 3 or
one more than a multiple of 3