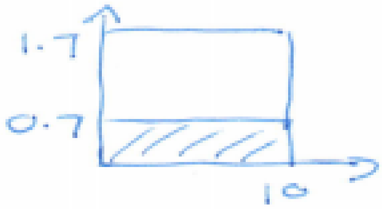
1 Given that

$$\int_0^{10} f(x)\,dx = 7$$



deduce the value of

$$\int_0^{10} \left(f(x) + 1\right) dx$$

up 1 unit

$1.7 \times 10 = 17$

Circle your answer.

[1 mark]

-3 7 8 17

outside the box

6 Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x}\,dx \qquad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom	$\int \frac{1}{x}\,dx = \ln x$
Josh	$\int \frac{1}{x}\,dx = k\ln x$
Floella	$\int \frac{1}{x}\,dx = \ln Ax$
Georgia	$\int \frac{1}{x}\,dx = \ln x + c$

6 (a) (i) Explain what is wrong with Tom's answer.

[1 mark]

No constant of integration

6 (a) (ii) Explain what is wrong with Josh's answer.

[1 mark]

Constant of integration should have been added, not multiplied

6 (b) Explain why Floella and Georgia's answers are equivalent.

[2 marks]

$\ln ab = \ln a + \ln b$ (addition law)

$\therefore \ln Ax = \ln A + \ln x$

and $\ln A$ is a number which we could make c

outside the box

16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\frac{dy}{dx}$

Simplify your answer.

[3 marks]

$u = e^{-x}$ $\quad v = \sin x + \cos x$

$\frac{du}{dx} = -e^{-x}$ $\quad \frac{dv}{dx} = \cos x - \sin x$

$\frac{dy}{dx} = e^{-x}(\cos x - \sin x) - e^{-x}(\sin x + \cos x)$

$= e^{-x}(\cos x - \sin x - \sin x - \cos x)$

$= e^{-x}(-2\sin x)$

$= -2e^{-x}\sin x$

16 (b) Hence, show that

$$\int e^{-x}\sin x \, dx = a e^{-x}(\sin x + \cos x) + c$$

where a is a rational number.

[2 marks]

using part a)

$\int -2e^{-x}\sin x \, dx = e^{-x}(\sin x + \cos x) + c$

$\div$ by -2

$\int e^{-x}\sin x \, dx = -\frac{1}{2}e^{-x}(\sin x + \cos x) + c$

5 Use integration by substitution to show that

$$\int_{-\frac{1}{4}}^{6} x\sqrt{4x+1}\,\mathrm{d}x = \frac{875}{12}$$

Fully justify your answer.

[6 marks]

let $u = 4x + 1 \Rightarrow x = \frac{u-1}{4}$

$\frac{du}{dx} = 4 \Rightarrow dx = \frac{du}{4}$

limits $x = 6$, $u = 25$

$x = -\frac{1}{4}$, $u = 0$

$$\int_0^{25} \frac{u-1}{4} \times u^{\frac{1}{2}} \times \frac{du}{4}$$

$$= \frac{1}{16}\int_0^{25} u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$$

$$= \frac{1}{16}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_0^{25}$$

$$= \frac{1}{16}\left[\left(1250 - \frac{250}{3}\right) - (0-0)\right]$$

$$= \frac{1}{16}\left[\frac{3500}{3}\right] = \frac{875}{12}$$

as required

5 An arithmetic sequence has first term a and common difference d.

The sum of the first 16 terms of the sequence is 260

5 (a) Show that $4a + 30d = 65$

[2 marks]

$S_n = \frac{n}{2}(2a + (n-1)d)$ $n = 16, S_{16} = 260$

$260 = 8(2a + 15d)$

$260 = 16a + 120d$

$\div$ by 4

$65 = 4a + 30d$ (1)

(as required)

5 (b) Given that the sum of the first 60 terms is 315, find the sum of the first 41 terms.

[3 marks]

$S_{60} = 315$

$\therefore 315 = 30(2a + 59d)$

$315 = 60a + 1770d$

$\div$ by 15

$21 = 4a + 118d$ (2)

Solving (1) and (2) simultaneously

$4a + 30d = 65$ (1)

$4a + 118d = 21$ (2)

(2) − (1) $88d = -44$

$d = -0.5$

in (1) $4a + 30 \times -0.5 = 65$

$4a - 15 = 65$

$4a = 80$

$a = 20$

$S_{41} = \frac{41}{2}(2 \times 20 + 40 \times -0.5)$

$= 410$

5 (c) S_n is the sum of the first n terms of the sequence.

Explain why the value you found in part **(b)** is the maximum value of S_n

[2 marks]

1st term = a = 20

2nd term = $a + d$ = 19.5

41st term = $a + 40d$

= $20 + 40 \times -0.5$

= 0

42nd term = $a + 41d$

= $20 + 41 \times -0.5$

= -0.5

As all the terms after the 41st term are negative, maximum value for S_n is when $n = 41$

7 (a) Express $\dfrac{4x+3}{(x-1)^2}$ in the form $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$

[3 marks]

$$\frac{4x+3}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$4x+3 = A(x-1) + B$

let $x = 1$, $7 = B$

Equate coefficients of x, $4 = A$

$$\therefore \frac{4x+3}{(x-1)^2} = \frac{4}{x-1} + \frac{7}{(x-1)^2}$$

7 (b) Show that

$$\int_3^4 \frac{4x+3}{(x-1)^2}\,dx = p + \ln q$$

where p and q are rational numbers.

[5 marks]

$$\int_3^4 \frac{4}{x-1} + 7(x-1)^{-2}\,dx$$

$$= \left[4\ln(x-1) - 7(x-1)^{-1}\right]_3^4$$

$$= \left(4\ln(4-1) - \frac{7}{4-1}\right) - \left(4\ln(3-1) - \frac{7}{3-1}\right)$$

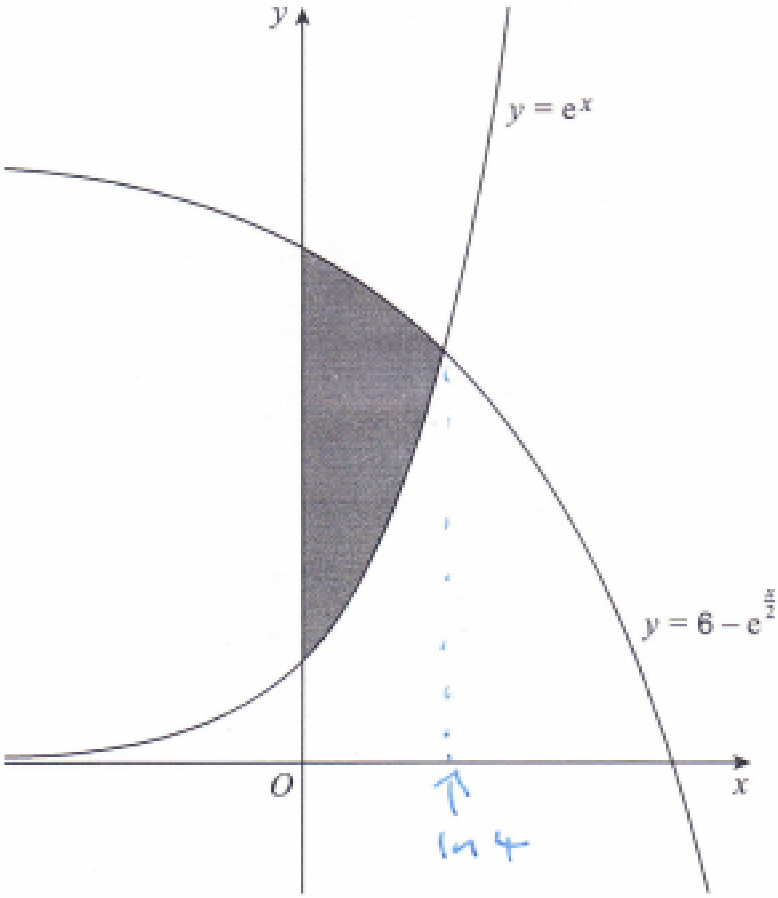
$$= 4\ln 3 - \frac{7}{3} - 4\ln 2 + \frac{7}{2}$$

$$= \ln 3^4 - \ln 2^4 + \frac{7}{6}$$

$$= \ln 81 - \ln 16 + \frac{7}{6}$$

$$= \ln\frac{81}{16} + \frac{7}{6}$$

15 The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.



Show that the exact area of the shaded region is

$$6 \ln 4 - 5$$

Fully justify your answer.

[10 marks]

Lines meet when

$$e^x = 6 - e^{\frac{x}{2}}$$

$$e^x + e^{\frac{x}{2}} - 6 = 0$$

$$(e^{\frac{x}{2}} + 3)(e^{\frac{x}{2}} - 2) = 0$$

Either $e^{\frac{x}{2}} + 3 = 0$ or $e^{\frac{x}{2}} - 2 = 0$

$e^{\frac{x}{2}} = -3$

$\frac{x}{2} = \ln(-3)$ $\qquad \frac{x}{2} = \ln 2$

$x = 2 \ln 2$

impossible $\qquad x = \ln 4$

Shaded area

$$= \int_0^{\ln 4} \left(6 - e^{\frac{x}{2}}\right) - e^x \, dx$$

$$= \left[6x - 2e^{\frac{x}{2}} - e^x \right]_0^{\ln 4}$$

$$= \left(6\ln 4 - 2e^{\frac{\ln 4}{2}} - e^{\ln 4}\right)$$
$$- (0 - 2 - 1)$$

$$= 6\ln 4 - 2e^{\ln 4^{\frac{1}{2}}} - 4$$
$$- 0 + 2 + 1$$
$$= 6\ln 4 - 2 \times 4^{\frac{1}{2}} - 4 + 2 + 1$$
$$= 6\ln 4 - 4 - 4 + 2 + 1$$
$$= 6\ln 4 - 5$$

(as required)