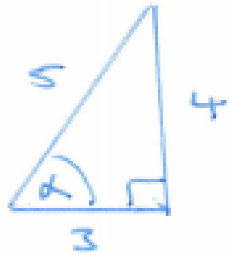
1.



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

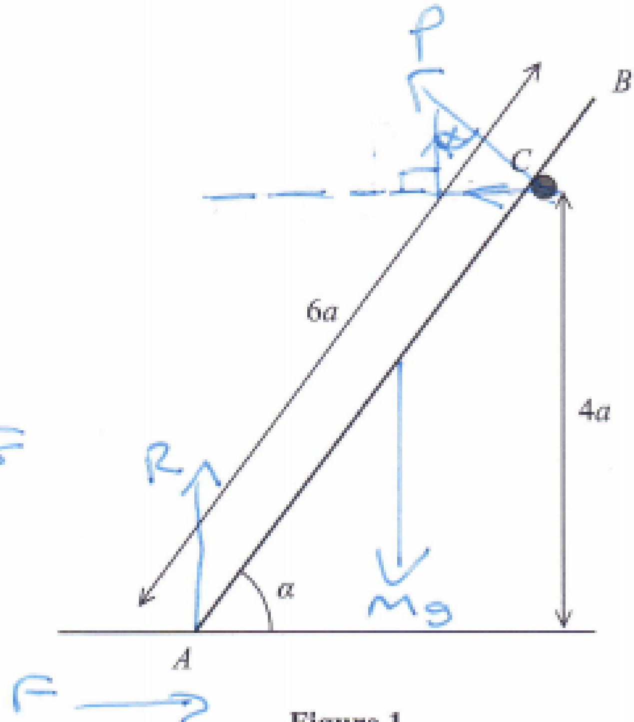
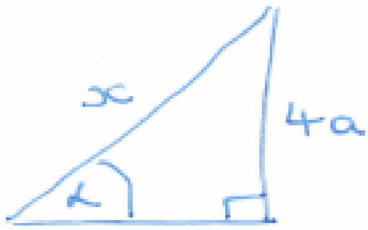


Figure 1



$\sin\alpha = \frac{4a}{x}$

$x = \frac{4a}{\sin\alpha}$

$x = \frac{4a}{\frac{4}{5}}$

$x = 5a$

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

a) $R(\rightarrow)$ $\quad F = P\sin\alpha$ $\quad$ 1

$R(\uparrow)$ $\quad R + P\cos\alpha - Mg = 0$ $\quad$ 2

$M(A)$ $\circlearrowright^{+ve}$ $\quad 3a\cos\alpha \times Mg - 5a \times P = 0$ $\quad$ 3

$$3\cancel{a} \times \frac{3}{5} \times Mg = 5\cancel{a}P$$

$$\frac{9}{25}Mg = P$$

(as required)

b) in 2

$$R = Mg - P\cos\alpha$$

$$R = Mg - \frac{9}{25}Mg \times \frac{3}{5}$$

$$R = Mg - \frac{27}{125}Mg$$

$$R = \frac{98}{125}Mg$$

1 gives $F = P\sin\alpha$

$$F = \frac{9}{25}Mg \times \frac{4}{5}$$

$$F = \frac{36}{125}Mg$$

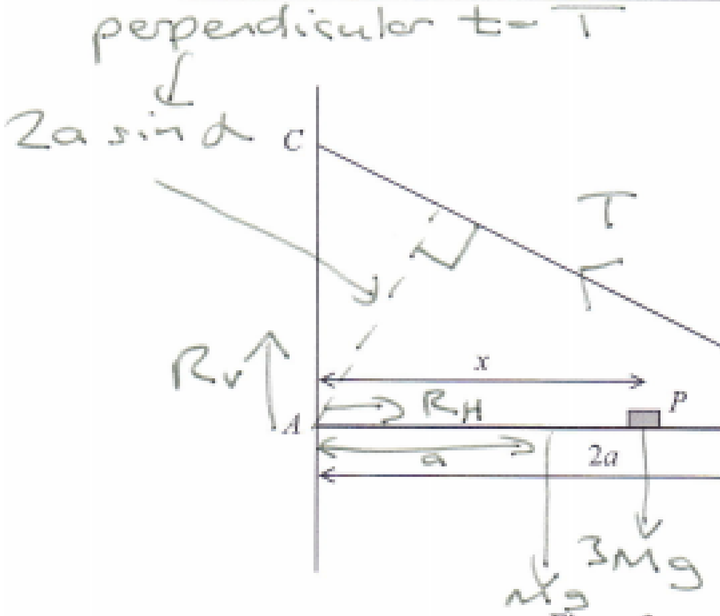
limiting friction

$$F = \mu R$$

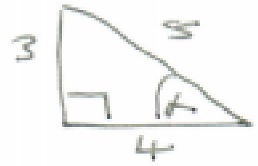
$$\mu = \frac{F}{R} = \frac{\frac{36}{125}Mg}{\frac{98}{125}Mg}$$

$$\mu = \frac{36}{98} = \frac{18}{49}$$

2.



tan α = $\frac{3}{4}$



sin α = $\frac{3}{5}$

cos α = $\frac{4}{5}$

Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

a) $M(A)$ $\circlearrowright$ +ve

$$2a\sin\alpha \times T = a \times Mg + x \times 3Mg$$

$$2a \times \frac{3}{5} \times T = aMg + 3xMg$$

$$\frac{6}{5}a \times T = aMg + 3xMg$$

$$T = \frac{5}{6a}(aMg + 3xMg)$$

$$T = \frac{5Mg(a+3x)}{6a}$$ (as required)

b) R ($\longrightarrow$)

$$R_H = T\cos\alpha$$

$$2\cancel{Mg} = \frac{5\cancel{Mg}(a+3x)}{6a} \times \frac{4}{5}$$

$$\frac{2 \times 6a \times 5}{4 \times 5} = a + 3x$$

$$3a = a + 3x$$

$$2a = 3x$$

$$x = \frac{2}{3}a$$

c) $M(B)$ $\circlearrowleft$ +ve

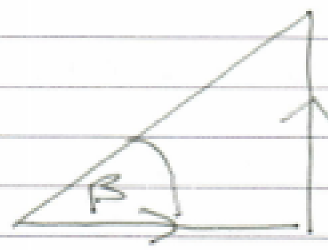
$$2a \times R_V = (2a - x) \times 3Mg + a \times Mg$$

$$2a R_V = (2a - \tfrac{2}{3}a) \times 3Mg - aMg$$

$$2a R_V = 4aMg + aMg$$

$$2a R_V = 5a Mg$$

$$R_V = \frac{5}{2} Mg$$



$R_V = \frac{5}{2} Mg$

β

$R_H = 2Mg$

$$\tan \beta = \frac{\frac{5}{2}}{2} = \frac{5}{4}$$

c) If $T > 5Mg$ rope will break

$$\cancel{5}\frac{Mg(3x+a)}{6a} \leq \cancel{5}\cancel{M}g$$

$$3x + a < 6a$$

$$3x < 5a$$

$$x \leq \frac{5}{3}a$$

For rope to not break

distance from A of the block cannot be more than $\frac{5a}{3}$ from A

6 -

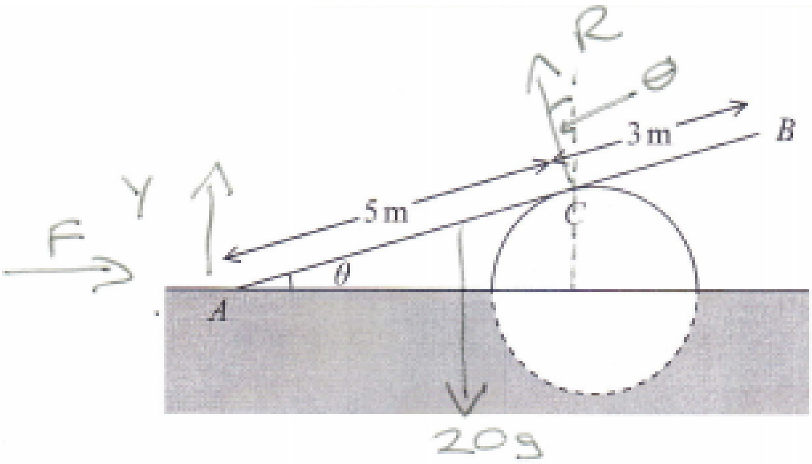


Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at A.

(9)

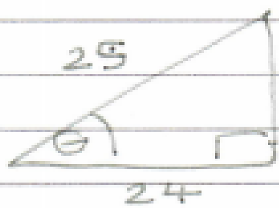
The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

a) Ramp AB is a tangent to the cylindrical drum, only touching at C.



$\sin\theta = \frac{7}{25}$

$\cos\theta = \frac{24}{25}$

b) $m(A) \circlearrowleft +ve$ $\quad 4\cos\theta \times 20g - 5R = 0$ ①

$$\frac{4 \times \frac{24}{25} \times 20g}{5} = R$$

$$R = \frac{384}{5}g = 150.528\text{ N}$$

$R(\uparrow)$ $\quad R\cos\theta + Y = 20g$ ②

$$Y = 20g - 150.528 \times \frac{24}{25}$$

$$Y = 51.49312\text{ N}$$

$R(\rightarrow)$ $\quad F = R\sin\theta$

$$F = 150.528 \times \frac{7}{25}$$

$$F = 42.14784\text{ N}$$

Resultant force at A

$$= \sqrt{51.49312^2 + 42.14784^2}$$

$$= 66.54308$$

$$= 66.5\text{ N (3 sf)}$$

c) If centre of mass closer to A, using ①

$4\cos\theta$ would become smaller, so R would be smaller.

length 4cm would reduce