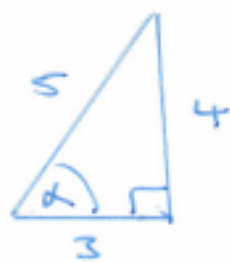


1.



$$\sin \alpha = \frac{4}{5}$$

$$\cos \alpha = \frac{3}{5}$$

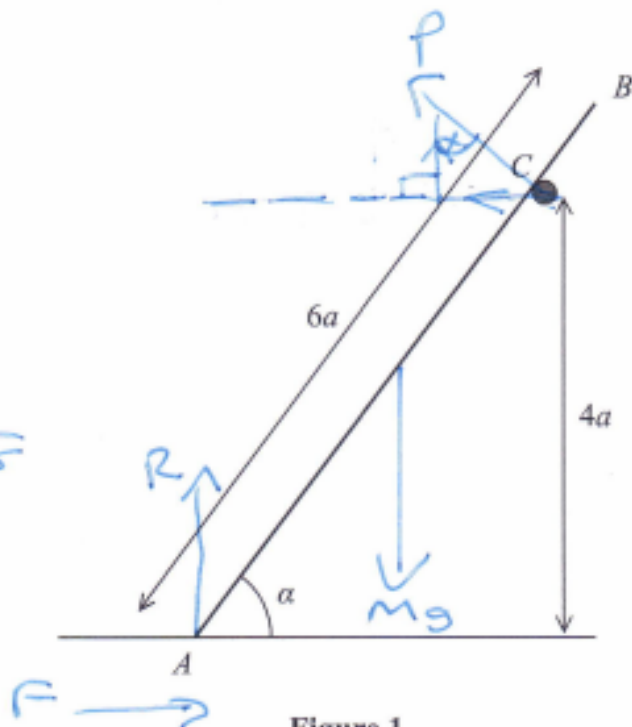


Figure 1



$$\sin \alpha = \frac{4a}{x}$$

$$x = \frac{4a}{\sin \alpha}$$

$$x = \frac{4a}{\frac{4}{5}}$$

$$x = 5a$$

A ladder  $AB$  has mass  $M$  and length  $6a$ .

The end  $A$  of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point  $C$ .

The point  $C$  is at a vertical height  $4a$  above the ground.

The vertical plane containing  $AB$  is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle  $\alpha$ , where  $\sin \alpha = \frac{4}{5}$ , as shown in Figure 1.

The coefficient of friction between the ladder and the ground is  $\mu$ .

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at  $C$  is  $\frac{9Mg}{25}$  (3)

(b) Hence, or otherwise, find the value of  $\mu$ . (7)

$$a) \quad R (\rightarrow) \quad F = P \sin \alpha \quad (1)$$

$$R (\uparrow) \quad R + P \cos \alpha - Mg = 0 \quad (2)$$

$$m(A) \quad (\curvearrowright) \quad 3a \cos \alpha \times Mg - 5a \times P = 0 \quad (3)$$

$$3\alpha \times \frac{3}{5} \times Mg = 5\alpha P$$

$$\frac{9}{25} Mg = P \quad (\text{as required})$$

b) in (2)

$$R = Mg - P \cos \alpha$$

$$R = Mg - \frac{9}{25} Mg \times \frac{4}{5}$$

$$R = Mg - \frac{36}{125} Mg$$

$$R = \frac{98}{125} Mg$$

(1) gives  $F = P \sin \alpha$

$$F = \frac{9}{25} Mg \times \frac{4}{5}$$

$$F = \frac{36}{125} Mg$$

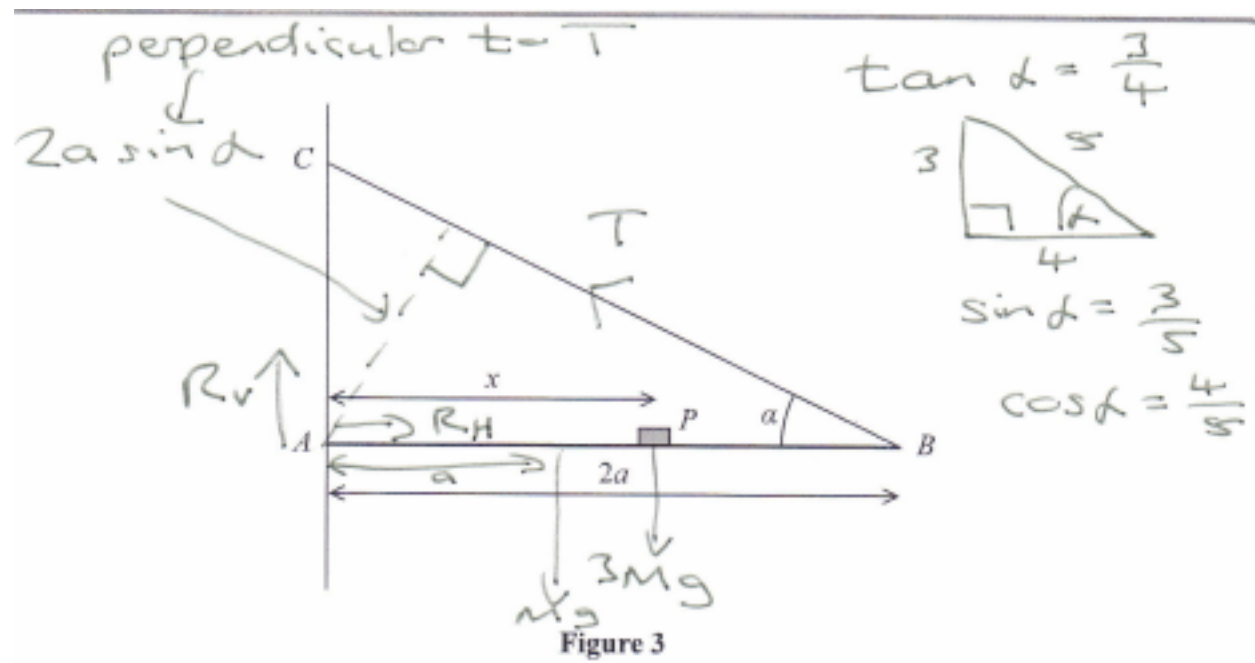
Limiting friction

$$F = \mu R$$

$$\mu = \frac{F}{R} = \frac{\frac{36}{125} Mg}{\frac{98}{125} Mg}$$

$$\mu = \frac{36}{98} = \frac{18}{49}$$

2.



A plank,  $AB$ , of mass  $M$  and length  $2a$ , rests with its end  $A$  against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at  $B$  and the other end is attached to the wall at the point  $C$ , which is vertically above  $A$ .

A small block of mass  $3M$  is placed on the plank at the point  $P$ , where  $AP = x$ . The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is  $\alpha$ , where  $\tan \alpha = \frac{3}{4}$ , as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is  $\frac{5Mg(3x + a)}{6a}$  (3)

The magnitude of the horizontal component of the force exerted on the plank at  $A$  by the wall is  $2Mg$ .

(b) Find  $x$  in terms of  $a$ . (2)

The force exerted on the plank at  $A$  by the wall acts in a direction which makes an angle  $\beta$  with the horizontal.

(c) Find the value of  $\tan \beta$  (5)

The rope will break if the tension in it exceeds  $5Mg$ .

(d) Explain how this will restrict the possible positions of  $P$ . You must justify your answer carefully. (3)

a)  $m(A)$   $\curvearrowright$  +ve

$$2a \sin \alpha \times T = a \times Mg + x \times 3Mg$$

$$2a \times \frac{3}{5} \times T = aMg + 3xMg$$

$$\frac{6}{5} a \times T = aMg + 3xMg$$

$$T = \frac{5}{6a} (aMg + 3xMg)$$

$$T = \frac{5Mg(a + 3x)}{6a}$$

(a is required)

b)  $R$  ( $\rightarrow$ )

$$R_H = T \cos \alpha$$

$$2Mg = \frac{5Mg(a + 3x)}{6a} \times \frac{4}{5}$$

$$\frac{2 \times 6a \times 5}{4 \times 5} = a + 3x$$

$$3a = a + 3x$$

$$2a = 3x$$

$$x = \frac{2}{3} a$$

c)  $m(B)$   $\curvearrowright$  +ve

$$2a \times R_V = (2a - x) \times 3Mg + a \times Mg$$

$$2a R_V = (2a - \frac{2}{3}a) \times 3Mg + aMg$$

$$2a R_V = 4aMg + aMg$$

























































