YORKSHIRE MATHS TUTOR

1. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P
(2)

(b) show that $\mu = \frac{3}{4}$
(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane.
(1)

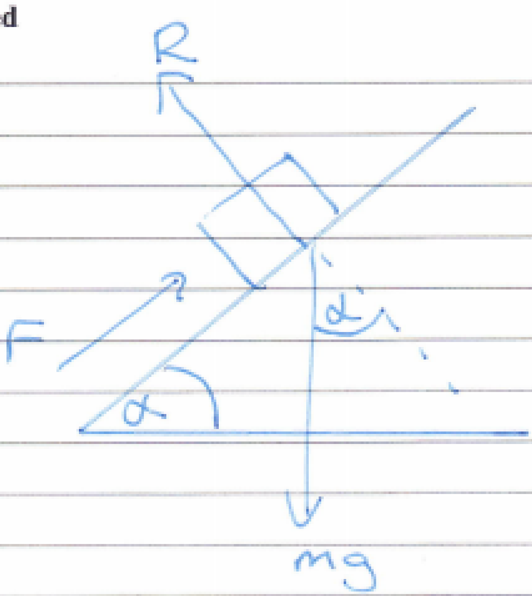
Brick Q is now projected with speed $0.5\,\text{m}\,\text{s}^{-1}$ down a line of greatest slope of the plane.

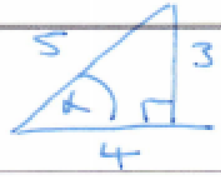
Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer.
(2)

Question 1 continued





$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$

a) $R(\nearrow)$ parallel to plane

$F - mg\sin\alpha = 0$ ①

$F = \frac{3}{5}mg$

$R(\nwarrow)$ perpendicular to plane

$R - mg\cos\alpha = 0$ ②

$R = \frac{4}{5}mg$

Friction $F = \mu R$

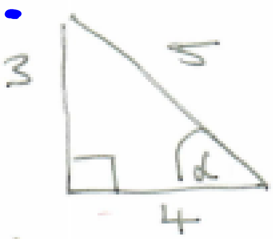
b) $\mu = \dfrac{F}{R} = \dfrac{\frac{3}{5}mg}{\frac{4}{5}mg} = \dfrac{3}{4}$

c) The forces acting on Q will still balance as the "mass" in the F = and R = equations will cancel each other

d) Q would slide with constant speed $0.5\,ms^{-1}$, no acceleration

As there is no resultant Force down the plane (equilibrium), acceleration will be zero

3.



$\sin\alpha = \frac{3}{5}$

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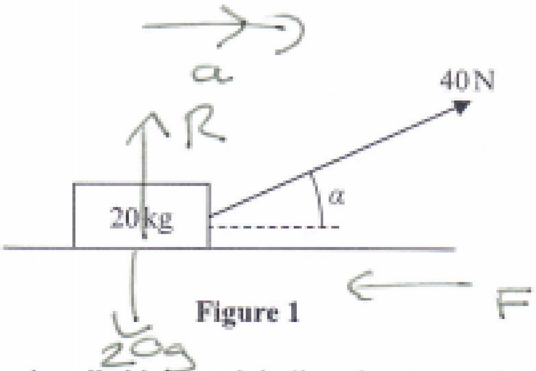


Figure 1

$\mu = 0.14$

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

a) Equation of motion $(\rightarrow)$

$20 \times a = 40\cos\alpha - F$ (1)

Equation of motion $(\uparrow)$

$m \times 0 = R + 40\sin\alpha - 20g$ (2)

Limiting friction $F = \mu R$ (3)

using (2)

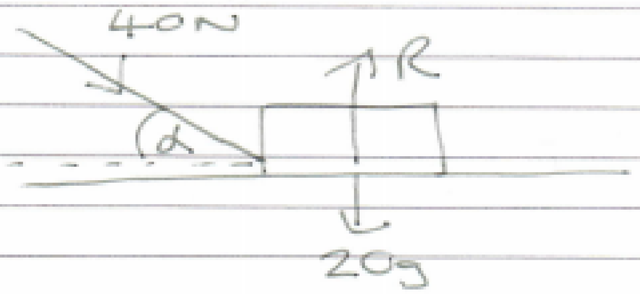
$$R = -40 \times \frac{3}{5} + 20 \times 9.8$$
$$R = 172 \text{ N}$$

in (3) $F = 0.14 \times 172$
$$F = 24.08 \text{ N}$$

in (1) $20a = 40 \times \frac{4}{5} - 24.08$
$$a = \frac{40 \times \frac{4}{5} - 24.08}{20}$$

$$a = 0.396 \text{ ms}^{-2}$$

b)



Only change to part a) is $40 \sin \alpha$ force now downwards instead of upwards

This means (2)
$$R = 40 \times \frac{3}{5} + 20 \times 9.8$$
$$R = 220$$

as R increases, as $F = \mu R$, Frictional force F also increases, which in (1) causes "a" to decrease

6.

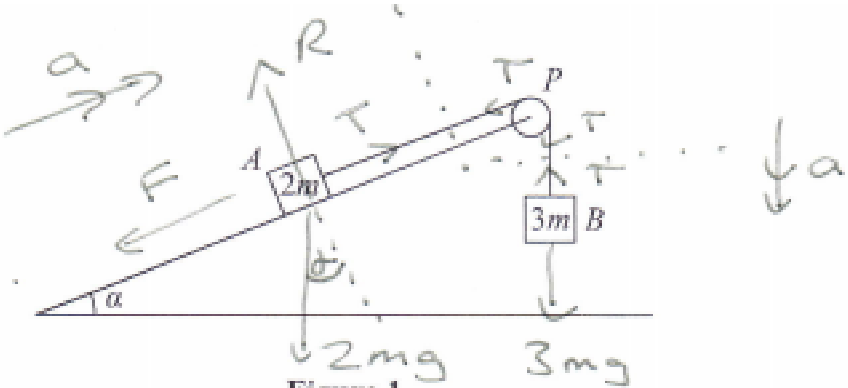


Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan \alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

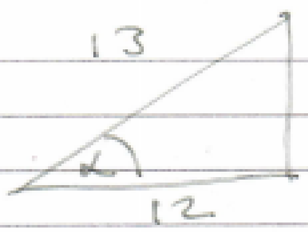
After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)



$\sin \alpha = \frac{5}{13}$

$\cos \alpha = \frac{12}{13}$

a) Equation of motion for A ($\nearrow$) parallel to plane

$$2m \times a = T - F - 2mg\sin\alpha \quad (1)$$

Equation of motion for B ($\downarrow$)

$$3m \times a = 3mg - T \quad (2)$$

Friction $\quad F = \mu R \quad (3)$

Equation of motion for A ($\nwarrow$) perpendicular to pl

$$2m \times 0 = R - 2mg\cos\alpha \quad (4)$$

(4) gives $R = 2mg \times \frac{12}{13} = \frac{24}{13}mg$

in (3) $\quad F = \frac{2}{3} \times \frac{24}{13}mg = \frac{16}{13}mg$

(2) gives $T = 3mg - 3ma$

Using (1) $\quad 2ma = (3mg - 3ma) - \frac{16}{13}mg - 2 \times \frac{5}{13}mg$

$$2ma + 3ma = 3mg - \frac{16}{13}mg - \frac{10}{13}mg$$

$$5\cancel{m}a = \cancel{m}g$$

$$a = \frac{1}{5}g$$

in (2) gives $\quad T = 3mg - 3m \times \frac{1}{5}g$

$$T = \frac{12mg}{5}$$ (as required)

3b) String goes slack ($T = 0N$)
– particle A moves up plane
when stops

Equation of motion parallel to plane
($\swarrow$)

$$2m \times a = 2mg \sin \alpha - F$$

but $F = \frac{16}{13} mg$

if it moves, then $2mg \sin \alpha$ must be $> \frac{16}{13} mg$

$\therefore$ $2mg \times \frac{5}{13}$

$\frac{10mg}{13}$

Force down plane is only $\frac{10}{13} mg$,
so will remain at rest.

c) Allow for friction in the pulley.
Allow for elasticity in the string.