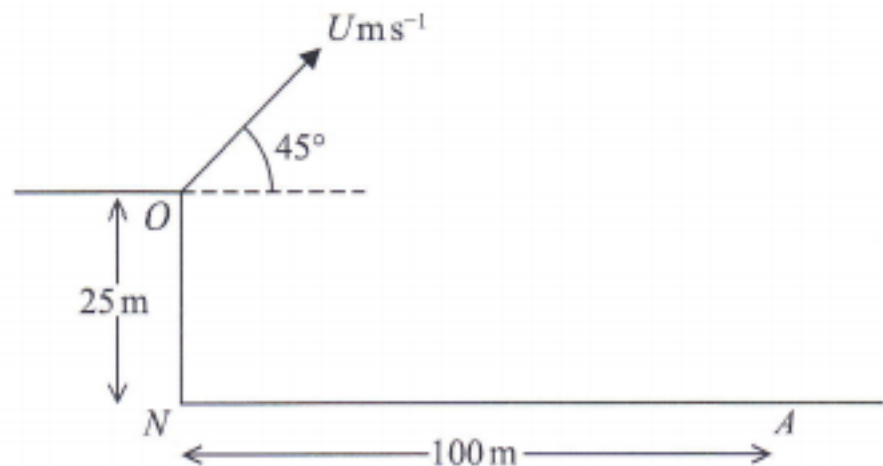


3.



**Figure 2**

A small ball is projected with speed  $U \text{ m s}^{-1}$  from a point  $O$  at the top of a vertical cliff.

The point  $O$  is 25 m vertically above the point  $N$  which is on horizontal ground.

The ball is projected at an angle of  $45^\circ$  above the horizontal.

The ball hits the ground at a point  $A$ , where  $AN = 100 \text{ m}$ , as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that  $U = 28$  (6)

(b) find the greatest height of the ball above the horizontal ground  $NA$ . (3)

In a refinement to the model of the motion of the ball from  $O$  to  $A$ , the effect of air resistance is included.

This refined model is used to find a new value of  $U$ .

(c) How would this new value of  $U$  compare with 28, the value given in part (a)? (1)

(d) State one further refinement to the model that would make the model more realistic. (1)

a) ( $\rightarrow$ ) motion  $100 = U \cos 45^\circ \times t$  ①

( $\uparrow$ ) +ve  $s = -25 \text{ m}$

$u = U \sin 45$

$v =$

$a = -9.8 \text{ m s}^{-2}$

$t =$

$$s = ut + \frac{1}{2} at^2$$

$$-25 = u \sin 45^\circ \times t + \frac{1}{2} (-9.8) t^2 \quad (2)$$

$$(1) \text{ gives } u = \frac{100}{\cos 45^\circ \times t}$$

in (2)

$$-25 = \frac{100}{t} \times \frac{\sin 45^\circ}{\cos 45^\circ} \times t - 4.9 t^2$$

$$-25 = 100 - 4.9 t^2$$

$$4.9 t^2 - 125 = 0$$

$$\begin{cases} \tan 45^\circ = 1 \\ \frac{\sin 45^\circ}{\cos 45^\circ} = \tan 45^\circ \end{cases}$$

$$t = 5.050762723, -5.050762723$$

$$\text{in (1)} \quad u = \frac{100}{5.050762723 \times \cos 45^\circ}$$

$$u = 28 \quad (\text{as required})$$

$$b) \quad s = ?$$

$$u = 28 \sin 45^\circ \text{ m s}^{-1}$$

$$v = 0 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = ?$$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - (28 \sin 45^\circ)^2}{2 \times -9.8}$$

$$s = 20 \text{ m}$$

$$\text{Height above ground} = 20 + 25 = 45 \text{ m}$$

c) To reach point A the new value of  $u$  would have to be larger than 28 to compensate for wind resistance.

d) Account for effects of wind  
or

Use more accurate value for  $g$

or

Account for spin of the ball

5.

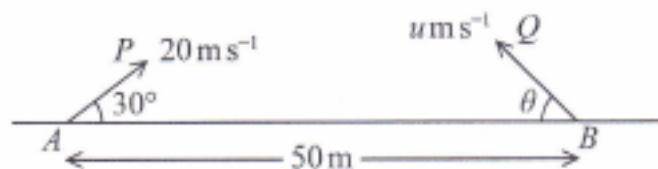


Figure 3

The points  $A$  and  $B$  lie 50 m apart on horizontal ground.

At time  $t = 0$  two small balls,  $P$  and  $Q$ , are projected in the vertical plane containing  $AB$ .

Ball  $P$  is projected from  $A$  with speed  $20 \text{ ms}^{-1}$  at  $30^\circ$  to  $AB$ .

Ball  $Q$  is projected from  $B$  with speed  $u \text{ ms}^{-1}$  at angle  $\theta$  to  $BA$ , as shown in Figure 3.

At time  $t = 2$  seconds,  $P$  and  $Q$  collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of  $P$  at the instant before it collides with  $Q$ .

(6)

(b) Find

(i) the size of angle  $\theta$ ,

(ii) the value of  $u$ .

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

a) A

$$v = u + at$$

$$v = 20 \sin 30^\circ + -9.8 \times 2$$

$$v = -9.6 \text{ ms}^{-1}$$

s

$$u = 20 \sin 30^\circ$$

v

$$a = -g$$

$$t = 2$$

horizontal velocity

$$= 20 \cos 30^\circ = 10\sqrt{3} \text{ ms}^{-1}$$

$$\text{overall velocity} = \sqrt{(-9.6)^2 + (10\sqrt{3})^2}$$

$$= 19.8 \text{ ms}^{-1} \text{ at } t = 2$$

at this time vertical height at collision

$$s = ut + \frac{1}{2}at^2$$

$$s = 20 \sin 30^\circ \times 2 + \frac{1}{2} \times -9.8 \times 2^2$$

$$s = 0.4 \text{ m}$$

b) for A horizontal motion

$$s = 20 \cos 30^\circ \times 2 = 20\sqrt{3} \text{ m}$$

when they collide

$\therefore$  horizontal distance moved by B is  $50 - 20\sqrt{3} \text{ m}$

for B  $u \cos \theta \times 2 = 50 - 20\sqrt{3}$  ①

$$s = 0.4 \text{ m} \leftarrow \text{height of collision}$$

$\uparrow$  +ve  $v = u \sin \theta$

$$a = -g$$

$$t = 2$$

$$s = ut + \frac{1}{2} at^2$$

$$0.4 = 2u \sin \theta + \frac{1}{2} \times -9.8 \times 2^2 \quad \text{②}$$

$$\text{① gives } u = \frac{50 - 20\sqrt{3}}{2 \cos \theta}$$

sub in ② gives

$$0.4 = \frac{2(50 - 20\sqrt{3})}{2} \frac{\sin \theta}{\cos \theta} - 19.6$$

$$\frac{0.4 + 19.6}{(50 - 20\sqrt{3})} = \tan \theta$$

$$\theta = 52.477568^\circ$$

$$\therefore u = \frac{50 - 20\sqrt{3}}{2 \cos(52.477568^\circ)} = 12.6085 \text{ ms}^{-1}$$

(i)  $\theta = 52.5^\circ$  (3 sf)

(ii)  $u = 12.6 \text{ ms}^{-1}$  (3 sf)

c) Wind is a limitation

6.

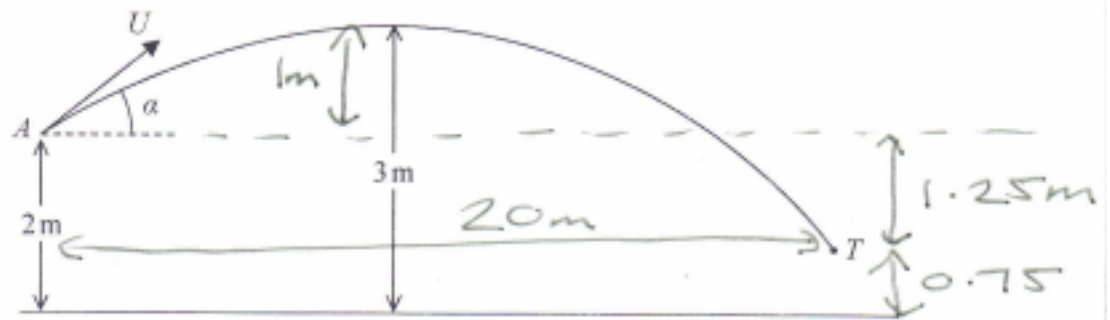


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point  $A$ , the ball is 2 m above horizontal ground and is moving with speed  $U$  at an angle  $\alpha$  above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point  $T$ , as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

Using the model,

- (a) show that  $U^2 = \frac{2g}{\sin^2 \alpha}$ . (2)

The point  $T$  is at a horizontal distance of 20 m from  $A$  and is at a height of 0.75 m above the ground. The ball reaches  $T$  without hitting the ground.

- (b) Find the size of the angle  $\alpha$  (9)

- (c) State one limitation of the model that could affect your answer to part (b). (1)

- (d) Find the time taken for the ball to travel from  $A$  to  $T$ . (3)



a) horizontal motion ( $\rightarrow$ )  
constant horizontal velocity

$$S = U \cos \alpha \times t \quad (1)$$

vertical motion  $\uparrow$

$$S = 1\text{m}$$

$$v = u \sin \alpha$$

$$v = 0 \quad (\text{when highest point reached})$$

$$a = -g$$

$$t =$$

$$v^2 = u^2 + 2as$$

$$0 = u^2 \sin^2 \alpha + 2x - g \times 1$$

$$2g = u^2 \sin^2 \alpha$$

$$\frac{2g}{\sin^2 \alpha} = u^2 \quad (\text{as required})$$

b) horizontal motion ( $\rightarrow$ ) using ①

$$s = u \cos \alpha \times t$$

$$20 = u \cos \alpha \times t \quad \text{①}$$

vertical motion ( $\uparrow$ )

$$s = -1.25 \text{ m (see diagram)}$$

$$u = u \sin \alpha$$

$$v = ?$$

$$a = -g$$

$$t = \frac{20}{u \cos \alpha} \quad (\text{from ①})$$

$$s = ut + \frac{1}{2}at^2$$

$$-1.25 = u \sin \alpha \times \frac{20}{u \cos \alpha} + \frac{1}{2} \times -g \times \left( \frac{20}{u \cos \alpha} \right)^2$$

$$-1.25 = 20 \tan \alpha + \frac{1}{2} \times -g \times \frac{400}{u^2 \cos^2 \alpha}$$

$$-1.25 = 20 \tan \alpha - \frac{1960}{u^2 \cos^2 \alpha}$$

↑ but from a)

$$u^2 = \frac{2g}{\sin^2 \alpha}$$

$$-1.25 = 20 \tan \alpha - 1960$$

$$\frac{2g}{\sin^2 \alpha} \times \cos^2 \alpha$$

$$-1.25 = 20 \tan \alpha - 100 \tan^2 \alpha$$

$$100 \tan^2 \alpha - 20 \tan \alpha - 1.25 = 0$$

$$\tan \alpha = \frac{20 \pm \sqrt{(-20)^2 - 4 \times 100 \times -1.25}}{2 \times 100}$$

$$\tan \alpha = \frac{1}{4} \quad \text{or} \quad \tan \alpha = -\frac{1}{20}$$

$$\alpha = 14.036^\circ$$

$$\alpha = -2.86^\circ$$

↑  
impossible

$$\alpha = 14^\circ \text{ (to nearest degree)}$$

c) limitations could be wind effects or air resistance

d) first find  $u$

$$u^2 = \frac{2g}{\sin^2(14.036^\circ)} = \frac{1666}{5}$$

$$u = \frac{7\sqrt{170}}{5}$$

Horizontal motion ①

$$20 = \frac{7\sqrt{170}}{5} \times t$$

$$t = \frac{20}{\frac{7\sqrt{170}}{5}} = 1.09566 = 1.1 \text{ seconds (1dp)}$$