2.

[In this question position vectors are given relative to a fixed origin O]

At time t seconds, where $t \geqslant 0$, a particle, P, moves so that its velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

a) $\underset{\sim}{v} = \begin{pmatrix} 6t \\ -5t^{\frac{3}{2}} \end{pmatrix}$

Differentiating

$\underset{\sim}{a} = \begin{pmatrix} 6 \\ -7.5t^{\frac{1}{2}} \end{pmatrix}$

at $t = 4$, $\underset{\sim}{a} = \begin{pmatrix} 6 \\ -7.5 \times 4^{\frac{1}{2}} \end{pmatrix}$

$\underset{\sim}{a} = \begin{pmatrix} 6 \\ -15 \end{pmatrix}$

$= 6\underset{\sim}{i} - 15\underset{\sim}{j}$

b) $\underset{\sim}{v} = \begin{pmatrix} 6t \\ -5t^{\frac{3}{2}} \end{pmatrix}$

integrating

$\underset{\sim}{s} = \begin{pmatrix} 3t^2 \\ -2t^{\frac{5}{2}} \end{pmatrix} + c$

at $t = 0$ at $\begin{pmatrix} -20 \\ 20 \end{pmatrix}$

$\Rightarrow \underset{\sim}{s} = \begin{pmatrix} 3t^2 - 20 \\ -2t^{\frac{5}{2}} + 20 \end{pmatrix}$

when $t = 4$, $\underset{\sim}{s} = \begin{pmatrix} 3 \times 4^2 - 20 \\ -2 \times 4^{\frac{5}{2}} + 20 \end{pmatrix} = \begin{pmatrix} 28 \\ -44 \end{pmatrix}$

$= 28\underset{\sim}{i} - 44\underset{\sim}{j}$

4.

At time t seconds, where $t \geqslant 0$, a particle P moves in the x-y plane in such a way that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB.

(6)

$\underset{\sim}{v} = t^{-\frac{1}{2}} \underset{\sim}{i} - 4t \underset{\sim}{j}$

Integrate $\underset{\sim}{v}$ to get $\underset{\sim}{s}$

$\underset{\sim}{s} = 2t^{\frac{1}{2}} \underset{\sim}{i} - \frac{4t^2}{2} \underset{\sim}{j} + c$

$\underset{\sim}{s} = 2t^{\frac{1}{2}} \underset{\sim}{i} - 2t^2 \underset{\sim}{j} + c$

at $t = 1$, $\underset{\sim}{s} = 2 \times 1^{\frac{1}{2}} \underset{\sim}{i} - 2 \times 1^2 \underset{\sim}{j}$
$= 2\underset{\sim}{i} - 2\underset{\sim}{j} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

at $t = 4$, $\underset{\sim}{s} = 2 \times 4^{\frac{1}{2}} \underset{\sim}{i} - 2 \times 4^2 \underset{\sim}{j}$
$= 4\underset{\sim}{i} - 32\underset{\sim}{j} = \begin{pmatrix} 4 \\ -32 \end{pmatrix}$

$|AB| = \sqrt{(-32 - -2)^2 + (4-2)^2}$
$= \sqrt{(-30)^2 + 2^2} = \sqrt{904}$
$= 2\sqrt{226}$ m

6. (i) At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geqslant 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is $5\,\mathrm{m\,s^{-1}}$

a) $\underset{\sim}{a} = (1-4t)\underset{\sim}{i} + (3-t^2)\underset{\sim}{j}$

integrating

$\underset{\sim}{v} = (t - 2t^2)\underset{\sim}{i} + (3t - \frac{1}{3}t^3)\underset{\sim}{j} + c$

at $t = 0$, $\underset{\sim}{v} = 36\underset{\sim}{i}\ \mathrm{ms^{-1}}$

$\underset{\sim}{v} = \begin{pmatrix} t - 2t^2 \\ 3t - \frac{1}{3}t^3 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} 36 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$

$\Rightarrow x = 36,\ y = 0$

$\underset{\sim}{v} = \begin{pmatrix} t - 2t^2 \\ 3t - \frac{1}{3}t^3 \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix}$

at $t = 4$, $v = \begin{pmatrix} 4 - 2\times 4^2 \\ 3\times 4 - \frac{1}{3}\times 4^3 \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix}$

$v = \begin{pmatrix} -28 \\ -\frac{28}{3} \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ -\frac{28}{3} \end{pmatrix}$

$\underset{\sim}{v} = 8\underset{\sim}{i} - \frac{28}{3}\underset{\sim}{j}$

b) perpendicular to $\underset{\sim}{i}$

when $\underset{\sim}{i}$ component of $\underset{\sim}{v}$ is zero

$$t - 2t^2 + 36 = 0$$
$$2t^2 - t + 36 = 0$$
$$(2t - 9)(t + 4) = 0$$
$$2t - 9 = 0 \quad \text{or} \quad t + 4 = 0$$
$$t = 4.5 \qquad t = -4$$

impossible

$\therefore$ $t = 4.5$ seconds

(ii) $\underset{\sim}{r} = \begin{pmatrix} t^2 - t \\ 3t \end{pmatrix}$ $\downarrow$ differentiate

$$\underset{\sim}{v} = \begin{pmatrix} 2t - 1 \\ 3 \end{pmatrix}$$

$$(2t - 1)^2 + 3^2 = 5^2$$

$$4t^2 - 4t + 1 + 9 = 25$$
$$4t^2 - 4t - 15 = 0$$
$$(2t + 3)(2t - 5) = 0$$
$$2t + 3 = 0 \quad \text{or} \quad 2t - 5 = 0$$
$$t = -1.5 \quad \text{or} \quad t = 2.5$$

impossible

$\therefore$ $t = 2.5$ seconds