

2.

[In this question position vectors are given relative to a fixed origin  $O$ ]

At time  $t$  seconds, where  $t \geq 0$ , a particle,  $P$ , moves so that its velocity  $\mathbf{v} \text{ m s}^{-1}$  is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When  $t = 0$ , the position vector of  $P$  is  $(-20\mathbf{i} + 20\mathbf{j})\text{m}$ .

(a) Find the acceleration of  $P$  when  $t = 4$

(3)

(b) Find the position vector of  $P$  when  $t = 4$

(3)

Differentiating

$$\mathbf{v} = \begin{pmatrix} 6t \\ -5t^{\frac{3}{2}} \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} 6 \\ -7.5t^{\frac{1}{2}} \end{pmatrix}$$

at  $t = 4$ ,  $\mathbf{a} = \begin{pmatrix} 6 \\ -7.5 \times 4^{\frac{1}{2}} \end{pmatrix}$

$$\mathbf{a} = \begin{pmatrix} 6 \\ -15 \end{pmatrix}$$

$$= 6\mathbf{i} - 15\mathbf{j}$$

Integrating

$$\mathbf{v} = \begin{pmatrix} 6t \\ -5t^{\frac{3}{2}} \end{pmatrix}$$

$$\mathbf{s} = \begin{pmatrix} 3t^2 \\ -2t^{\frac{5}{2}} \end{pmatrix} + \mathbf{c}$$

at  $t = 0$  at  $\begin{pmatrix} -20 \\ 20 \end{pmatrix}$

$$\Rightarrow \mathbf{s} = \begin{pmatrix} 3t^2 - 20 \\ -2t^{\frac{5}{2}} + 20 \end{pmatrix}$$

when  $t = 4$ ,  $\mathbf{s} = \begin{pmatrix} 3 \times 4^2 - 20 \\ -2 \times 4^{\frac{5}{2}} + 20 \end{pmatrix} = \begin{pmatrix} 28 \\ -44 \end{pmatrix}$

$$= 28\mathbf{i} - 44\mathbf{j}$$

4.

At time  $t$  seconds, where  $t \geq 0$ , a particle  $P$  moves in the  $x$ - $y$  plane in such a way that its velocity  $\mathbf{v}$  m s<sup>-1</sup> is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When  $t = 1$ ,  $P$  is at the point  $A$  and when  $t = 4$ ,  $P$  is at the point  $B$ .

Find the exact distance  $AB$ .

(6)

$$\underline{\mathbf{v}} = t^{-\frac{1}{2}}\underline{\mathbf{i}} - 4t\underline{\mathbf{j}}$$

Integrate  $\underline{\mathbf{v}}$  to get  $\underline{\mathbf{s}}$

$$\underline{\mathbf{s}} = 2t^{\frac{1}{2}}\underline{\mathbf{i}} - \frac{4t^2}{2}\underline{\mathbf{j}} + \mathbf{c}$$

$$\underline{\mathbf{s}} = 2t^{\frac{1}{2}}\underline{\mathbf{i}} - 2t^2\underline{\mathbf{j}} + \mathbf{c}$$

$$\text{at } t=1, \underline{\mathbf{s}} = 2 \times 1^{\frac{1}{2}}\underline{\mathbf{i}} - 2 \times 1^2\underline{\mathbf{j}} \\ = 2\underline{\mathbf{i}} - 2\underline{\mathbf{j}} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\text{at } t=4, \underline{\mathbf{s}} = 2 \times 4^{\frac{1}{2}}\underline{\mathbf{i}} - 2 \times 4^2\underline{\mathbf{j}} \\ = 4\underline{\mathbf{i}} - 32\underline{\mathbf{j}} = \begin{pmatrix} 4 \\ -32 \end{pmatrix}$$

$$|AB| = \sqrt{(-32 - -2)^2 + (4 - 2)^2}$$

$$= \sqrt{(-30)^2 + 2^2} = \sqrt{904}$$

$$= 2\sqrt{226} \text{ m}$$

6.

- (i) At time  $t$  seconds, where  $t \geq 0$ , a particle  $P$  moves so that its acceleration  $\mathbf{a}$   $\text{ms}^{-2}$  is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when  $t = 0$ , the velocity of  $P$  is  $36\mathbf{i}$   $\text{ms}^{-1}$

- (a) Find the velocity of  $P$  when  $t = 4$

(3)

- (b) Find the value of  $t$  at the instant when  $P$  is moving in a direction perpendicular to  $\mathbf{i}$

(3)

- (ii) At time  $t$  seconds, where  $t \geq 0$ , a particle  $Q$  moves so that its position vector  $\mathbf{r}$  metres, relative to a fixed origin  $O$ , is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of  $t$  at the instant when the speed of  $Q$  is  $5$   $\text{ms}^{-1}$

a)  $\underline{\mathbf{a}} = (1 - 4t)\underline{\mathbf{i}} + (3 - t^2)\underline{\mathbf{j}}$   
 integrating  
 $\underline{\mathbf{v}} = (t - 2t^2)\underline{\mathbf{i}} + (3t - \frac{1}{3}t^3)\underline{\mathbf{j}} + \underline{\mathbf{c}}$   
 at  $t = 0$ ,  $\underline{\mathbf{v}} = 36\underline{\mathbf{i}}$   $\text{ms}^{-1}$

$$\underline{\mathbf{v}} = \begin{pmatrix} t - 2t^2 \\ 3t - \frac{1}{3}t^3 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 36 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow x = 36, y = 0$$

$$\underline{\mathbf{v}} = \begin{pmatrix} t - 2t^2 \\ 3t - \frac{1}{3}t^3 \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix}$$

$$\text{at } t = 4, \underline{\mathbf{v}} = \begin{pmatrix} 4 - 2 \times 4^2 \\ 3 \times 4 - \frac{1}{3} \times 4^3 \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix}$$

$$\underline{\mathbf{v}} = \begin{pmatrix} -28 \\ -\frac{28}{3} \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ -\frac{28}{3} \end{pmatrix}$$

$$\underline{\mathbf{v}} = 8\underline{\mathbf{i}} - \frac{28}{3}\underline{\mathbf{j}}$$



























































