1. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

$\underset{\sim}{s}$

$\underset{\sim}{u} = -2\underset{\sim}{i} + 2\underset{\sim}{j}\ \mathrm{ms^{-1}}$ $\qquad \underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$\underset{\sim}{v} =$

$\underset{\sim}{a} = 4\underset{\sim}{i} - 5\underset{\sim}{j}\ \mathrm{ms^{-2}}$ $\qquad \underset{\sim}{v} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$t = 2$

$\underset{\sim}{v} = \begin{pmatrix} -2+8 \\ 2-10 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$

$\underset{\sim}{v} = 6\underset{\sim}{i} - 8\underset{\sim}{j}\ \mathrm{ms^{-1}}$

b) $\underset{\sim}{s} = \lambda\underset{\sim}{i} - 4.5\underset{\sim}{j}$

$\underset{\sim}{u} = -2\underset{\sim}{i} + 2\underset{\sim}{j}$

$\underset{\sim}{v}$

$\underset{\sim}{a} = 4\underset{\sim}{i} - 5\underset{\sim}{j}$

$t = T$

$\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2$

$\begin{pmatrix} \lambda \\ -4.5 \end{pmatrix} = T\begin{pmatrix} -2 \\ 2 \end{pmatrix} + \frac{1}{2}T^2\begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$\underset{\sim}{i}$ components $\lambda = -2T + 2T^2$ (1)

$\underset{\sim}{j}$ components $-4.5 = 2T - 2.5T^2$ (2)

2 gives $1.5T^2 - 2T - 4.5 = 0$

Using calculator to solve quadratic

$T = 1.8$ or $T = -1$

b) as $T > 0$, $T = 1.8$

c) in 1

$\lambda = -2 \times 1.8 + 2 \times 1.8^2$

$\lambda = 2.88$

3.

[In this question **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.]*

A particle P moves with constant acceleration.
At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ — $\underset{\sim}{v} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$
At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\,\mathrm{m}$. — $\underset{\sim}{s} = \begin{pmatrix} 7 \\ -10 \end{pmatrix}$

(a) Show that the magnitude of the acceleration of P is $2.5\,\mathrm{m\,s^{-2}}$ $\quad t = 2$

(4)

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\,\mathrm{m\,s^{-2}}$

At the instant when P reaches the point B, the direction of motion of P is north east. $= \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) Find the time it takes for P to travel from A to B.

(4)

$\underset{\sim}{s} = \begin{pmatrix} 7 \\ -10 \end{pmatrix}$

$\underset{\sim}{u} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\underset{\sim}{v}$

$\underset{\sim}{a} = \begin{pmatrix} x \\ y \end{pmatrix}$

$t = 2$

$\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2$

$\begin{pmatrix} 7 \\ -10 \end{pmatrix} = 2\begin{pmatrix} 2 \\ -3 \end{pmatrix} + \frac{1}{2} \times 2^2 \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} 7 \\ -10 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} 2x \\ 2y \end{pmatrix}$

$7 - 4 = 2x$

$3 = 2x$

$x = 1.5$

$-10 = -6 + 2y$

$-4 = 2y$

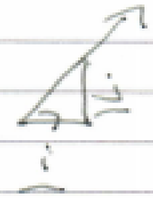
$y = -2 \quad \Rightarrow \quad \underset{\sim}{a} = \begin{pmatrix} 1.5 \\ -2 \end{pmatrix}$

$|\underset{\sim}{a}| = \sqrt{1.5^2 + (-2)^2} = 2.5\,\mathrm{m\,s^{-2}}$

as required

b) $\underset{\sim}{s}$

$\underset{\sim}{u} = \begin{pmatrix}5\\-7\end{pmatrix}$

$\underset{\sim}{v} = \lambda\begin{pmatrix}1\\1\end{pmatrix}$ → 

NE direction

$\underset{\sim}{a} = \begin{pmatrix}4\\8.8\end{pmatrix}$

$t = ?$

at point a, velocity (from a))

$\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix} + 2\begin{pmatrix}1.5\\-2\end{pmatrix} = \begin{pmatrix}2+3\\-3-4\end{pmatrix} = \begin{pmatrix}5\\-7\end{pmatrix}$$

this means $\underset{\sim}{u} = \begin{pmatrix}5\\-7\end{pmatrix}$ for $\overrightarrow{AB}$

$\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$$\lambda\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}5\\-7\end{pmatrix} + \begin{pmatrix}4\\8.8\end{pmatrix}t$$

$\lambda = 5 + 4t$ ①

$\lambda = -7 + 8.8t$ ②

② − ① $0 = -12 + 4.8t$

$t = \dfrac{12}{4.8} = 2.5$ seconds

4. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

a) $\underset{\sim}{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\underset{\sim}{u} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ at $t = 0$

$\underset{\sim}{v} = \lambda \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$t = T$

$\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a} \times t$

$\lambda \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + T \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\underset{\sim}{i}$'s $\quad 3\lambda = -1 + 2T \quad (1) \quad \times 3$

$\underset{\sim}{j}$'s $\quad -4\lambda = 4 - 3T \quad (2) \quad \times 2$

$9\lambda = -3 + 6T \quad (1)$

$\underline{-8\lambda = 8 - 6T} \quad (2)$

(1) + (2) $\quad \lambda = 5$

in (1) $\quad 2T = 1 + 3 \times \lambda$

$2T = 1 + 3 \times 5$

$2T = 16$

$T = 8$ seconds

b) $t = 0$ at point A

$\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2 \qquad t = 4$

$\underset{\sim}{s} = 4\begin{pmatrix} -1 \\ 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 \\ -3 \end{pmatrix} \times 4^2$

$\underset{\sim}{s} = \begin{pmatrix} -4 \\ 16 \end{pmatrix} + \begin{pmatrix} 16 \\ -24 \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$

$|AB| = \sqrt{12^2 + (-8)^2} = 4\sqrt{13}$ m

$= 14.4$ m (3 sf)