1. Tessa owns a small clothes shop in a seaside town. She records the weekly sales figures, £w, and the average weekly temperature, t °C, for 8 weeks during the summer.
The product moment correlation coefficient for these data is −0.915

(a) Stating your hypotheses clearly and using a 5% level of significance, test whether or not the correlation between sales figures and average weekly temperature is negative.
(3)

(b) Suggest a possible reason for this correlation.
(1)

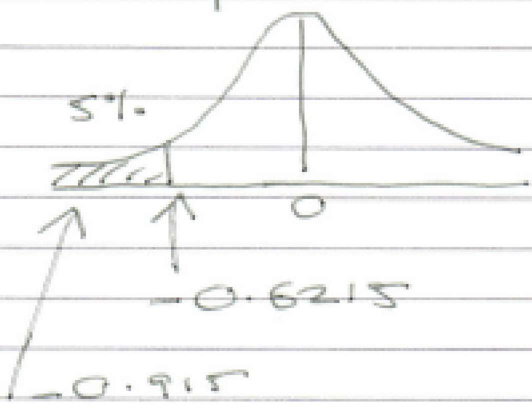
Tessa suggests that a linear regression model could be used to model these data.

(c) State, giving a reason, whether or not the correlation coefficient is consistent with Tessa's suggestion.
(1)

(d) State, giving a reason, which variable would be the explanatory variable.
(1)

a) PMCC = −0.915 ← negative correlation

$H_0: \rho = 0 \quad H_1: \rho < 0$



Sample level = 8 weeks
From table
Sample Level = 8
Level = 0.05
$\rho = \pm 0.6215$

As −0.915 < −0.6215 it is in the critical region so reject H_0
This mean we accept H_1, there is evidence of a negative correlation between w and t.

b) As temperature increases, people spend more time on beach instead of shopping in the town.

c) $r = -1$ is strong negative correlation. As PMCC $= -0.915$ this is close to -1 so consistent.

d) For every degree of temperature rise, sales drop by £171 per week.

5.

Barbara is investigating the relationship between average income (GDP per capita), x US dollars and average annual carbon dioxide (CO_2) emissions, y tonnes, for different countries.

She takes a random sample of 24 countries and finds the product moment correlation coefficient between average annual CO_2 emissions and average income to be 0.446

(a) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not the product moment correlation coefficient for all countries is greater than zero.

(3)

Barbara believes that a non-linear model would be a better fit to the data.
She codes the data using the coding $m = \log_{10} x$ and $c = \log_{10} y$ and obtains the model $c = -1.82 + 0.89m$

The product moment correlation coefficient between c and m is found to be 0.882

(b) Explain how this value supports Barbara's belief.

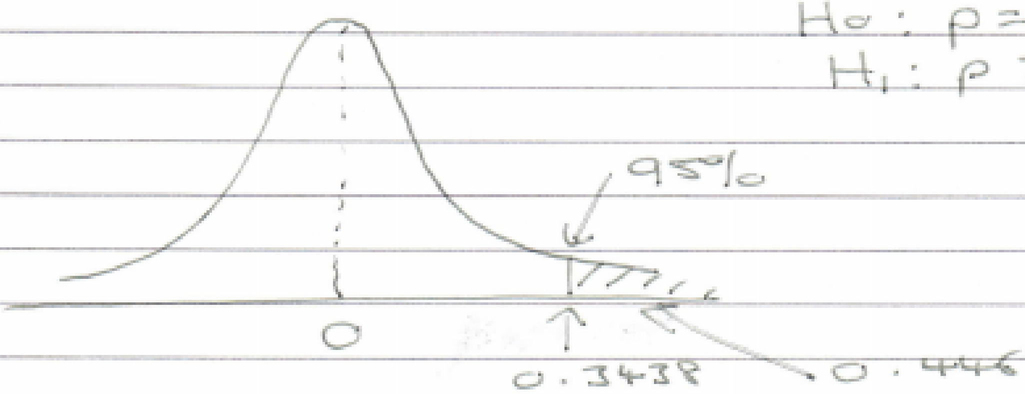
(1)

(c) Show that the relationship between y and x can be written in the form $y = ax^n$ where a and n are constants to be found.

(5)

a) $n = 24$ PMCC $= 0.446$

$H_0 : \rho = 0$
$H_1 : \rho > 0$



From PMCC table $n = 24$, 0.05
PMCC $= 0.3438$

From the diagram
0.446 is in the critical region
∴ REJECT H_0
So we accept H_1, $\rho > 0$,
the coefficient is greater than zero

b) $m = 0.882$ which is close to 1.
The PMCC is between -1 (strong negative correlation) and $+1$ (strong

positive correlation). As value close to 1 it mean there is a linear correlation between m and c.

c) $m = \log_{10} x \qquad c = \log_{10} y$

$$c = -1.82 + 0.89m$$

$$\log_{10} y = -1.82 + 0.89 \log_{10} x$$

$$\log_{10} y - 0.89 \log_{10} x = -1.82$$
$$\log_{10} y - \log_{10} x^{0.89} = -1.82$$

$$\log_{10} \frac{y}{x^{0.89}} = -1.82$$

$$\frac{y}{x^{0.89}} = 10^{-1.82}$$

$$y = 10^{-1.82} x^{0.89}$$

where $a = 10^{-1.82} = 0.0151$ (3sf)
$n = 0.89$

7.

The lifetime, L hours, of a battery has a normal distribution with mean 18 hours and standard deviation 4 hours.

Alice's calculator requires 4 batteries and will stop working when any one battery reaches the end of its lifetime.

(a) Find the probability that a randomly selected battery will last for longer than 16 hours. (1)

At the start of her exams Alice put 4 new batteries in her calculator.
She has used her calculator for 16 hours, but has another 4 hours of exams to sit.

(b) Find the probability that her calculator will not stop working for Alice's remaining exams. (5)

Alice only has 2 new batteries so, after the first 16 hours of her exams, although her calculator is still working, she randomly selects 2 of the batteries from her calculator and replaces these with the 2 new batteries.

(c) Show that the probability that her calculator will not stop working for the remainder of her exams is 0.199 to 3 significant figures. (3)

After her exams, Alice believed that the lifetime of the batteries was more than 18 hours. She took a random sample of 20 of these batteries and found that their mean lifetime was 19.2 hours.

(d) Stating your hypotheses clearly and using a 5% level of significance, test Alice's belief. (5)

a) $L \sim N(18, 4^2)$

$P(L > 16) = 1 - P(L \leq 16)$

Menu 7
2: Normal CD
Lower : 0
Upper : 16
$\sigma = 4$
$\mu = 18$

$\rightarrow p = 0.30853$

$1 - 0.30853$
$= 0.69146$

b) $P(L > 20 \mid P(L > 16)$

$P(L > 20) = P(L \leq 20)$

Menu 7
2: Normal CD
Lower : 0
Upper : 20
σ : 4
μ : 18

$\rightarrow p = 0.691459$

$1 - 0.691459$
$= 0.30854$

$$P(\underset{\text{OK}}{\text{one battery}}) = \frac{P(L > 20)}{P(L > 16)}$$

$$= \frac{0.30854}{0.69146} = 0.4462166$$

$$P(\text{all 4 work OK}) = (0.4462166)^4$$

$$= 0.039644$$

c) For new batteries, they only have to last for 4 hours

$$P(L > 4) = 1 - P(x < 4)$$
$$= 1 - 0.00022923$$
$$= 0.9997707$$

$p = 0.00022923$

Menu 7
2: Normal CD
Lower : 0
Upper : 4
$\sigma = 4$
$\mu = 18$

For end of exams

$$\underbrace{(0.9997707)^2}_{\text{2 new batteries}} \times \underbrace{(0.4462166)^2}_{\text{2 old batteries (part b)}}$$

$$= 0.19901$$
$$= 0.199 \text{ (3sf) as required}$$

d) Sample mean $\bar{L} \sim N(\mu, \frac{\sigma^2}{n})$

$H_0: \mu = 18 \qquad H_1: \mu > 18$

$L \sim N(18, 4^2)$

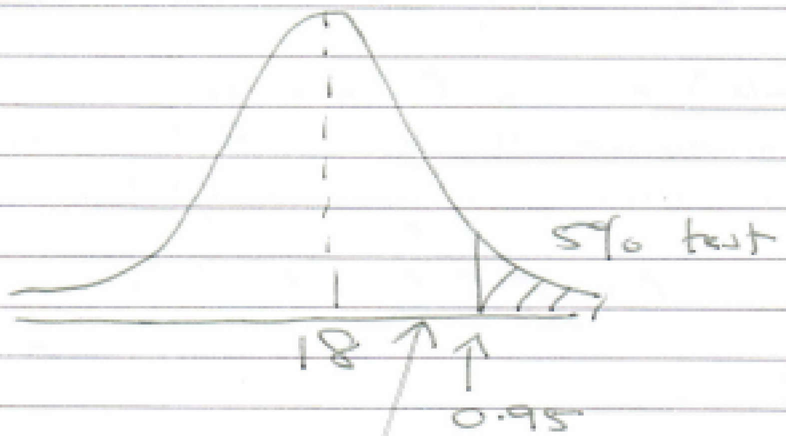
Sample mean $\bar{L} \sim N(18, \frac{4^2}{20})$

$\bar{L} \sim N(18, (\sqrt{\frac{4^2}{20}})^2)$

$\sigma = 0.8944$

Menu 7
Normal CD
Lower : 0
Upper : 19.2
σ : 0.8944
μ : 18

$\rightarrow p = 0.9101437$



test $p = 0.9101437$

Test is not in critical region, so we accept H_0. We reject Alice's belief that the lifetime of the batteries was more than 18 hours.