

1. Helen believes that the random variable  $C$ , representing cloud cover from the large data set, can be modelled by a discrete uniform distribution.

(a) Write down the probability distribution for  $C$ .

(2)

(b) Using this model, find the probability that cloud cover is less than 50%

(1)

Helen used all the data from the large data set for Hurn in 2015 and found that the proportion of days with cloud cover of less than 50% was 0.315

(c) Comment on the suitability of Helen's model in the light of this information.

(1)

(d) Suggest an appropriate refinement to Helen's model.

(1)

a) Daily mean cloud cover (page 12 in text book) is measured in 'oktas' or eighths of the sky covered.

$C$	0	1	2	3	4	5	6	7	8
$P(C=c)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

b) less than 50%  
 $P(C < 4) = \frac{4}{9}$

c) The probability 0.315 is lower than the expected value  $\frac{4}{9}$  (0.4) which suggests the model is not good.

d) Cloud cover will vary from month to month and season to season, so a non-uniform distribution may be a better model.

In an experiment a group of children each repeatedly throw a dart at a target. For each child, the random variable  $H$  represents the number of times the dart hits the target in the first 10 throws.

Peta models  $H$  as  $B(10, 0.1)$

- (a) State two assumptions Peta needs to make to use her model. (2)
- (b) Using Peta's model, find  $P(H \geq 4)$  (1)

For each child the random variable  $F$  represents the number of the throw on which the dart first hits the target.

Using Peta's assumptions about this experiment,

- (c) find  $P(F = 5)$  (2)

Thomas assumes that in this experiment no child will need more than 10 throws for the dart to hit the target for the first time. He models  $P(F = n)$  as

$$P(F = n) = 0.01 + (n - 1) \times \alpha$$

where  $\alpha$  is a constant.

- (d) Find the value of  $\alpha$  (4)
- (e) Using Thomas' model, find  $P(F = 5)$  (1)
- (f) Explain how Peta's and Thomas' models differ in describing the probability that a dart hits the target in this experiment. (1)

a)  $H \sim B(10, 0.1)$

- 1) Probability of each child hitting the target is the same
- 2) Dart throws are independent of each other.

b)  $P(H \geq 4) = 1 - P(X \leq 3)$

Menu T      2: Variable

↓       $X = 3$

1: Binomial CD       $n = 10$

$P = 0.1$

$P(X \leq 3) = 0.987204$

$$= 1 - 0.987204$$

$$= 0.012795$$

c)  $P(F=5) = 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.1$

4 misses                      1st hit

$$= 0.06561$$

d)  $n \quad P(F=n)$

1       $0.01 + (1-1)d = 0.01$

2       $0.01 + (2-1)d = 0.01 + d$

3       $0.01 + (3-1)d = 0.01 + 2d$

...

10      $0.01 + (10-1)d = 0.01 + 9d$

$$\therefore (10 \times 0.01) + d + 2d + 3d + 4d + 5d + 6d + 7d + 8d + 9d = 1$$

$$0.1 + 45d = 1$$

$$45d = 0.9$$

$$d = \frac{1}{50} = 0.02$$

e)  $P(F=5) = 0.01 + (5-1) \times 0.02$

$$= 0.09$$

f) Peter's model assumes probability of each dart hitting is constant.

Thomas's assumes probability of hitting increases with each throw.

3. The discrete random variable  $D$  has the following probability distribution

$d$	10	20	30	40	50
$P(D=d)$	$\frac{k}{10}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{40}$	$\frac{k}{50}$

where  $k$  is a constant.

(a) Show that the value of  $k$  is  $\frac{600}{137}$  (2)

The random variables  $D_1$  and  $D_2$  are independent and each have the same distribution as  $D$ .

(b) Find  $P(D_1 + D_2 = 80)$   
Give your answer to 3 significant figures. (3)

A single observation of  $D$  is made.

The value obtained,  $d$ , is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral  $Q$

(c) Find the exact probability that the smallest angle of  $Q$  is more than  $50^\circ$  (5)

$$4a) k \left( \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} + \frac{1}{50} \right) = 1$$

$$\frac{137}{600} k = 1$$

$$k = \frac{600}{137} \text{ (as required)}$$

b) For  $D_1 + D_2 = 80$

$D_1 = 30 \quad D_2 = 50$   
or  $D_1 = 50 \quad D_2 = 30$   
or  $D_1 = 40 \quad D_2 = 40$

$$= \frac{k}{30} \times \frac{k}{50} + \frac{k}{50} \times \frac{k}{30} + \frac{k}{40} \times \frac{k}{40}$$

$$= k^2 \left( \frac{1}{1500} + \frac{1}{1500} + \frac{1}{1600} \right)$$

$$= \left( \frac{600}{137} \right)^2 \left( \frac{1}{1500} + \frac{1}{1500} + \frac{1}{1600} \right)$$

$$= \frac{705}{18769} = 0.03756 = 0.0376 \text{ (3 sf)}$$

























































