2 The graph of $y = 5^x$ is transformed by a stretch in the y-direction, scale factor 5

State the equation of the transformed graph.

Circle your answer.

[1 mark]

$y = 5 \times 5^x$ $\qquad$ $y = 5^{\frac{x}{5}}$ $\qquad$ $y = \frac{1}{5} \times 5^x$ $\qquad$ $y = 5^{5x}$

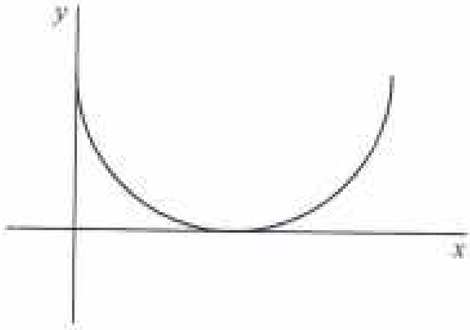
scale factor $\times 5$

$\therefore$ $y = 5 \times 5^x$

3 Determine which one of these graphs does **not** represent y as a function of x.

Tick (✓) **one** box.

[1 mark]



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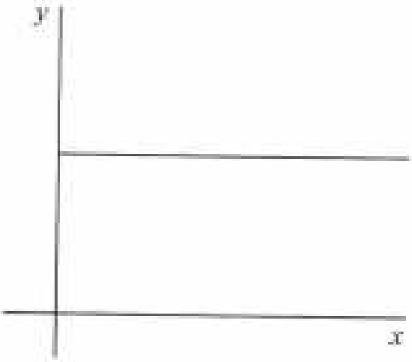

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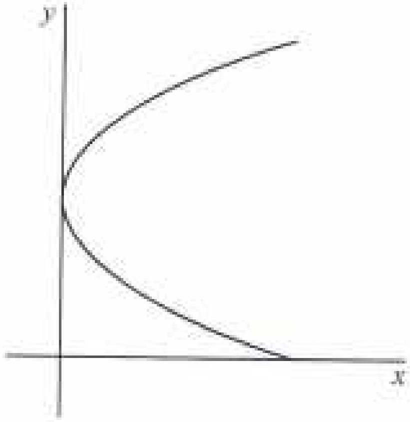
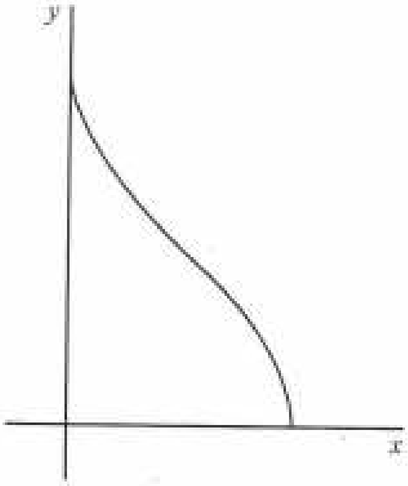


$f(x) = y^2 \ldots$

☐ ✓



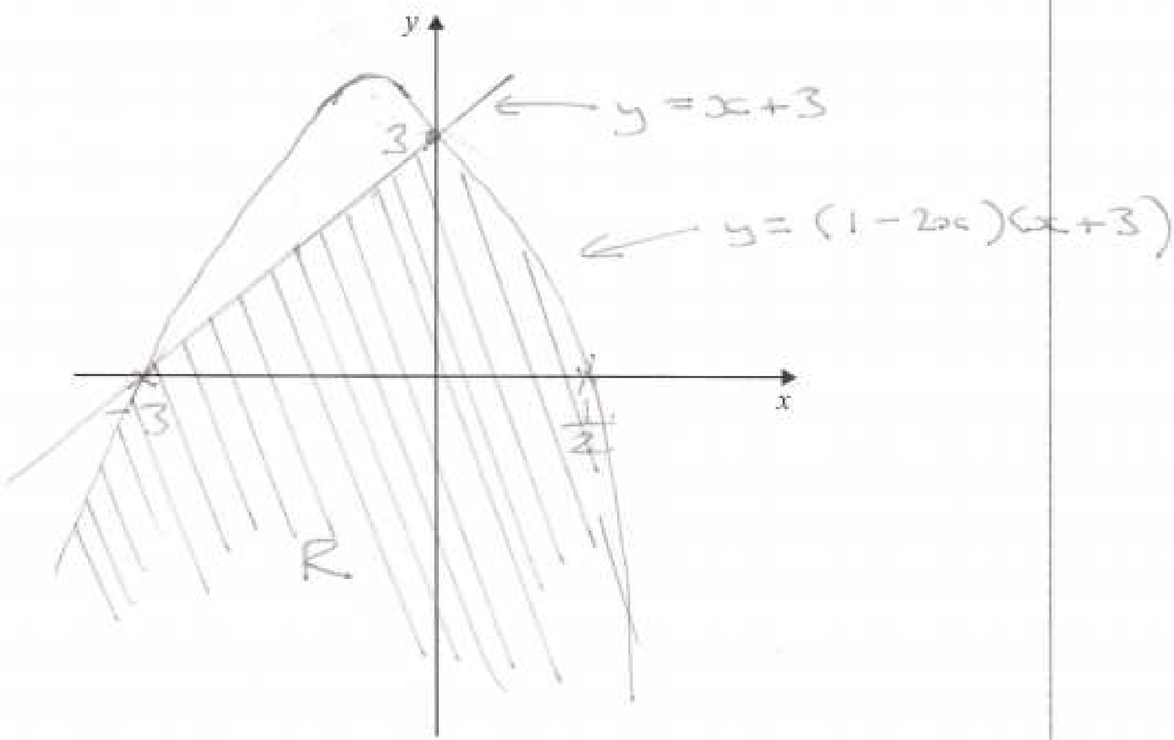
☐

4 Sketch the region defined by the inequalities

$$y \leq (1 - 2x)(x + 3) \text{ and } y - x \leq 3$$

Clearly indicate your region by shading it in and labelling it R.

[3 marks]



Turn over for the next question

$y = (1-2x)(x+3)$

critical values

$x = \frac{1}{2}$, $x = -3$

$y = x+3$

outside the box

4 $p(x) = 4x^3 - 15x^2 - 48x - 36$

4 (a) Use the factor theorem to prove that $x - 6$ is a factor of $p(x)$.

[2 marks]

$p(6) = 4 \times 6^3 - 15 \times 6^2 - 48 \times 6 - 36 = 0$

as $p(6) = 0$

$x - 6$ is a factor of $p(x)$

4 (b) (i) Prove that the graph of $y = p(x)$ intersects the x-axis at exactly one point.

[4 marks]

$$
\begin{array}{r l}
 & 4x^2 + 9x + 6 \\
x - 6 & \overline{)\,4x^3 - 15x^2 - 48x - 36} \\
 & -\ \underline{4x^3 - 24x^2} \\
 & 9x^2 - 48x \\
 & -\ \underline{9x^2 - 54x} \\
 & 6x - 36 \\
 & \underline{6x - 36} \\
 & 0
\end{array}
$$

$4x^2 + 9x + 6$

$b^2 - 4ac = 81 - 4 \times 4 \times 6 = -15$

as $b^2 - 4ac < 0$ has no real roots

$\therefore$ Curve only intersects x-axis at $(6,0)$

4 (b) (ii) State the coordinates of this point of intersection.

[1 mark]

(6, 0)

Turn over for the next question

4 (a) Sketch the graph of

$$y = 4 - |2x - 6|$$

$y = 4 - (2x - 6)$

$y = 10 - 2x$, $y = 0$, $x = 5$

$(5, 0)$

or

$y = 4 - -(2x - 6)$

$y = -2 + 2x$

$y = 0$, $x = 1$

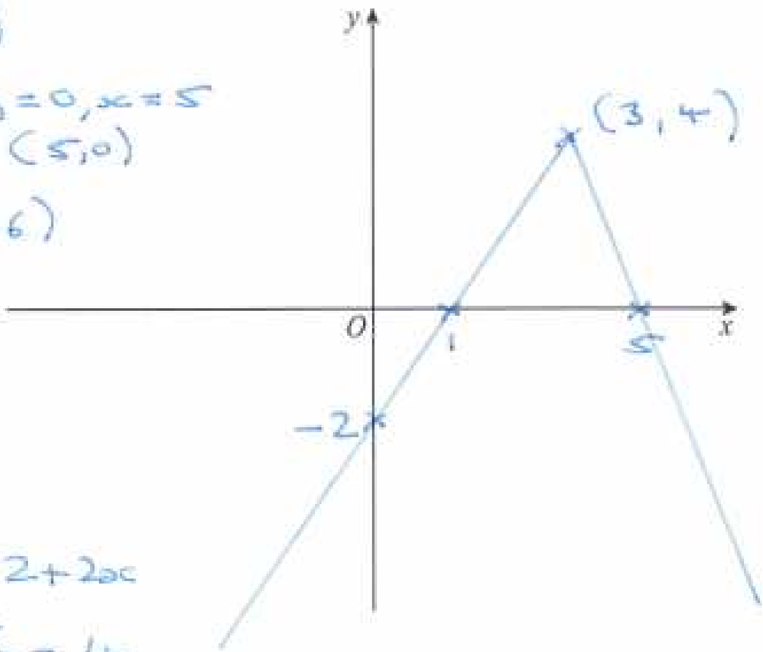
$(1, 0)$

meet when

$10 - 2x = -2 + 2x$

$12 = 4x$

$x = 3$, $y = 4$



[3 marks]

4 (b) Solve the inequality

$$4 - |2x - 6| > 2$$

[2 marks]

$4 - 2 > 2x - 6$ or $4 - 2 > -(2x - 6)$

$2 > 2x - 6$ $\quad$ $2 > -2x + 6$

$8 > 2x$ $\quad$ $2x > 6 - 2$

$4 > x$ $\quad$ $2x > 4$

$x < 4$ $\quad$ $x > 2$

$2 < x < 4$

9 Chloe is attempting to write $\frac{2x^2+x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1 $\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+2)^2}$

Step 2 $2x^2+x \equiv A(x+2)^2 + B(x+1)$

Step 3 Let $x=-1 \Rightarrow A=1$
Let $x=-2 \Rightarrow B=-6$

Answer $\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} - \frac{6}{(x+2)^2}$

9 (a) (i) By using a counter example, show that the answer obtained by Chloe cannot be correct.

[2 marks]

substitute values in both equations to check equivalence

$x=1$ $\frac{2\times1^2+1}{(1+1)\times(1+2)^2} = \frac{1}{6}$

$x=1$ $\frac{1}{1+1} - \frac{6}{(1+2)^2} = \frac{1}{3}$

$\therefore \frac{2x^2+x}{(x+1)(x+2)^2} \neq \frac{1}{x+1} - \frac{6}{(x+2)^2}$

9 (a) (ii) Explain her mistake in Step 1.

[1 mark]

Chloe needed a 3rd term $\frac{C}{x+2}$ also

9 (b) Write $\dfrac{2x^2 + x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

[4 marks]

$$\frac{2x^2+x}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$2x^2 + x = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$$

$x = -1$, $1 = A$

$x = -2$, $6 = -C \Rightarrow C = -6$

Equate coefficients of x^2

$2 = A + B$

$2 = 1$ … $B = 1$

$$\frac{2x^2+x}{(x+1)(x+2 \ldots} \ldots + \frac{1}{x+2} - \frac{6}{(x+2)^2}$$

6 The function f is defined by

$$f(x) = \frac{1}{2}(x^2+1),\ x \geq 0$$

6 (a) Find the range of f.

[1 mark]

at $x = 0$, $f(x) = \frac{1}{2}$

range $f(x) \geq \frac{1}{2}$

6 (b) (i) Find $f^{-1}(x)$

[3 marks]

$y = \frac{1}{2}x^2 + \frac{1}{2}$

$y - \frac{1}{2} = \frac{1}{2}x^2$

$2y - 1 = x^2$

$\sqrt{2y-1} = x$

$f^{-1}(x) = \sqrt{2x-1}$

6 (b) (ii) State the range of $f^{-1}(x)$

[1 mark]

Domain is range of $f(x)$,

so for $x \geq \frac{1}{2}$

$f^{-1}(x) \geq 0$

6 The function f is defined by

$$f(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f.

[1 mark]

at $x = 0$, $f(x) = \frac{1}{2}$

range $f(x) \geqslant \frac{1}{2}$

6 (b) (i) Find $f^{-1}(x)$

[3 marks]

$y = \frac{1}{2}x^2 + \frac{1}{2}$

$y - \frac{1}{2} = \frac{1}{2}x^2$

$2y - 1 = x^2$

$\sqrt{2y-1} = x$

$f^{-1}(x) = \sqrt{2x-1}$

6 (b) (ii) State the range of $f^{-1}(x)$

[1 mark]

Domain is range of $f(x)$,

so for $x \geqslant \frac{1}{2}$

$f^{-1}(x) \geqslant 0$

6 (c) State the transformation which maps the graph of $y = f(x)$ onto the graph of $y = f^{-1}(x)$

[1 mark]

Reflection in line $y = x$

6 (d) Find the coordinates of the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$

[2 marks]

$\frac{1}{2}(x^2 + 1) = \sqrt{2x - 1}$

cannot solve this

Must meet at line $y = x$

$\therefore \quad x = \frac{1}{2}(x^2 + 1)$

$2x = x^2 + 1$

$0 = x^2 - 2x + 1$

$0 = (x - 1)(x - 1)$

$x = 1$ or $x = 1$

$\therefore \quad y = 1$

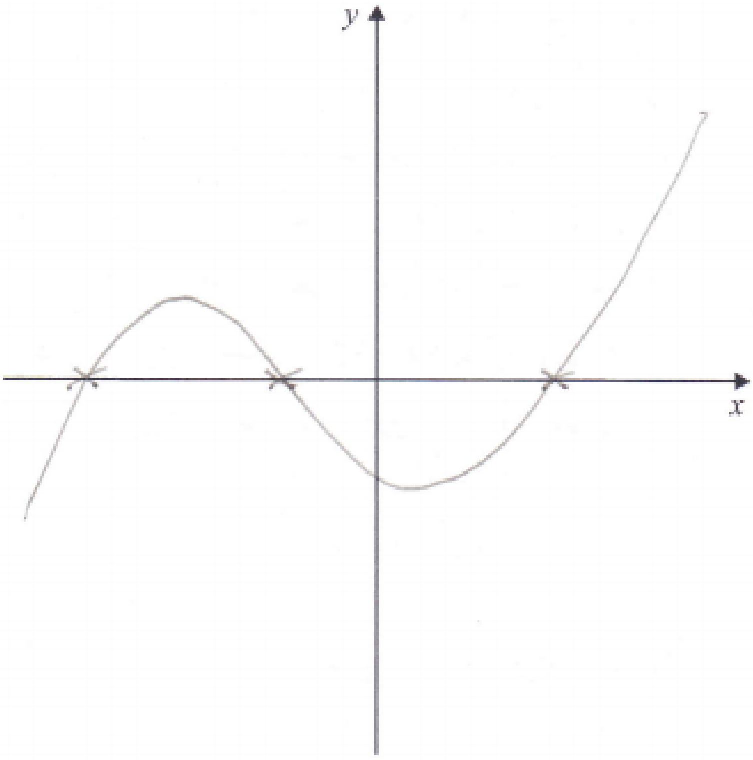
Coordinate of intersection is

$(1, 1)$

Turn over for the next question

7 (a) Sketch the graph of any cubic function that has **both** three distinct real roots **and** a positive coefficient of x^3

[2 marks]



7 (b) The function $f(x)$ is defined by

$$f(x) = x^3 + 3px^2 + q$$

where p and q are constants and $p > 0$

7 (b) (i) Show that there is a turning point where the curve crosses the y-axis.

[3 marks]

$f'(x) = 3x^2 + 6px$

at turning point, $f'(x) = 0$

$0 = 3x^2 + 6px$

$0 = 3x(x + 2p)$

turning point at $x = 0$ and $x = -2p$

where crosses y-axis

$f''(x) = 6x + 6p$ at $x = 0$,

$f''(x) > 0$ as $p > 0$

minimum pt. at at $x = 0$

outside the box

7 (b) (ii) The equation $f(x) = 0$ has three distinct real roots.

By considering the positions of the turning points find, in terms of p, the range of possible values of q.

[5 marks]

at $x = -2p$

$f''(x) = -12p + 6p = -6p$

$\therefore f''(x) < 0$ maximum pt

at $x = 0$, $f(x) = 0 + 0 + q$ $(0, q)$

at $x = -2p$, $f(x) = (-2p)^3 + 3p(-2p)^2 + q$

$\therefore 0 = -8p^3 + 12p^3 + q$

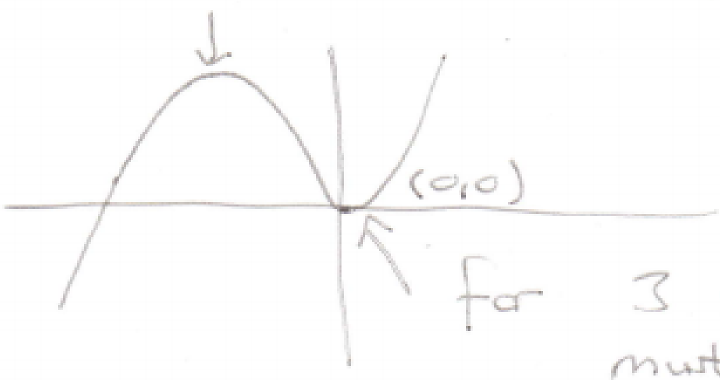
$0 = 4p^3 + q$

$q = -4p^3$

$\therefore$ range $q < -4p^3$

$(-2p, 4p^3+q)$

Turn over for the next question



for 3 roots, minimum point must have $q<0$