A-LEVEL Mathematics

Paper 1
Mark scheme

Practice paper – Set 1

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme has been prepared for practice papers and has not, therefore, been through the process of standardising that would take place for live papers.

Further copies of this mark scheme are available from allaboutmaths.aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the paper
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.1b	B1	$\text{cosec}\,\theta = -\frac{5}{3}$
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	4
	Total		**1**	
3	Circles correct answer	AO1.1b	B1	$-\frac{7}{4}$
	Total		**1**	
4 (a)	Uses $l = r\theta$ to form an equation	AO1.1a	M1	$2r + 1.5r = 10.5$ $3.5r = 10.5$ $r = 3$
	Obtains the correct answer	AO1.1b	A1	
	Total		**2**	
4 (b)	Uses $A = \frac{1}{2}r^2\theta$ with 'their' r	AO1.1a	M1	$A = \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 3^2 \times 1.5$ $= 6.75(\text{cm}^2)$
	Obtains correct answer	AO1.1b	A1F	
	Total		**2**	
5	Begins disproof by counter-example, by choosing suitable values for a and b	AO3.1a	B1	Let $a = -1$ and $b = -2$ so $a > b$ Then $a^2 = 1$ and $b^2 = 4$ so $a^2 \not> b^2$ $\therefore a > b \not\Rightarrow a^2 > b^2$
	Completes rigorous argument	AO2.1	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6	Begins argument by comparing the size of two consecutive terms of the sequence	AO3.1a	M1	$u_{n+1} - u_n = a \times 2^{-(n+1)} - a \times 2^{-n}$ $= a(2^{-n} \times 2^{-1} - 2^{-n})$ $= a \times 2^{-n}(2^{-1} - 1)$ $= a \times 2^{-n} \times -\frac{1}{2}$ > 0 since both a and $-\frac{1}{2} < 0$, and $2^{-n} > 0$ $\therefore\ u_{n+1} > u_n$ for all n Hence the sequence is increasing.
	Simplifies ‘their’ expression to a form that enables the required deduction to be made	AO1.1b	A1	
	Deduces $u_{n+1} - u_n > 0$ or $u_{n+1} > u_n$ with some reasoning	AO2.2a	R1	
	Completes fully correct argument with no algebraic slips and fully explains why $u_{n+1} > u_n$	AO2.1	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7 (a)	Demonstrates $p(-\frac{1}{2})=0$	AO1.1b	B1	$p(-\frac{1}{2})=6(-\frac{1}{2})^3-19(-\frac{1}{2})^2+9(-\frac{1}{2})+10$ $=0$ $p\left(-\frac{1}{2}\right)=0\Rightarrow\ 2x+1$ is a factor
	Constructs rigorous mathematical proof (to achieve this mark, the students must clearly substitute $-\frac{1}{2}$ and state that $p(-\frac{1}{2})=0$ and clearly state that this implies that $2x+1$ is a factor)	AO2.1	R1	
	Total		**2**	
7 (b)	Factorises the numerator and denominator (this mark is achieved for any reasonable attempt at factorisation through the selection of an appropriate method eg long division, inspection or repeated use of factor theorem)	AO1.1a	M1	$\frac{3x^2-6x}{6x^3-19x^2+9x+10}$ $=\frac{3x(x-2)}{(2x+1)(3x^2-11x+10)}$ $=\frac{3x(x-2)}{(2x+1)(3x-5)(x-2)}$ $=\frac{3x}{(2x+1)(3x-5)}$
	Finds second factor in denominator or fully factorises numerator (PI by complete factorisation)	AO1.1b	A1	
	Finds fully correct factorised expression	AO1.1b	A1	
	Obtains completely correct solution	AO1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7 (c)	Identifies one value of x for which their answer is invalid.	AO3.2b	B1	$x = 2$ $x = -\frac{1}{2}$ $x = \frac{5}{3}$
	Identifies all three values correctly with no extra values CAO	AO3.2b	B1	
	Total		**2**	
8 (a)	Translates rate of change into $\frac{dN}{dt}$	AO3.3	M1	$\frac{dN}{dt} = kN(1500 - N)$
	Translates the product using $N(1500 - N)$	AO3.3	M1	
	Forms differential equation correctly with correct notation $\frac{dN}{dt} = kN(1500 - N)$	AO1.1b	A1	
	Total		**3**	
8 (b)	Gives a relevant criticism of the assumption	AO3.5b	E1	The number of students is unlikely to follow this model all the time, students may buy more at the weekends, so N will increase more on Mondays.
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9	Clearly states conditions which will ensure that the x axis will be a tangent: $b^2 - 4ac = 0$ Or min value is 0	AO2.4	E1	If $x^2 - 6kx + 9k^2 = 0$ has equal roots, the x-axis is a tangent. $b^2 - 4ac = (-6k)^2 - 4 \times 1 \times 9k^2$ $= 36k^2 - 36k^2$ $= 0$ Since the discriminant is zero for all values of k, the x-axis is always a tangent to $y = x^2 - 6kx + 9k^2$
	Attempts to form quadratic expression in k, complete the square/factorise or differentiates (allow one error)	AO1.1a	M1	
	Correct discriminant in k and simplified to 0 or correctly factorised expression Or finds x for $\frac{dy}{dx} = 0$	AO1.1b	A1	
	Completes correct argument (in the right direction) with no errors explaining independence of k	AO2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
10 (a)	Recalls the correct derivative	AO1.2	B1	$3^x \ln 3$
	Total		**1**	
10 (b)	Selects an appropriate method for integrating, which could lead to a correct exact solution (this could be indicated by an attempt at a substitution or attempting to write the integrand in the form $f'(x)\left(f(x)\right)^n$	AO3.1a	M1	Let $u = 3^x$ then $\frac{du}{dx} = 3^x \ln 3$ and $\frac{1}{\ln 3}\frac{du}{dx} = 3^x$ $I = \int_9^{27} \frac{1}{u-8}\frac{1}{\ln 3}\frac{du}{dx}\,dx$ $= \frac{1}{\ln 3}\int_9^{27}\frac{1}{u-8}\,du$ $= \frac{1}{\ln 3}\left[\ln(u-8)\right]_9^{27}$ $= \frac{\ln 19 - \ln 1}{\ln 3}$ $= \frac{\ln 19}{\ln 3}$
	Correctly writes integrand in a form which can be integrated (condone missing or incorrect limits)	AO1.1b	A1	
	Integrates 'their' expression (allow one error)	AO1.1a	M1	
	Substitutes correct limits corresponding to 'their' method	AO1.1a	M1	
	Obtains the values of a and b	AO1.1b	A1	
	Completes correct solution, which is clear, easy to follow and contains no slips. Substitution should be clearly stated in exact form and change of variable or solution by direct inspection should be achieved correctly with correct use of symbols and connecting language	AO2.1	R1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
11	Applies the product rule twice to find the second derivative	AO3.1a	M1	$y = xe^{\frac{x}{2}}$ $\frac{dy}{dx} = e^{\frac{x}{2}} + \frac{1}{2}xe^{\frac{x}{2}}$ $\frac{d^2y}{dx^2} = \frac{1}{2}e^{\frac{x}{2}} + \frac{1}{2}\left(e^{\frac{x}{2}} + \frac{1}{2}xe^{\frac{x}{2}}\right)$ $= e^{\frac{x}{2}} + \frac{1}{4}xe^{\frac{x}{2}}$ $= e^{\frac{x}{2}}\left(1 + \frac{1}{4}x\right)$ $\frac{d^2y}{dx^2} = 0 \Leftrightarrow e^{\frac{x}{2}}\left(1 + \frac{1}{4}x\right) = 0$ $\therefore$ only possible point of inflection is when $x = -4$ When $x = 0, \frac{d^2y}{dx^2} = 1 > 0$ and $x = -5, \frac{d^2y}{dx^2} = -0.205... < 0$ $\therefore$ Since $y = xe^{\frac{x}{2}}$ is continuous, $\frac{d^2y}{dx^2}$ changes sign at the point where $x = -4$ So the coordinates of the point of inflection are $\left(-4, -\frac{4}{e^2}\right)$
	Obtains correct $\frac{dy}{dx}$	AO1.1b	A1	
	Obtains correct $\frac{d^2y}{dx^2}$	AO1.1b	A1	
	Equates 'their' $\frac{d^2y}{dx^2} = 0$	AO3.1a	M1	
	Obtains x coordinate from 'their' equation	AO1.1b	A1	
	Begins to construct arguments by showing that $\frac{d^2y}{dx^2} > 0$ and $\frac{d^2y}{dx^2} < 0$ for two points either side of 'their' x	AO2.1	R1	
	Explains reasoning fully. Clearly states that $\frac{d^2y}{dx^2}$ must change sign at $x = -4$	AO2.4	E1	
	States coordinates at the point of inflection in an exact form	AO2.5	A1	
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
12 (a)	Deduces that $a = 1$	AO2.2a	B1	$a = 1$
	Total		**1**	
12 (b)(i)	Obtains $\mathrm{h}(x)$	AO1.1b	B1	$\mathrm{h}(x) = \sqrt{1-2^x}$
	Total		**1**	
12 (b)(ii)	Deduces correct max value for x	AO2.2a	B1	$\{x \in \mathbb{R} : x \leq 0\}$
	Correctly states the greatest possible domain of h using set notation	AO2.5	B1	
	Total		**2**	
11 (b)(iii)	Deduces the upper and lower bounds for h	AO2.2a	B1	$\{x \in \mathbb{R} : 0 \leq x < 1\}$
	Correctly states the range of h in any correct form eg [0,1), $0 \leq \mathrm{h}(x) < 1$	AO1.1b	B1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13	Translates rate of increase of volume into $\frac{dV}{dt} = 10$	AO3.3	B1	$\frac{dA}{dt} = \frac{dA}{dV} \times \frac{dV}{dt}$
	Uses the chain rule to connect rates of change volume and area	AO3.1b	M1	$\frac{dV}{dr} = 4\pi r^2$ $\frac{dA}{dr} = 8\pi r$
	Uses the chain rule or writes A in terms of V to find $\frac{dA}{dV}$	AO3.1b	M1	$\therefore \frac{dA}{dV} = \frac{2}{r}$
	Obtains correct derivative for $\frac{dA}{dV}$	AO1.1b	A1	$\frac{dA}{dt} = \frac{dA}{dV} \times \frac{dV}{dt}$ $= \frac{2}{r} \times 10$
	Uses diameter/radius in 'their' model	AO3.4	M1	When diameter = 8 $\frac{dA}{dt} = \frac{20}{4} = 5$
	Clearly states correct answer with correct units in context	AO3.2a	A1	The rate of increase of surface area is 5 cm^2/s
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
14 (a)	Completes the square twice or applies standard formula for C_1	AO3.1a	M1	$x^2+y^2-8x-14y=-40$ $\Rightarrow (x-4)^2-16+(y-7)^2-49=-40$ $\Rightarrow (x-4)^2+(y-7)^2=25$ C_1: Radius = 5 and centre (4, 7) C_2: Radius = 7 and centre (16, 12) Distance between centres, d $d^2=(16-4)^2+(12-7)^2$ $=169$ $d=13$ Since the sum of the radii 5 + 7 = 12 is less than the distance between the centres, the circles do not intersect.
	Obtains correct equation	AO1.1b	A1	
	Obtains correct radius and correct coordinates of centre for C_1 Follow through 'their' equation	AO1.1b	A1F	
	States correct radius and correct coordinates of centre for C_2	AO1.1b	B1	
	Uses a method to find distance between the centres	AO3.1a	M1	
	Obtains correct distance	AO1.1b	A1	
	Demonstrates clearly that the sum of the radii of the two circles is less than the distance between 'their' centres and deduces that the circles do not overlap	AO2.2a	A1	
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
14 (a) (Alt)	Attempts simultaneous equations and expands one bracket correctly	AO3.1a	M1	$x^2+y^2-8x-14y=-40$ $x^2+y^2-32x-24y=-351$ $24x+10y=311$ $y=\dfrac{311-24x}{10}$ $x^2+\left(\dfrac{311-24x}{10}\right)^2-8x-14\left(\dfrac{311-24x}{10}\right)=-40$ $x^2+\dfrac{96721-14928x+576x^2}{100}-8x-\dfrac{4354-336x}{10}+40=0$ $100x^2+96721-14928x+576x^2-800x-43540+3360x+4000=0$ $676x^2-12368x+57181=0$ $b^2-4ac=-1650000<0$ $\therefore$ no real roots so circles do not intersect
	Obtains correct equation	AO1.1b	A1	
	Eliminates squared terms	AO1.1b	A1f	
	Writes x in terms of y OE	AO1.1b	B1	
	Eliminates x or y to form quadratic	AO3.1a	M1	
	Obtains correct simplified quadratic	AO1.1b	A1	
	Demonstrates the quadratic has no real solutions and deduces that the circles do not intersect	AO2.2a	A1	
	Total		**7**	
14 (b)	Uses the fact that maximum distance is along the line of centres PI by a diagram	AO3.1a	M1	Max distance = $d+5+7$ Max distance = 25
	Obtains maximum distance Follow through ‘their’ distance and radius for C_1	AO1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
15 (a)	Recalls and uses definitions of sec and cot	AO1.2	B1	$\dfrac{\cos x}{\sec x+1}+\dfrac{\cos x}{\sec x-1}$
	Performs some correct algebraic manipulation and uses an identity to commence proof (at least two lines of argument)	AO2.1	R1	$\equiv \dfrac{\cos x(\sec x-1)+\cos x(\sec x+1)}{\sec^2 x-1}$ $\equiv \dfrac{2\cos x\sec x}{\sec^2 x-1}$
	Concludes a rigorous mathematical argument to prove given identity **AG** Must start with one side and through clear logical steps arrive at the other side. In order to be sufficiently clear, each line should be a single step, unless clear further explanation is given	AO2.1	R1	$\equiv \dfrac{2\cos x\sec x}{\tan^2 x}$ $\equiv \dfrac{2}{\tan^2 x}$ $\equiv 2\cot^2 x$
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15 (b)	Rearranges so that the identity from part **(a)** can be used to write quadratic equation in cot x or cot θ	AO3.1a	M1	let $x=\left(2\theta+\frac{\pi}{3}\right)$ $\frac{\pi}{3}\le x\le\frac{13\pi}{3}$ $\frac{\cos x}{\sec x+1}=\cot x-\frac{\cos x}{\sec x-1}$ $\Rightarrow\frac{\cos x}{\sec x+1}+\frac{\cos x}{\sec x-1}-\cot x=0$ $\Rightarrow 2\cot^2 x-\cot x=0$ $\Rightarrow \cot x=0$ or $\cot x=\frac{1}{2}$ $x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},$ 1.11, 4.25, 7.39, 10.53 $\theta=\frac{\pi}{12},\frac{7\pi}{12},\frac{13\pi}{12},\frac{19\pi}{12},$ 0.0300, 1.60, 3.17, 4.74
	Obtains correct quadratic equation	AO1.1b	A1	
	Solves 'their' quadratic to obtain values for cot x	AO1.1a	M1	
	Obtains at least 2 correct values for x from each of 'their' 2 values of cot x	AO1.1b	A1F	
	Obtains all correct values for θ with no extra values CAO	AO1.1b	A1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
16 (a)(i)	Models the cost with an expression of the form $ar^2 + brh$	AO3.3	M1	$C = 2\pi r^2 \times \frac{5}{4} p + 2\pi rhp$
	Attempt to eliminate h, using volume equation, to form a model for the cost in one variable	AO3.3	M1	$\pi r^2 h = 400$ $h = \frac{400}{\pi r^2}$ $C = \frac{5}{2}\pi r^2 p + \frac{800p}{r}$
	Obtains a correct equation to model cost in terms of r AG	AO3.1b	A1	
	Total		**3**	
16 (a)(ii)	Uses the model to find minimum (at least one term correctly differentiated and expression equated to zero)	AO3.4	M1	$\frac{\mathrm{d}C}{\mathrm{d}r} = 5\pi rp - \frac{800p}{r^2} = 0$
	Correct equation from differentiating	AO1.1b	A1	$r^3 = \frac{800}{5\pi}$ $r = \sqrt[3]{\frac{160}{\pi}}$
	Obtains correct value for r with correct units in context	AO3.2a	A1	radius of can $= 3.7$ cm
	Obtains the second derivative or obtains the gradient values either side of 'their' r	AO1.1a	M1	$\frac{\mathrm{d}^2C}{\mathrm{d}r^2} = 5\pi p + \frac{1600p}{r^3}$ $r > 0 \Rightarrow \frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$ therefore minimum
	Performs a correct test of 'their' solution to justify that a minimum value for C has been found Shows that second derivative > 0 or tests gradient/values either side with a correct concluding statement	AO2.1	R1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
16 (b)	Explains any relevant assumption	AO3.5b	E1	Edges do not overlap to form joints. There is no cost in forming the joints. Thickness of material is insignificant.
	Total		**1**	
16 (c)	Explains what part of the model could cause the additional cost	AO3.5a	E1	Cutting circles would cause significant waste material as they do not tessellate
	Total		**1**	
17	Separates variables, at least one side correct	AO3.1a	M1	$\frac{1}{(3-x)}\frac{\mathrm{d}x}{\mathrm{d}t}=t$
	Obtains correct separation PI	AO1.1b	A1	$\int\frac{1}{(3-x)}\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t=\int t\mathrm{d}t$ $\int\frac{1}{(3-x)}\mathrm{d}x=\int t\mathrm{d}t$
	Integrates 'their' expressions at least one of 'their' sides correct	AO1.1a	M1	$-\ln(3-x)=\frac{t^2}{2}+c$
	Obtains correct integral (condone missing + c) CAO	AO1.1b	A1	$t=0,\ x=0.65$ $\Rightarrow c=-\ln 2.35=-0.8544$
	Substitutes initial conditions to find c	AO1.1a	M1	$\ln 2.35-\ln(3-x)=\frac{t^2}{2}$
	Obtains a correct solution to model the depth ACF	AO1.1b	A1	$x=1.85\Rightarrow\frac{t^2}{2}=\ln 2.35-\ln 1.15$ $t=1.20$ $(t=1.19553...)$
	Uses model to find correct time Award ft from correct substitution into incorrect equation but only if all three M1 marks have been awarded	AO3.4	A1F	
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
18	Identifies step 3	AO1.2	B1	Step 3
	Explains that q is not necessarily prime	AO2.4	E1	It is not necessarily true that q is a prime number
	Total		**2**	
19 (a)	Uses $\cos\theta \approx 1 - \frac{1}{2}\theta^2$	AO1.1a	M1	$\frac{\cos\theta - 1}{\theta} \approx \frac{1 - \frac{1}{2}\theta^2 - 1}{\theta}$ $\approx -\frac{1}{2}\theta$
	Obtains correct answer	AO1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
19 (b)	Writes gradient of chord as $\frac{\cos(x+h)-\cos(x)}{h}$	AO2.1	M1	$f'(x) = \lim_{h\to 0}\left[\frac{\cos(x+h)-\cos(x)}{h}\right]$ $= \lim_{h\to 0}\left[\frac{\cos x\cos h - \sin x \sin h - \cos(x)}{h}\right]$ $= \lim_{h\to 0}\left[\frac{\cos x(\cos h - 1)}{h} - \frac{\sin x \sin h}{h}\right]$ $= \lim_{h\to 0}\left[\frac{(\cos h - 1)}{h}\right]\cos x - \lim_{h\to 0}\left[\frac{\sin h}{h}\right]\sin x$ $= 0 \times \cos x - 1 \times \sin x$ $= -\sin x$
	Uses $\cos(A+B)$ identity, replacing $\cos(x+h)$, to commence argument. At least two lines of argument seen	AO1.1a	M1	
	Obtains correct two term expression involving cos h and sin h	AO1.1b	A1	
	Deduce what happens as $h \to 0$, for one part of 'their' expression using the limit of $\frac{\sin h}{h}$ Or by using small angles approximations	AO2.2a	R1	
	Deduce what happens as $h \to 0$, for second part of 'their' expression using the limit of $\frac{\cos h - h}{h}$ Or by using small angles approximations	AO2.2a	R1	
	Completes a rigorous argument leading to the correct exact value, with all the steps in the method clearly shown. Must include link to $f'(x)$ as the limit as h tends to 0	AO2.1	R1	
	Total		**6**	
	TOTAL		**100**	