



A-level Mathematics

Paper 3
Mark scheme

Practice paper – Set 1

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme has been prepared for practice papers and has not, therefore, been through the process of standardising that would take place for live papers.

Further copies of this mark scheme are available from allaboutmaths.aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

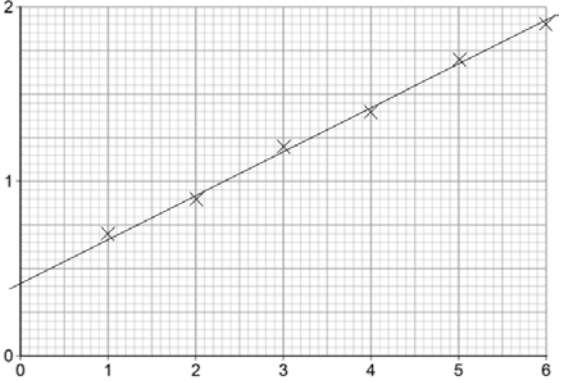
Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.1b	B1	$y = \ln(5x + 2)$
	Total		1	
2	Circles correct answer	AO2.2a	R1	16
	Total		1	
3 (a)	Writes the expression in a form that may be expanded	AO1.1a	M1	$\sqrt{16+x} = 4\left(1 + \frac{x}{16}\right)^{\frac{1}{2}}$
	Correctly expands at least three terms	AO1.1b	A1	$\approx 4\left(1 + \binom{1}{2}\left(\frac{x}{16}\right) + \frac{\binom{1}{2}\binom{-1}{2}}{2!}\left(\frac{x}{16}\right)^2 + \frac{\binom{1}{2}\binom{-1}{2}\binom{-3}{2}}{3!}\left(\frac{x}{16}\right)^3\right)$
	Obtains correct expansion with all terms simplified	AO1.1b	A1	$= 4\left(1 + \frac{x}{32} - \frac{x^2}{2048} + \frac{x^3}{65536}\right)$ $= 4 + \frac{x}{8} - \frac{x^2}{512} + \frac{x^3}{16384}$
	Total		3	
3 (b)	States correct range of values	AO2.3	B1	$ x < 16$
	Total		1	

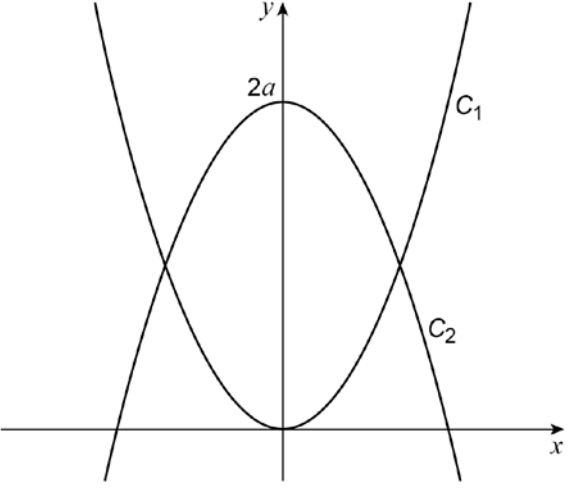
Q	Marking instructions	AO	Marks	Typical solution
4 (a)	Takes logs of both sides	AO1.1a	M1	$y = ab^x$
	Uses an appropriate log rule	AO1.1a	M1	$\log_{10} y = \log_{10} (ab^x)$ $\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^x$ $\Rightarrow \log_{10} y = \log_{10} a + x \log_{10} b$
	Uses a second appropriate log rule to complete demonstration of linear form. Only award if clear with no slips in algebra	AO2.1	R1	If we let $y = \log_{10} y$ and rewrite $y = (\log_{10} b)x + \log_{10} a$ This has the form $Y = mx + c$, which is a linear relationship
	Explains clearly how the derived expression gives a linear relationship. This could include a comparison with the structure of $y = mx + c$ or using a substitution $Y = \log_{10} y$	AO2.4	E1	
	Total		4	

Q	Marking instructions	AO	Marks	Typical solution																					
4 (b)(i)	Calculates correct values for $\log_{10}y$ At least 5 correct PI by correct points	AO1.1a	M1	<table border="1"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>y</td> <td>5</td> <td>8</td> <td>15</td> <td>26</td> <td>47</td> <td>85</td> </tr> <tr> <td></td> <td>0.7</td> <td>0.9</td> <td>1.2</td> <td>1.4</td> <td>1.7</td> <td>1.9</td> </tr> </table>	x	1	2	3	4	5	6	y	5	8	15	26	47	85		0.7	0.9	1.2	1.4	1.7	1.9
	x	1	2	3	4	5	6																		
y	5	8	15	26	47	85																			
	0.7	0.9	1.2	1.4	1.7	1.9																			
Plots points and draws appropriate straight-line graph	AO1.1b	A1																							
Total			2																						
4 (b)(ii)	Obtains vertical intercept c and attempts 10^c Condone AFWW 0.4 to 0.45 for intercept Or Obtains gradient m and attempts 10^m Condone AWRT 0.25 for gradient	AO1.1a	M1	Gradient = 0.249 Intercept = 0.4 $a = 10^{0.4} = 2.5$ $b = 10^{0.249} = 1.8$																					
	Obtains correct value for a	AO1.1b	A1																						
	Obtains correct value for b	AO1.1b	A1																						
Total			3																						

Q	Marking instructions	AO	Marks	Typical solution
4 (c)	Substitutes $x = 6.5$ into 'their' model	AO3.4	M1	$y = 2.5 \times 1.8^{6.5}$ $= 114$
	Follow through 'their' a and b Explains that the model predicts a larger value, suggesting that it would need to be modified	AO3.2b	A1F	
	Total		2	

Q	Marking instructions	AO	Marks	Typical solution
5	Translates finding the area into a definite integral	AO3.1a	M1	$y = x^2 \sin x$ $\int x^2 \sin x dx$
	Solves $y = 0$ to find limits of integration	AO2.2a	R1	$u = x^2 \quad u' = 2x$ $v' = \sin x \quad v = -\cos x$
	Uses integration by parts correctly	AO1.1a	M1	$\int x^2 \sin x dx = -x^2 \cos x - \int 2x(-\cos x) dx$ $= -x^2 \cos x + 2 \int x \cos x dx$
	Obtains correct integral after first application	AO1.1b	A1	$u = x \quad u' = 1$ $v' = \cos x \quad v = \sin x$
	Uses integration by parts for a second time correctly	AO1.1a	M1	$I = -x^2 \cos x + 2(x \sin x - \int \sin x dx)$ $= -x^2 \cos x + 2x \sin x + 2 \cos x$
	All notation including dx correct	AO2.5	B1	$\text{area} = \int_0^\pi x^2 \sin x dx$
	Obtains fully correct integral must have correct limits	AO1.1b	A1	$= \left[-x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^\pi$ $= (-\pi^2 \cos \pi + 2\pi \sin \pi + 2 \cos \pi) - (0^2 \cos 0 + 0 \sin 0 + 2 \cos 0)$
	Substitutes 'their' limits into 'their' integral	AO1.1a	M1	$= (\pi^2 - 2) - (2)$ $= \pi^2 - 4$
Completes argument to show the required result	AO2.1	R1		
	Total		9	

Q	Marking instructions	AO	Marks	Typical solution
6	Identifies and clearly defines variables	AO3.1b	B1	Let t = time in years M = mass
	Sets up an exponential model, including the initial mass and a constant	AO3.3	M1	$M = M_0 e^{-kt}$ When $t = 10$, $M = 0.8M_0$ $0.8 = e^{-10k}$
	Uses given condition to determine the constant	AO3.1b	M1	$-10k = \ln 0.8$ $k = -\frac{1}{10} \ln 0.8$
	Obtains correct constant	AO1.1b	A1	$k = 0.022314\dots$ when $M = 0.1M_0$ $0.1 = e^{-0.022314t}$
	Uses 'their' model to find t	AO3.4	M1	$t = -\frac{1}{0.022314} \ln 0.1 = 103.18$
	Obtains correct number of years ft 'their' model	AO3.2a	A1F	It will take another 93.2 years
	Total		6	

Q	Marking instructions	AO	Marks	Typical solution
7 (a)	Sketches and labels C_1 correctly	AO1.2	B1	
	Sketches and labels C_2 correctly	AO1.1a	M1	
	Labels correct y-intercept	AO1.1b	A1	
	Total		3	

Q	Marking instructions	AO	Marks	Typical solution
7 (b)	Uses a correct method to find the correct area This could be integrating and subtracting Or Subtracting then integrating Or Finding $\frac{1}{4}$ of the required area and multiplying by 4	AO3.1a	M1	$A = 4 \left(a\sqrt{a} - \int_0^{\sqrt{a}} x^2 dx \right)$ $= 4 \left(a\sqrt{a} - \left[\frac{x^3}{3} \right]_0^{\sqrt{a}} \right)$ $= 4 \left(a\sqrt{a} - \frac{a\sqrt{a}}{3} \right)$ $= 4 \times \frac{2a\sqrt{a}}{3}$ $= \frac{8a^{\frac{3}{2}}}{3}$
	Integrates one term correctly	AO1.1b	A1	ALT $A = \int_{-\sqrt{a}}^{\sqrt{a}} 2a - x^2 - x^2 dx$
	Obtains correct expression	AO1.1b	A1	$= \int_{-\sqrt{a}}^{\sqrt{a}} 2a - 2x^2 dx$
	Obtains correct limits	AO1.1b	A1	$= \left[2ax - \frac{2}{3}x^3 \right]_{-\sqrt{a}}^{\sqrt{a}}$
	Substitutes 'their' limits	AO1.1a	M1	$= \left(2a\sqrt{a} - \frac{2}{3}\sqrt{a}^3 \right) - \left(2a(-\sqrt{a}) - \frac{2}{3}(-\sqrt{a})^3 \right)$
	Obtains correct area through completely correct argument AG	AO2.1	R1	$= \left(\frac{6}{3}a^{\frac{3}{2}} - \frac{2}{3}a^{\frac{3}{2}} \right) + \left(\frac{6}{3}a^{\frac{3}{2}} - \frac{2}{3}a^{\frac{3}{2}} \right)$ $= \frac{4}{3}a^{\frac{3}{2}} + \frac{4}{3}a^{\frac{3}{2}}$ $= \frac{8}{3}a^{\frac{3}{2}}$
Total			6	
7 (c)	Forms and solves correct equation	AO1.1a	M1	$\frac{8}{3}a^{\frac{3}{2}} = 72$
	Obtains correct value for a CAO	AO1.1b	A1	$a^{\frac{3}{2}} = 27$ $a = 9$
Total			2	

Q	Marking instructions	AO	Marks	Typical solution
8 (a)	Squares the terms of the sequence PI by stating correct first term and ratio	AO3.1a	M1	a, ar, ar^2, \dots initial sequence $a^2, (ar)^2, (ar^2)^2, \dots$ terms squared $a^2, a^2r^2, a^2r^4 \Rightarrow$ 1st term a^2 , ratio r^2 $\frac{a^2}{1-r^2} = 2 \left(\frac{a}{1-r} \right)$ $\Rightarrow a^2(1-r) = 2a(1-r^2)$ $\Rightarrow a(1-r) = 2(1-r)(1+r)$ Assuming $a \neq 0$ $\Rightarrow a = 2(1+r)$ AG Since $r < 1 \Rightarrow 1-r \neq 0$
	Obtains/states correct first term and ratio	AO1.1b	A1	
	Forms an equation using both sums to infinity	AO3.1a	M1	
	Shows clear and correct algebraic reasoning to reach the required result. Factorising must be seen AG	AO2.1	R1	
	Explains why cancelling $1-r$ and a is allowed	AO2.4	E1	
	Total		5	
8 (b)	Uses $-1 < r < 1$ to start to find an inequality for a	AO2.1	M1	$-1 < r < 1$ $\Rightarrow 0 < r+1 < 2$ $\Rightarrow 0 < 2(r+1) < 4$ $\therefore a \in (0, 4)$
	Obtains correct set for a	AO2.2a	A1	
	Total		2	

Q	Marking instructions	AO	Marks	Typical solution
9	Ticks the correct box	AO1.2	B1	Approximately 95% of the values are within three standard deviations of the mean
	Total		1	
10	Ticks the correct box	AO1.2	B1	The data for paper 2 is negatively skewed
	Total		1	
11 (a)	Gives two criticisms in context. Allow examples that illustrate the issues Any one of A, B or C	AO2.3	E1	A: Not every member will be allocated 3 digits because $3 \times 350 > 1000$ B: Only random numbers with 3 different digits will be chosen so not every member can be chosen.
	Any two of A, B or C Condone incorrect extras	AO2.3	E1	C: Repeated numbers are possible so same member could be selected twice.
	Total		2	
11 (b)	Gives clear method to achieve a simple random sample. The method suggested here uses the Ranint function on a calculator Any one of A, B or C oe	AO3.5c	E1	A: Allocate every member a number from 1 to 350 B: Use a random number generator to choose a random integer from 1 to 350 C: Continue until 20 different numbers have been identified and select the corresponding members
	Any two of A, B or C	AO3.5c	E1	
	Any three of A, B or C Condone incorrect extras	AO3.5c	E1	
	Total		3	

Q	Marking instructions	AO	Marks	Typical solution
12	Uses area of rectangles to find either total number of cars or number of cars ≥ 30 mph correctly	AO1.1a	M1	Number of cars $= (10 \times 5) + (5 \times 15) + (5 \times 20) + (5 \times 10) + (25 \times 5) = 400$ Number cars ≥ 30 mph $= (5 \times 10) + (25 \times 5) = 175$
	Obtains correct probability accept ACF	AO1.1b	A1	$P(\text{speed} \geq 30) = \frac{175}{400}$
Total			2	
13 (a)	Finds the probabilities for $x = 1$ to 3 using given $f(x)$ at least 1 correct	AO1.1a	M1	$P(X = 1) = \left(\frac{2}{3}\right)^3 + \left(\frac{1}{3}\right)^4 = \frac{25}{81}$ $P(X = 2) = \frac{49}{243}$
	Uses $\sum p_i = 1$ (at least 2 correct p_i s)	AO1.1b	M1	$P(X = 3) = \frac{97}{729}$ $k = 1 - \left[\frac{25}{81} + \frac{49}{243} + \frac{97}{729} \right]$
	Obtains correct exact value for k CAO	AO1.1b	A1	$= \frac{260}{729}$
Total			3	
13 (b)	Correctly finds $P(3 \leq X \leq 4)$ ACF	AO1.1b	B1	$\frac{119}{243}$
Total			1	

Q	Marking instructions	AO	Marks	Typical solution
14 (a)(i)	States weak/some/moderate (nothing stronger than moderate) negative correlation, condone no context. Allow 'no correlation'	AO2.5	B1	Weak negative correlation between purchased quantities of brown bread and purchased quantities of white bread
Total			1	
14 (a)(ii)	Infers James is incorrect	AO2.2b	E1	James is incorrect because the scatter graph does not show trend over the time period of the LDS
	Uses their knowledge of the LDS to explain that trend varies over time	AO3.2b	dE1	Trends vary over time
Total			2	
14 (b)	Calculates a relevant ratio to give a comparison of 2014 and 2011	AO1.1b	B1	Change = $13 - 11 = 2$ Change ratio = $\frac{(13-11)}{11} \equiv \frac{2}{11}$
	Calculates a ratio with rounding considered in both numerator and denominator	AO2.3	M1	Using rounding $\frac{(13.5 \rightarrow 12.5) - (11.5 \rightarrow 10.5)}{(10.5 \rightarrow 11.5)}$ Min % = $\frac{1}{11.5} \times 100 = 8.70\%$
	Produces a convincing argument to show that 11% is justified or states a calculation that gives 11%	AO3.1b	A1	Max % $\frac{3}{10.5} \times 100 = 28.6\%$ $\therefore 11\%$ is within this range so a possible value and hence justified.
Total			3	

Q	Marking instructions	AO	Marks	Typical solution
15 (a)(i)	Uses calculator to find correct probability AWRT 0.106	AO1.1b	B1	0.106
	Total		1	
15 (a)(ii)	Uses calculator to find correct probability AWRT 0.586	AO1.1b	B1	0.586
	Total		1	
15 (b)	Identifies that 30 is a discrete value or T is a continuous/Normal distribution	AO2.4	E1	30 is a discrete value but T is a continuous distribution So $P(T = 30) = 0$ as T takes an infinite number of possible values
	Expresses clearly the idea that $P(\text{exact value}) = 0$ as there are an infinite number of possible values or area above that point would be zero (OE)	AO2.4	E1	
	Total		2	
15 (c)	States correct answer (using inverse normal function on calculator) 41.2 to 41.3 AFWW	AO1.1b	B1	$y = 41.2$
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
15 (d)	Finds $P(T < 0)$ from calculator	AO1.1b	B1	$P(T < 0) = 0.0000884$ As $P(T < 0)$ is so small the model could be valid, although in reality it should be 0
	Makes appropriate comment supporting the model, comparing the probability found to zero	AO3.5a	E1	
Total			2	
15 (e)	Recognises the need to find $P(30 - 8 \leq T \leq 30 + 8)$	AO3.1b	M1	$P(22 \leq T \leq 38)$ $= 0.683$ $X \sim B(9, 0.683)$ $P(X \geq 5) = 0.879$
	Obtains correct probability from calculator (AWRT 0.683)	AO1.1b	A1	
	Selects and uses binomial model use of $B(9, p)$ for 'their' p seen	AO3.3	M1	
	Obtains correct value from calculator (AWRT 0.879)	AO1.1b	A1	
Total			4	

Q	Marking instructions	AO	Marks	Typical solution
16 (a)	States both hypotheses using correct language	AO2.5	B1	$H_0: p = 0.7$ $H_1: p \neq 0.7$ (2 tailed test, 5% level)
	States model used PI by 0.0491 or 0.0208 seen	AO1.1a	M1	$X \sim B(28, 0.7)$ (X = no. who rate app as excellent) $P(X \leq 15) = 0.0491$
	Evaluates probability by calculator, AWRT 0.049	AO1.1b	A1	As $0.0491 > 0.025$ we do not reject H_0
	Evaluates Binomial model by comparing $P(X \leq 15)$ with 0.025 and infers H_0 is accepted/not rejected	AO2.2b	B1	There is insufficient evidence to suggest that the proportion of customers who rate the app as excellent has changed.
	Concludes correctly in context requires 'not sufficient' or 'no' oe	AO3.2a	E1	
	Total		5	
16 (b)	States why Derek thinks that the value of p has fallen to below 55%	AO2.4	E1	$\frac{15}{28} \times 100\% = 53.8\% < 55\%$
	States valid reason why Derek's conclusion is incorrect	AO2.4	E1	Test only confirms that the value of p is unlikely to have changed, not that it has fallen or fallen below a specific value So Derek is incorrect
	Total		2	

Q	Marking instructions	AO	Marks	Typical solution
17 (a)(i)	States clearly the meaning of $A' \cap B$ in context oe	AO3.2a	B1	Ellie does not order nachos as a starter and does order a chicken burrito as a main course.
	Total		1	
17 (a)(ii)	States the event in terms of A and B oe	AO3.1b	B1	$A \cup B$
	Total		1	
17 (b)(i)	Uses $P(B A)$ formula and solves for $P(A)$ ACF	AO1.1b	B1	$P(B A) = \frac{P(B \cap A)}{P(A)} = \frac{9}{13}$ $\rightarrow P(A) = \frac{\frac{9}{20}}{\frac{9}{13}} = \frac{13}{20} = 0.65$
	Total		1	
17 (b)(ii)	Uses expression to find $P(B)$ or shows probabilities on a Venn diagram or	AO1.1a	M1	$P(B) = P(A' \cap B) + P(A \cap B)$ $= \frac{1}{4} + \frac{9}{20} = \frac{7}{10}$ $P(A B) = \frac{\frac{9}{20}}{\frac{7}{10}} = \frac{9}{14} = 0.643$
	Obtains correct value of $P(B)$	AO1.1b	A1	
	Uses $P(A B)$ formula and achieves correct result ACF	AO1.1b	A1	
	Total		3	
17 (c)	States 'no' with a valid reason, some comparison of numerical values required	AO2.4	E1	$P(A \cap B) = \frac{9}{20}$ $P(A)P(B) = \frac{13}{20} \times \frac{7}{10} = \frac{91}{200}$ <p>$P(A \cap B) \neq P(A)P(B)$ so A and B are not independent</p>
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
18	States both hypotheses using correct terminology	AO2.5	B1	$H_0: \mu = 8$ $H_1: \mu > 8$ (1 tailed test, 1% level)
	Finds sample mean	AO1.1b	B1	$\bar{x} = \frac{492.3}{60} = 8.205$
	Considers $\bar{X} \sim N\left(8, \frac{\sqrt{0.2}}{\sqrt{60}}\right)$	AO1.1a	M1	$P(X > 8.205) = 1.92 \times 10^{-4}$ $1.92 \times 10^{-4} < 0.01$ we reject H_0 The evidence suggests that the mean sugar content of the bar has increased
	Obtains correct probability $P(X > 8.205)$	AO1.1b	A1	
	Infers H_0 is rejected by comparison of probability and significance level	AO2.2b	A1	
	Concludes correctly in context, not too definitive	AO3.2a	E1	
	Total		6	
	TOTAL		100	