A-LEVEL
Mathematics

Paper 2
Mark scheme

Practice papers – Set 1

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme has been prepared for practice papers and has not, therefore, been through the process of standardising that would take place for live papers.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the paper
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.1b	B1	$y=\sqrt[3]{x}=x^{\frac{1}{3}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3x^{\frac{2}{3}}}$
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	$n+1$
	Total		**1**	
3	Splits integrand into partial fractions and uses a suitable method to find constant numerators	AO3.1a	M1	$\frac{2}{(x+1)(x+3)}\equiv\frac{A}{(x+1)}+\frac{B}{(x+3)}$ $\Rightarrow 2\equiv A(x+3)+B(x+1)$ let $x=-1\Rightarrow 2=2A$ $\Rightarrow A=1$ let $x=-3\Rightarrow 2=-2B$ $\Rightarrow B=-1$ $\int_1^3\frac{2}{(x+1)(x+3)}\mathrm{d}x=\int_1^3\frac{1}{(x+1)}-\frac{1}{(x+3)}\mathrm{d}x$ $=\left[\ln(x+1)-\ln(x+3)\right]_1^3$ $=(\ln 4-\ln 6)-(\ln 2-\ln 4)$ $=\ln\left(\frac{4\times4}{6\times2}\right)$ $=\ln\left(\frac{4}{3}\right)$
	Obtains both correct numerators	AO1.1b	A1	
	Integrates 'their' partial fractions one term correct	AO1.1a	M1	
	Integrates 'their' partial fractions both terms correct Follow through provided integrand is of the form $\frac{A}{(x+1)}+\frac{B}{(x+3)}$	AO1.1b	A1F	
	Substitutes limits	AO1.1a	M1	
	Completes correct argument to find correct result. Must be clear with no errors	AO2.1	R1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
4 (a)	Starts a rigorous argument by showing that $\mathrm{f}(1)<0$ and $\mathrm{f}(2)>0$ Both attempted at least one evaluated correctly. f must be clearly defined or substitution of values must be explicit.	AO2.1	R1	$\mathrm{f}(x)=\mathrm{e}^{x}-3x$ $\mathrm{f}(1)=\mathrm{e}^{1}-3\times1=-0.281...<0$ $\mathrm{f}(2)=\mathrm{e}^{2}-3\times2=1.389...>0$
	Explains reasoning fully to complete argument. Evaluations above need to be of opposite sign and ‘change of sign’ OE seen and reference to x-values 1 and 2 and reference to continuous function	AO2.4	E1	Change of sign and $\mathrm{f}(x)$ is continuous so a root must lie between $x=1$ and $x=2$
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4 (b)(i)	Uses Newton-Raphson, must have formula or correct x_2	AO1.1a	M1	$x_{n+1} = x_n - \dfrac{e^{x_n} - 3x_n}{e^{x_n} - 3}$ $x_2 = 2 - \dfrac{e^2 - 3\times 2}{e^2 - 3}$
	Obtains both correct values	AO1.1b	A1	$x_2 = 1.684$ $x_3 = 1.543$
	Total		**2**	
4 (b)(ii)	Deduces the value of α	AO2.2a	B1	$\alpha = 1.512$
	Total		**1**	
4 (c)	States the method fails to find α	AO2.4	E1	When $x_1 = 1$ the method fails to find α. There is a second root to which the method converges.
	Infers the method converges to a second root	AO2.2b	E1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5	Forms simultaneous equations for a and d using S_n	AO3.1a	M1	$\frac{20}{2}\left(2a+\left(20-1\right)d\right)=15$ $\Rightarrow 4a+38d=3$
	Obtains two correct equations	AO1.1b	A1	$\frac{40}{2}\left(2a+\left(40-1\right)d\right)=250$ $\Rightarrow 4a+78d=25$
	Uses a correct method to solve 'their' equations with a or d correct	AO1.1a	M1	$d=0.55, a=-4.475$
	Obtains values for a and d from 'their' equations	AO1.1b	A1F	$-4.475+(n-1)\times 0.55<100$
	Forms an inequality using 'their' a and d for u_n < 100 (or uses an equation)	AO1.1a	M1	$n<190.95...$
	Solves 'their' inequality or equation	AO1.1b	A1F	
	Deduces the number of terms of the sequence less than **100**	AO2.2a	A1F	There are 190 terms that are less than 100.
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
6 (a)	Uses compound angle formula to set up two equations for $R\cos\alpha$ and $R\sin\alpha$ PI by Pythagoras and $\tan\alpha$	AO1.1a	M1	$R\cos\theta\cos\alpha - R\sin\theta\sin\alpha \equiv \sqrt{3}\cos\theta - 2\sin\theta$ $R\cos\alpha = \sqrt{3}$ $R\sin\alpha = 2$ $R^2 = 2^2 + \left(\sqrt{3}\right)^2$ $R = \sqrt{7}$ $= 2.65$ $\tan\alpha = \frac{2}{\sqrt{3}}$ $\alpha = 0.857$ $\sqrt{3}\cos\theta - 2\sin\theta \equiv \sqrt{7}\cos(\theta + 0.857)$
	Obtains correct R or α	AO1.1b	A1	
	Obtains correct R and α and writes answer in the correct form	AO1.1b	A1	
	Total		**3**	
6 (b)(i)	Uses compound angle formula	AO3.1a	M1	$x = 8\cos t\cos\frac{\pi}{6} - 8\sin t\sin\frac{\pi}{6} - 2\sqrt{3}\cos t$ $= 8\cos t \times \frac{\sqrt{3}}{2} - 8\sin t \times \frac{1}{2} - 2\sqrt{3}\cos t$ $= 4\sqrt{3}\cos t - 4\sin t - 2\sqrt{3}\cos t$ $= 2\sqrt{3}\cos t - 4\sin t$
	Recalls and uses exact values for cos $\frac{\pi}{6}$ and sin $\frac{\pi}{6}$	AO1.2	B1	
	Completes clear and correct argument to show given result AG	AO2.1	R1	
	Total		**3**	
6 (b)(ii)	Deduces max distance using 'their' R	AO2.2a	B1F	$x = 2\left(\sqrt{3}\cos t - 2\sin t\right)$ $= 2\sqrt{7}\cos(t + 0.857)$ max distance $= 2\sqrt{7}$
	Total		**1**	
6 (b)(iii)	Deduces earliest time using 'their' α	AO2.2a	B1F	$t = \pi - 0.857$ $= 2.28$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7 (a)	Uses x or y coordinate to set up and solve an equation to find possible values for θ	AO3.1a	M1	$2\cos\theta = \sqrt{3}$ $\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ If $\theta = \frac{11\pi}{6}$ $y = 3\sin\frac{22\pi}{6} = -\frac{3\sqrt{3}}{2} < 0$ $\therefore \theta \neq \frac{11\pi}{6}$ $\therefore \theta = \frac{\pi}{6} \quad \left(\Rightarrow x = \sqrt{3} \text{ and } y = \frac{3\sqrt{3}}{2}\right)$
	Obtains value(s) for θ	AO1.1b	A1	
	Gives clear justification of why $\theta = \frac{\pi}{6}$	AO2.4	E1	
	Uses a correct method to find $\frac{dy}{dx}$	AO3.1a	M1	$\frac{dy}{d\theta} = 6\cos 2\theta$ $\frac{dx}{d\theta} = -2\sin\theta$ $\frac{dy}{dx} = \frac{6\cos 2\theta}{-2\sin\theta}$ when $\theta = \frac{\pi}{6}$ $\frac{dy}{dx} = -3\frac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{6}\right)} = -3$ $\therefore$ gradient of normal $m = \frac{1}{3}$ $y - \frac{3\sqrt{3}}{2} = \frac{1}{3}\left(x - \sqrt{3}\right)$ $3y - \frac{9\sqrt{3}}{2} = x - \sqrt{3}$ $3y - x = \frac{7\sqrt{3}}{2}$
	Obtains correct value for $\frac{dy}{dx}$	AO1.1b	A1	
	Obtains correct value for gradient of normal	AO1.1b	A1F	
	Obtains equation of normal in any correct form	AO1.1a	M1	
	Completes correct argument to show given result. Must be clear with no errors	AO2.1	R1	
	Total		**8**	

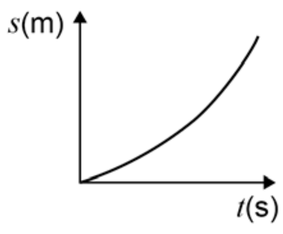
Q	Marking instructions	AO	Marks	Typical solution
7 (b)	Uses trigonometric identity to partially eliminate θ	AO3.1	M1	$y = 3\sin 2\theta$ $= 6\sin\theta\cos\theta$ $= (3\sin\theta)(2\cos\theta)$ $= (3\sin\theta)(x)$
	Uses second identity to complete elimination of θ	AO3.1	M1	$y^2 = (9\sin^2\theta)x^2$ $= 9(1-\cos^2\theta)x^2$ $= 9\left(1-\frac{x^2}{4}\right)x^2$ $4y^2 = 9(4-x^2)x^2$ $4y^2 + 9x^4 - 36x^2 = 0$
	Deduces correct values of a, b and c Accept integer multiples	AO1.1b	A1	
	Completes correct full solution with no algebraic slips	AO2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8	Uses implicit differentiation	AO3.1a	M1	$y^2 - 12xy + 3x^2 + 44 = 0$ $\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 12y - 12x\frac{\mathrm{d}y}{\mathrm{d}x} + 6x = 0$ Stationary points occur when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x = 2y$ $\Rightarrow y^2 - 24y^2 + 12y^2 + 44 = 0$ $\Rightarrow y = \pm 2$ Two solutions $\Rightarrow$ two stationary points The stationary points lie on the line $x = 2y$ and this line passes through the origin.
	Differentiates correctly	AO1.1b	A1	
	States that stationary points occur when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	AO2.4	R1	
	Uses $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to find x in terms of y (or vice versa)	AO1.1a	M1	
	Obtains $x = 2y$ oe	AO1.1b	A1	
	Deduces x or y values	AO2.2a	A1F	
	Correctly concludes argument to show that there are two stationary points	AO2.1	R1	
	States that the stationary points lie on the line and explains that this line also passes through the origin	AO2.4	R1	
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
9	Circles correct answer	AO1.1b	B1	$\mathbf{p} - \mathbf{q} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ $\lvert \mathbf{p} - \mathbf{q} \rvert = \sqrt{(-3)^2 + 4^2}$ Circles 5
	Total		**1**	
10	Circles correct answer	AO1.1b	B1	$a = \frac{dv}{dt}$ Circles $-3\sin(3t)$
	Total		**1**	
11 (a)(i)	Sums the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ correctly	AO1.1b	B1	$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ $5\mathbf{i} + 2\mathbf{j}$
	Total		**1**	
11 (a)(ii)	Uses a trig expression with appropriate values	AO1.1a	M1	$\tan\theta = \frac{2}{5}$ $\theta = \tan^{-1}\left(\frac{2}{5}\right)$ $= 21.8°$
	Obtains correct angle given to nearest 0.1º	AO1.1b	A1	
	Total		**2**	
11 (b)	Obtains correct force using ‘their’ part **(a)(i)**	AO1.1b	B1F	$-\mathbf{i} + 3\mathbf{j} = 5\mathbf{i} + 2\mathbf{j} + \mathbf{F}_3$ $\mathbf{F}_3 = -6\mathbf{i} + \mathbf{j}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12 (a)	States that reaction force at *C* is zero (PI)	AO1.2	B1	$R_C = 0$
	Uses correct forces to form a moment equation (PI) about any point (may include reaction at *C*)	AO1.1a	M1	$mg \times 0.6 = 6g \times 0.5$
	Obtains correct value for m	AO1.1b	A1	$m = 5$
	Total		**3**	
12 (b)	Selects a method by either taking moments about *C* or taking moments about another point and resolving vertically	AO3.1b	M1	Let d = distance from A $2g \times (d-1) + 6g \times 1 = \frac{13g}{2} \times 1.5$
	Obtains correct moments equation about *C* or correct simultaneous equations.	AO1.1b	A1	$2d - 2 + 6 = 9.75$ $d = 2.875$
	Solves to find either R_C ,distance from A or another relevant distance	AO1.1a	M1	**Alternative solution** Taking moments about A: $2g \times d + 6g \times 2 = R_c \times 1 + \frac{13g}{2} \times 2.5$
	Obtains correct value for the distance from *A*	AO1.1b	A1	Resolving vertically: $\frac{13g}{2} + R_c = 8g$ $R_c = \frac{3g}{2}$ $d = 2.875$
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
13	Integrates both components with at least one correct	AO1.1a	M1	$\mathbf{s} = \int \left(1 - 2t^2\right) \mathrm{d}t\,\mathbf{i} + \int \left(2t\right) \mathrm{d}t\,\mathbf{j}$
	Obtains correct terms (condone missing constants)	AO1.1b	A1	$\mathbf{s} = \left(t - \frac{2}{3}t^3 + c\right)\mathbf{i} + \left(t^2 + k\right)\mathbf{j}$
	Evaluates both constants (or uses definite integration) using 'their' expression for **s**	AO3.4	M1	$t = 1, r = 2\mathbf{i}$ $2 = 1 - \frac{2}{3} + c \Rightarrow c = \frac{5}{3}$ $0 = 1 + k \Rightarrow k = -1$
	Obtains correct expression for **s** at time t	AO1.1b	A1	$\mathbf{s} = \left(t - \frac{2}{3}t^3 + \frac{5}{3}\right)\mathbf{i} + \left(t^2 - 1\right)\mathbf{j}$
	Obtains 'their' correct displacement by substituting $t = 2$ into 'their' expression for **s**	AO1.1b	A1F	$\mathbf{s} = -\frac{5}{3}\mathbf{i} + 3\mathbf{j}$
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
14 (a)	Forms one correct equation for involving U and T	AO1.1a	M1	$a = \frac{3U - U}{T}$ $0.68 = \frac{2U}{T}$
	Obtains two correct equations	AO1.1b	A1	$s = \frac{1}{2}(U + 3U)T$ $17 = 2UT$
	Eliminates either U or T	AO1.1a	M1	$2U = 0.68T$ $\Rightarrow 17 = 0.68T^2$
	Obtains 'their' correct value for U	AO1.1b	A1F	$T = 5$
	Obtains 'their' correct value for T ft provided both M1s obtained	AO1.1b	A1F	$U = 1.7$
	Total		**5**	
14 (b)	Correct shape from (0, 0)	AO3.4	B1	
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15 (a)	Forms two equations of motion at least one correct (condone not converting g to kg)	AO3.3	M1	$0.85g - T = 0.85a$ $T - 0.65g = 0.65a$
	Obtains two correct equations of motion	AO3.3	A1	Adding: $0.85g - 0.65g = 0.85a + 0.65a$
	Uses model, solves 'their' equations to find acceleration and time	AO3.4	M1	a = 1.3 m s^{-2} T = 7.2 N
	Obtains correct values of acceleration and tension to 2sf, including units	AO3.2a	A1F	
	States clear assumption concerning the peg being smooth	AO3.3	E1	
	Total		**5**	
15 (b)	States one correct comment	AO3.5b	E1	The inextensibility of the string means that the masses have the same magnitude of acceleration.
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
16 (a)	Uses friction as $F = \mu R$	AO1.1b	B1	$F = \frac{1}{3}R$
	Resolves forces in two different directions to obtain equations involving *P* and *R* (allow *F* in the wrong direction)	AO3.3	M1	Resolving perpendicular: $R = 2g\cos\theta$ Resolving parallel: $P + F = 2g\sin\theta$
	Obtains correct equations	AO3.3	A1	
	Uses 'their' model and solves equations to find P	AO3.4	M1	$R = 2g \times \frac{4}{5} = \frac{8g}{5}$ $P + \frac{1}{3} \times \frac{8g}{5} = 2g \times \frac{3}{5}$
	Completes a rigorous argument that results with correct *P* **AG** (Only award if completely correct and easy to follow)	AO2.1	R1	$P = \frac{2}{3}g$
	Total		**5**	
16 (b)	Uses *F* = *ma* down the plane to obtain an expression for *a*	AO3.4	M1	$2g\sin\theta - \mu R = 2a$ $\frac{6g}{5} - \frac{1}{3} \times 2g \times \frac{4}{5} = 2a$
	Obtains correct value for a	AO1.1b	A1	$a = \frac{g}{3}$
	Uses kinematics formula and 'their' a to find an expression for v	AO3.4	M1	$v^2 = 0^2 + 2 \times \frac{g}{3} \times \frac{1}{6}$
	Obtains correct value in terms of g (need not be simplified)	AO1.1b	A1F	$v = \sqrt{\frac{g}{9}} = \frac{\sqrt{g}}{3}$
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
17	Obtains one of $\overrightarrow{AB}$, $\overrightarrow{AC}$ or $\overrightarrow{BC}$	AO1.1a	M1	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ \sqrt{3}-2 \\ 4 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 8+4\sqrt{3} \\ -1 \\ 8+4\sqrt{3} \end{pmatrix}$ $\overrightarrow{AC} = \left(2+\sqrt{3}\right)\begin{pmatrix} 4 \\ \sqrt{3}-2 \\ 4 \end{pmatrix} = \left(2+\sqrt{3}\right)\overrightarrow{AB}$ $\therefore \overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel Since the vectors are parallel and have a common point *A*, *B* and *C* must be collinear.
	Obtains second correct vector	AO1.1b	A1	
	Deduces correct scale factor for ‘their’ vectors and states that they must be parallel	AO2.2a	A1	
	Constructs correct argument to show collinearity Must state parallel vectors and common point to achieve this mark	AO2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
18 (a)	Forms an equation for the horizontal displacement	AO3.1b	M1	$45.4 = (28.5\cos 30°)t$
	Obtains correct time	AO1.1b	A1	t = 1.84
	Forms an equation for the vertical displacement	AO3.1b	M1	$h = (28.5\sin 30)t - \frac{1}{2}\times 9.81t^2$
	Substitutes in 'their' time	AO1.1a	M1	h = 9.61
	Obtains correct value for the vertical height at 'their' time	AO1.1b	A1F	9.61 > 9.5 So the ball will clear the tree
	Compares 'their' height with the known height of the tree and gives a conclusion	AO3.2a	E1F	
	Total		**6**	
18 (b)	Explains that the horizontal velocity has been assumed constant and this is unlikely to be true or considers the actual shape and size of the tree in comparison with the difference between the height of the ball and tree	AO3.5c	E1	It has been assumed there are no resistive forces acting on the ball. This is unlikely to be true in reality. Air resistance would slow the ball down
	Total		**1**	
	TOTAL		**100**	