Surname	
Other Names	
Candidate Signature	

YORKSHIRE MATHS TUTOR

Centre Number						Candidate Number				

Examiner Comments	Total Marks
Yorkshire Maths Tutor	

MATHEMATICS

A LEVEL PAPER 3

CM

Bronze Set A (Edexcel Version)

Time allowed: 2 hours

Instructions to candidates:

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer ALL of the questions.
- You must write your answer for each question in the spaces provided.
- You may use a calculator.

Information to candidates:

- Full marks may only be obtained for answers to ALL of the questions.
- The marks for individual questions and parts of the questions are shown in round brackets.
- There are 12 questions in this question paper. The total mark for this paper is 100.

Advice to candidates:

- You should ensure your answers to parts of the question are clearly labelled.
- You should show sufficient workings to make your working clear to the Examiner.
- Answers without working may not gain full credit.

A2/M/P3

1 0 3 3 1 3 1 3 8 0 0 0 4

Section A: Statistics

1 Amir is investigating daily mean windspeed using the large data set. He catagorises the available data from an overseas location in 2015 according to the Beaufort scale.

The Beaufort scale assigns a number between 0 and 12 to each windspeed, where 0 represents calm winds and 12 represents hurricanes.

Amir believes that the random variable W, representing the daily mean windspeed on the Beaufort scale, can be modelled using a discrete uniform distribution.

(a) Write down the probability distribution of W. **(2)**

(b) Using the model, find $P(2 < W \leq 7)$. **(1)**

The table above shows Amir's catagorisation of the data from his overseas location according to the Beaufort scale.

Beaufort number	0	1	2	3	4	5	6	≥ 7
Windspeed (knots)	< 1	1–3	4–6	7–10	11–16	17–21	22–27	> 28
Frequency	0	0	63	84	37	0	0	0

(c) Comment on the suitability of Amir's model in the light of this information. **(1)**

Amir also collects information about the daily mean temperature, t °C, for this same region. He finds that $\bar{t} = 15.2$.

(d) Using your knowledge of the large data set, suggest, giving reasons, the location that gave rise to these data. **(1)**

a)

w	P(W=w)	w	P(W=w)
0	$\frac{1}{13}$	7	$\frac{1}{13}$
1	$\frac{1}{13}$	8	$\frac{1}{13}$
2	$\frac{1}{13}$	9	$\frac{1}{13}$
3	$\frac{1}{13}$	10	$\frac{1}{13}$
4	$\frac{1}{13}$	11	$\frac{1}{13}$
5	$\frac{1}{13}$	12	$\frac{1}{13}$
6	$\frac{1}{13}$		

Question 1 continued

b) $P(2 < w \leq 7) = \frac{5}{13}$

c) Not suitable. Probability of each Beaufort number is not equal.

d) High average windspeeds so could be close to the sea. Lower temperature - could be Perth.

TOTAL 5 MARKS

2 The number of atoms in a radioactive substance, n, is measured at various times, t minutes, after the start of an experiment. The table below shows the data collected.

Time (t)	1	3	4	6	10
Atoms (n)	320	160	100	63	13
log(n)	2.51	2.20	2.00	1.80	1.11

The data are coded according to the change of variables $x = t$ and $y = \log(n)$.

(a) Complete the table above.
Give all your values to two decimal places. **(1)**

(b) (i) State what is measured by the product moment correlation coefficient. **(1)**

(ii) Calculate the product moment correlation coefficient for the coded data. **(1)**

(c) With reference to part (b), explain why an exponential model is a good fit for these data. **(1)**

The equation of the regression line for y on x is $y = -0.152x + 2.656$.

(d) Express n in terms of t. Give your answer in the form $n = pq^t$, where the constants p and q should be determined to three significant figures. **(3)**

b)(i) Whether there is a linear relationship between 2 variables

(ii) Classwiz Calculator r=-0.997247903

Menu 6
2: y = a + bx
enter time in x, log(n) in y column
$\boxed{\text{OPT}}$
4: Regression Calc

c) r=-0.997 is almost perfect negative correlation for log(n) against t which indicates an exponential curve.

Question 2 continued

d) $y = -0.152x + 2.656$

$n = pq^t$

$\log n = \log pq^t$

$\log n = \log p + t \log q$

$\log p = 2.656$

$p = 10^{2.656} = 452.8975$

$p = 453$ (3sf)

$\log q = -0.152$

$q = 10^{-0.152} = 0.704693$

$q = 0.705$ (3sf)

$\therefore \quad n = 453 \times 0.705^t$

TOTAL 7 MARKS

3 Chris is a teacher at a school wants to investigate the amount of time spent, in minutes, by students on their homework.

One morning, Chris stands at the school gates and asks a sample of 45 of the students how long they spent doing homework the previous evening.

(a) State the sampling technique used by Chris. **(1)**

Chris summarises his data in a grouped frequency table and represents it on the histogram in Figure 1 below.

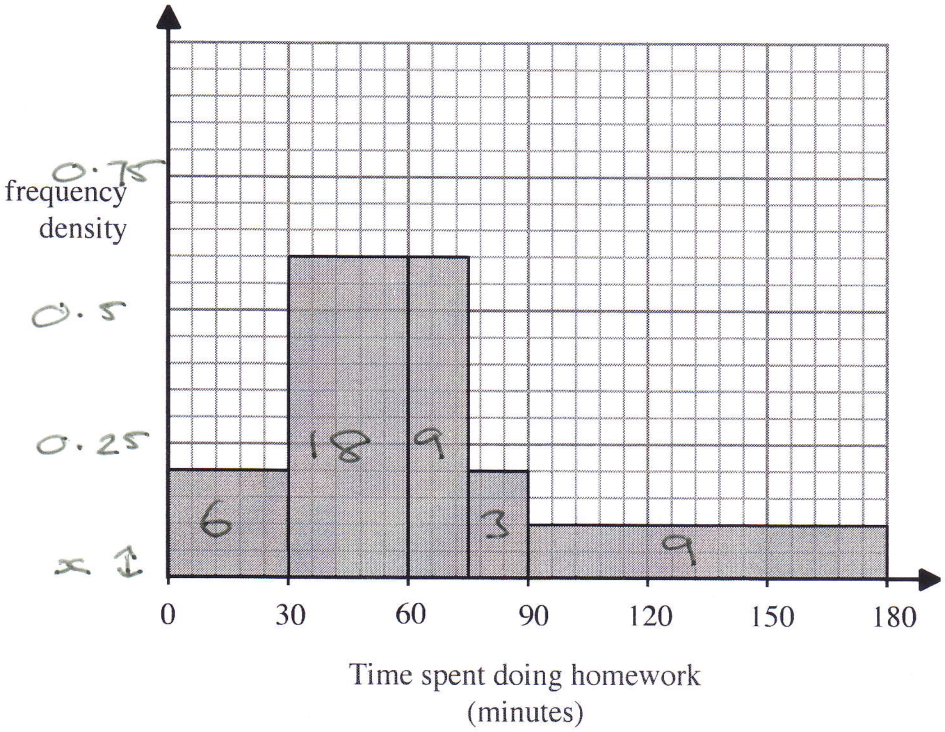


Figure 1

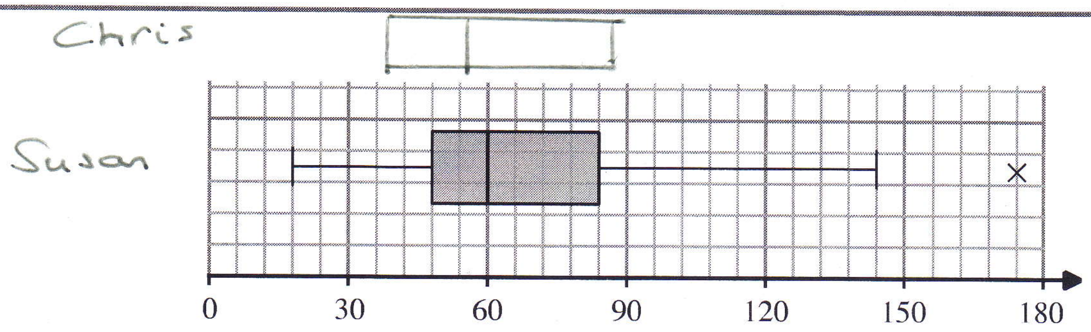
(b) Find an estimate for the mean of Chris's data. **(3)**

Chris also calculates the quartiles of his data as

$$Q_1 = 38 \qquad Q_2 = 56 \qquad Q_3 = 88$$

Susan is a teacher at another school. She also collects data on the time spent doing homework through a survey given to all 1100 students at her school.

The box-plot in Figure 2 on Page 7 shows her data.



Time spent doing homework
(minutes)

Figure 2

(c) Explain why it is better to use the median and interquartile range to compare Chris and Susan's data as opposed to the mean and standard deviation. **(1)**

(d) Suggest, giving suitable reasons, whether students spend more time doing homework at Chris' school or Susan's school. **(2)**

(e) State **one** limitation when comparing Chris and Susan's data. **(1)**

a) Opportunity (or convenience) sampling

b) Area shaded grey on histogram represents 45 people
let x be one unit on frequency density axis

$(30 \times 4x) + (30 \times 12x) + (15 \times 12x) + (15 \times 4x)$
$+ (90 \times 2x) = 45$
$900x = 45$
$x = 0.05$

$30 \times 4x = 6$
$30 \times 12x = 18$
$15 \times 12x = 9$
$15 \times 4x = 3$
$90 \times 2x = 9$

Time	f	Midpt	f×x
0-30	6	15	90
30-60	18	45	810
60-75	9	67.5	607.5
75-90	3	82.5	247.5
90-180	9	135	1215
	45		2970

Estimated mean $= \frac{2970}{45} = 66$

Question 3 continued

c) Susan's data has an outlier, therefore it will be better to use the median and IQR to compare the data, as the outlier will affect the mean and standard deviation.

d) Susan's data has a higher median and also a lower spread for the IQR (more consistency, which suggests students spend more time at Susan's school doing homework.

e) Chris only used a very small sample of 45 students.

TOTAL 8 MARKS

4 Entry to a certain university is determined by a national test. The test is sat and the scores on this test are normally distributed with a mean μ and standard deviation σ.

The probability that a randomly selected student scores at least 50 marks in the test is 0.804.

About 11% of students score less than 40 marks.

$1 - 0 \cdot 804$
$= 0 \cdot 196$

(a) Find the mean and standard deviation of the test. **(4)**

To be admitted into the university, a student must score better than at least 70% of the students that took the test.

Adam took the test and scored 102 marks.

(b) Will Adam be admitted into the university? **(2)**

In another test, the marks obtained by students M last year were normally distributed with a mean of 160 and standard deviation of 50.

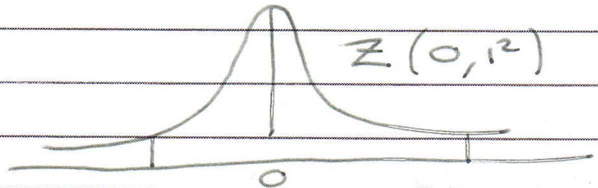
A random sample of 12 students who took the test this year is selected. The mean mark scored by these students was 140.

(c) Stating your hypotheses clearly and using a 5% level of significance, test whether or not the mean mark obtained by students this year has reduced. **(5)**

a) Using Z distribution

$P(Z > 50) = \frac{50 - \mu}{\sigma} = -0 \cdot 85599$ ①

$P(Z < 40) = \frac{40 - \mu}{\sigma} = -1 \cdot 2265$ ②



0·11 → p= -1·2265

0·196 → p= -0·85599

Menu 7
3. Inverse Normal
Area=0·11
σ=1
μ=0

① ⇒ $\frac{50 - \mu}{-0 \cdot 85599} = \sigma$

sub in ② $\frac{40 - \mu}{\left(\frac{50 - \mu}{-0 \cdot 85599}\right)} = -1.2265$

Question 4 continued

$$-0.85599\frac{(40-\mu)}{50-\mu} = -1.2265$$

$$-0.85599 \quad (40-\mu) = -1.2265(50-\mu)$$

$$-34.2396+0.85599\mu = -61.325+1.2265\sigma$$

$$-34.2396+61.325 = 1.2265\mu - 0.85599$$

$$27.0854 = 0.37051\mu$$

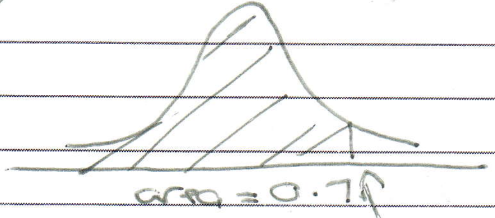
$$\mu = 73.10302$$

$$\Rightarrow \sigma = \frac{50-73.10302}{-0.85599} = 26.9898$$

$$\therefore \sigma = 27.0 \quad (3sf)$$

$$\mu = 73.1$$

b) $X \sim N(73.1, 27^2)$



Menu 7
3 Inverse Normal
$\sigma = 27$
$\mu = 73.1$

Adam scored 102, well above the 87.25 required, so will get into university

c) $\bar{M} \sim N(160, \frac{50^2}{12})$ Sample of 12

sample mean

$s.d = \sqrt{\frac{50^2}{12}} = 14.4337$

$H_0: \mu = 160 \quad H_1: \mu < 160$

5% significance level

TOTAL 11 MARKS

Menu 7

3. Inverse Normal

Area = 0·05

$\sigma = 14·4337$

$\mu = 160$

136·258
critical value

as 140 is not in the critical region, accept Ho

There is no evidence to suggest that the mean has reduced from 160.

5 The results of a survey show that at a college, 29% of students own a bike.

A sample of 8 students from the college is taken.

The random variable X represents the number of students in the sample that own a bike.

(a) Define a suitable distribution for the random variable X. **(1)**

(b) Find $P(X \geq 4)$. **(1)**

Another sample of n students from the college is taken. The probability that at least one of the students owns a bike is greater than 0.976.

(c) Find the least possible value of n. **(3)**

At another large college, the probability that a student owns a bike is 48%.

A sample of 200 students at the college is taken.

(d) Giving a justification for your choice, use a suitable approximation to estimate the probability that fewer than 88 students own a bike. **(4)**

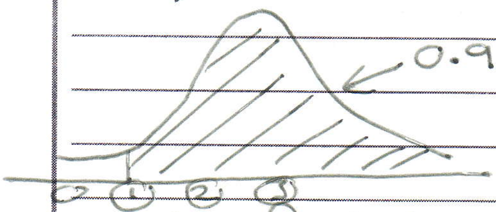
a) $X \sim B(8, 0.29)$

b) $P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.82365$

$= 0.1763$

Menu 7
↓
1: Binomial CD
2: Variable
$x = 3$
$n = 8$
$p = 0.29$



c) Probability at least one owns a bike is $P(X=0) < 0.024$

$$P(X=0) = \underbrace{{}^nC_0}_{=1} \underbrace{0.29^0}_{=1} \times 0.71^n$$

$\therefore$ $P(X=0) < 0.024$

$0.71^n < 0.024$

$\log 0.71^n < \log 0.024$

$n \log(0.71) < \log 0.024$

$n > \dfrac{\log 0.024}{\log 0.71} \Rightarrow n > 10.789$

$n = 11$ is least value of n

Question 5 continued

d) approximate binomial with normal distribution as

$n = 200$ and $p = 0.48$

↑ (close to 0.5)

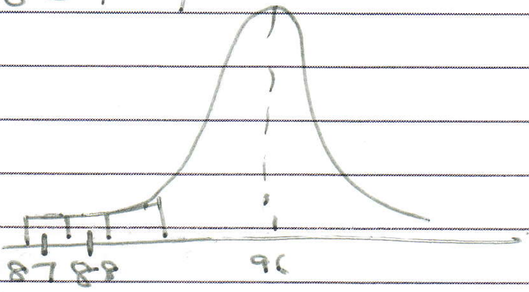
n large

$\mu = np = 200 \times 0.48 = 96$

$\sigma = \sqrt{np(1-p)} = \sqrt{96(0.52)} = 7.0654$

$X \sim N(96, 7.0654^2)$

$P(x < 88)$



Using continuity correction

$P(X < 87.5)$

Menu 7
2: Normal CD
Lower 0
Upper 87.5
$\sigma = 7.0654$
$\mu = 96$

$\rightarrow 0.11447$

$p = 0.114$ (3sf)

TOTAL 9 MARKS

6 Given that

$$P(A) = 0.37, \qquad P(B) = 0.63 \quad \text{and} \quad P(A \cup B) = 0.77$$

find

(a) $P(A \cap B)$ **(1)**

(b) $P(A' | B')$ **(2)**

The event C has $P(C) = 0.2$.

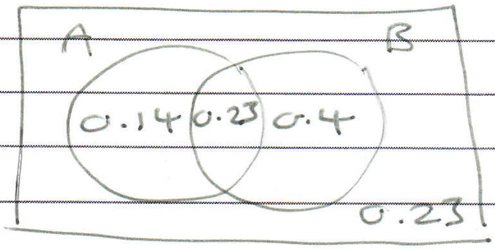
The events A and C are independent events and the events B and C are mutually exclusive.

(c) Find $P(A \cap C)$. **(1)**

(d) Draw a Venn diagram to illustrate the events A, B and C and the probabilities for each region. **(4)**

(e) Find $P\left(A | [B \cup C]'\right)$. **(2)**

a)



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \; P(A \cap B) = P(A) + P(B) - P(A \cup B)$

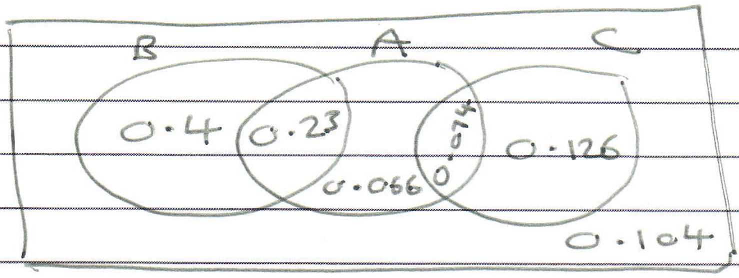
$= 0.37 + 0.63 - 0.77$

$= 0.23$

b) $P(A' | B') = \dfrac{0.23}{0.14 + 0.23}$ ← not A given B' ← not B

$= \dfrac{23}{37} = 0.6216$

Question 6 continued



c) If A and C Independent

$P(A) \times P(C) = P(A \cap C)$

$0.37 \times 0.2 = 0.074$

d) $0.2 - 0.074 = 0.126$

$P(C) - P(A \cap C)$

$0.066 = 0.37 - 0.23 - 0.074$

$0.104 = 1 - (0.4 + 0.23 + 0.066 + 0.074 + 0.126)$

e) $P(B \cup C)' = 0.066 + 0.104$

$\therefore P(A \mid P[B \cup C]') = \frac{0.066}{0.066 + 0.104}$

$= \frac{33}{85} = 0.38823$

TOTAL 10 MARKS

TOTAL FOR SECTION A IS 50 MARKS

Section B: Mechanics

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

7 [In this question, **i** and **j** are perpendicular unit vectors.]

Four forces F_1, F_2, F_3 and F_4 act on a particle P with mass 2 kg, where, in Newtons,

$$F_1 = \mathbf{i} + 2\mathbf{j}$$

$$F_2 = -3(\mathbf{i} - \mathbf{j})$$

$$F_3 = 4\mathbf{i} - \mathbf{j}$$

$$F_4 = a\mathbf{i} + b\mathbf{j}$$

and a and b are scalar constants.

Given that initially the particle does not move,

(a) find the values of a and b. **(3)**

The fourth force F_4 is now removed.

(b) Find the magnitude of the acceleration of the particle. **(4)**

a) $\underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3} + \underset{\sim}{F_4} = 0$

$$\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}-3\\3\end{pmatrix} + \begin{pmatrix}4\\-1\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}2+a\\4+b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$a = -2, \quad b = -4$

b) $\underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3} = \begin{pmatrix}2\\4\end{pmatrix}$

$\underset{\sim}{F} = m\underset{\sim}{a}$ $\qquad m = 2$

$$\underset{\sim}{a} = \frac{1}{2}\begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$$

$$|\underset{\sim}{a}| = \sqrt{1^2 + 2^2} = \sqrt{5} \text{ ms}^{-2}$$

8 A particle P moves on the x-axis.

The displacement of the particle, s m, from a fixed origin O at time t s is given by

$$s = at^3 + bt^2 + ct + d, \qquad t \geq 0$$

where a, b, c and d are constants.

At time $t = 0$, the particle is at the origin with velocity 5 m s^{-1}.

The initial acceleration of the particle is 4 m s^{-2}.

(a) Show that $d = 0$ and find the values of b and c. **(4)**

The particle attains its maximum velocity at time $t = 2$.

(b) Find the value of a. **(2)**

(c) Find the time(s) at which the particle is at rest. **(2)**

a) $\underset{\sim}{s} = at^3 + bt^2 + ct + d$ ①

integrating

$\underset{\sim}{v} = 3at^2 + 2bt + c$ ②

Using ① at $t=0$, $s=0$

$\therefore \; 0 = 0 + 0 + 0 + d$

$\Rightarrow d = 0$

Using ② at $t=0$, $v=5$

$5 = 0 + 0 + c$

$\Rightarrow c = 5$

Integrating ②

$\underset{\sim}{a} = 6at + 2b$

at $t=0$, $\underset{\sim}{a} = 4 \text{ ms}^{-2}$

$4 = 0 + 2b \quad \Rightarrow b = 2$

b) at $t=2$, max velocity means $\underset{\sim}{a} = 0$

$0 = 6a \times 2 + 2 \times 2$

$0 = 12a + 4 \quad \Rightarrow a = -\frac{1}{3}$

Question 8 continued

c) at rest, $\underset{\sim}{v} = 0$

$0 = 3\times -\frac{1}{3}t^2 + 2\times 2\times t + 5$

$0 = -t^2 + 4t + 5$

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$t = 5$ or $t = -1$

↑

impossible

TOTAL 8 MARKS

9

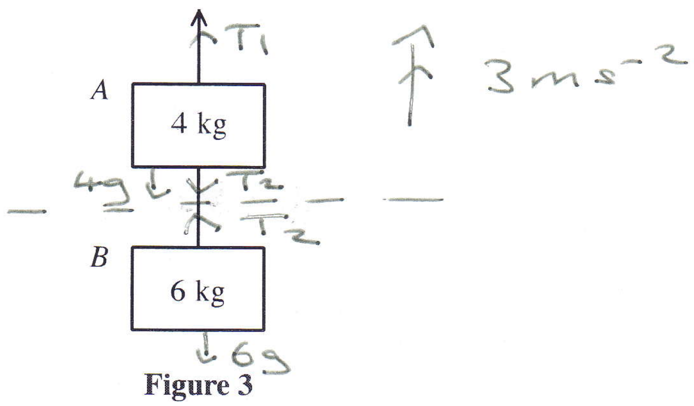


Figure 3

Two masses *A* and *B* are hung from another and are connected by a light inextensible string, as shown in Figure 3 above. The mass of *A* is 4 kg and the mass of *B* is 6 kg. The upper string is then pulled upwards so that the system accelerates upwards at 3 m s^{-2}.

Find the tension in the upper string and the tension in the lower string. **(4)**

Overall system

$10 \times 3 = T_1 + T_2 - T_2 - 4g - 6g$

$30 = T_1 + T_2 - 10g$ ①

$T_1 + T_2 - T_2 = 30 + 10g = 128\text{ N}$ (upper)

6 kg mass only

$T_2 - 6g = 6 \times 3$

$T_2 = 18 + 6g$

$T_2 = 76.8\text{ N}$ (lower

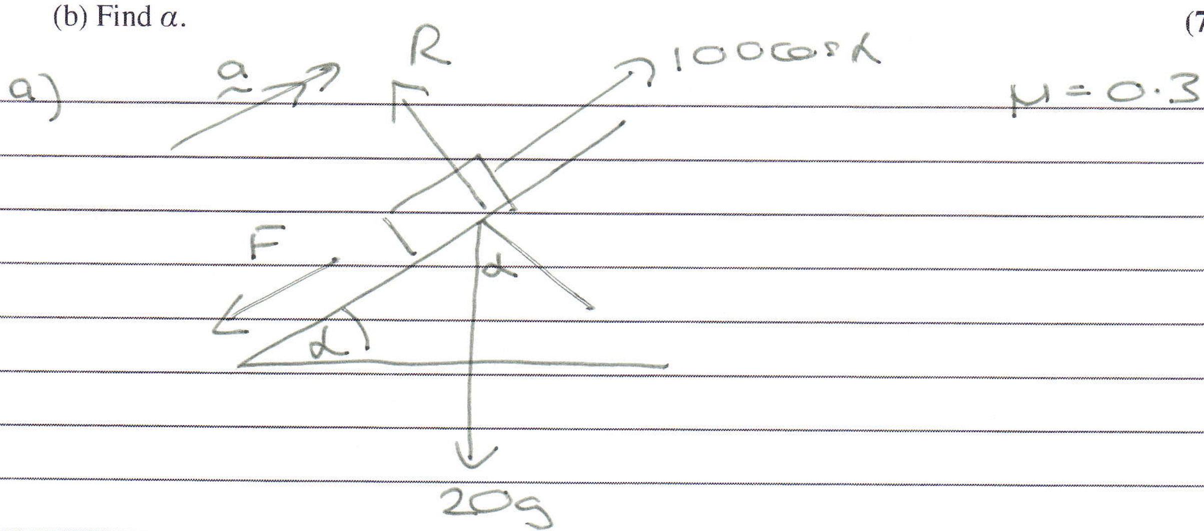
10 A child of mass 20 kg is sitting in a light sledge on a hill that is inclined at $\alpha°$ to the horizontal. His brother is trying to pull the sledge up the hill using a rope that is parallel to the slope of the hill. When the tension in the rope is $100\cos(\alpha)$, the sledge is on the point of moving up the slope. The coefficient of friction between the hill and the sledge is 0.3.

The child in the sledge is modelled as a particle, the hill is modelled a rough horizontal plane and the rope is modelled as a light inextensible string.

(a) Draw a diagram to illustrate this problem, showing all the forces acting on the sledge. **(2)**

(b) Find α. **(7)**

a)



b) Equation of motion parallel to plane ($\nearrow$)

$20\times0 = 100\cos\alpha - F - 20g\sin\alpha$ ①

Equation of motion perpendicular to plane ($\nwarrow$)

$20\times0 = R - 20g\cos\alpha$ ②

③ Limiting friction $F = \mu R$ ③

sub $F = 0.3R$ $\qquad F = 0.3R$

and $R = 20g\cos\alpha$ in ①

$0 = 100\cos\alpha - 0.3R - 20g\sin\alpha$

$0 = 100\cos\alpha - 0.3\times20g\cos\alpha - 20g\sin\alpha$

$20g\sin\alpha = 100\cos\alpha - 58.8\cos\alpha$

$196\sin\alpha = 41.2\cos\alpha$

$\frac{\sin\alpha}{\cos\alpha} = \frac{41.2}{196} \Rightarrow \tan\alpha = \frac{103}{490}$

$\alpha = 11.87°$

11

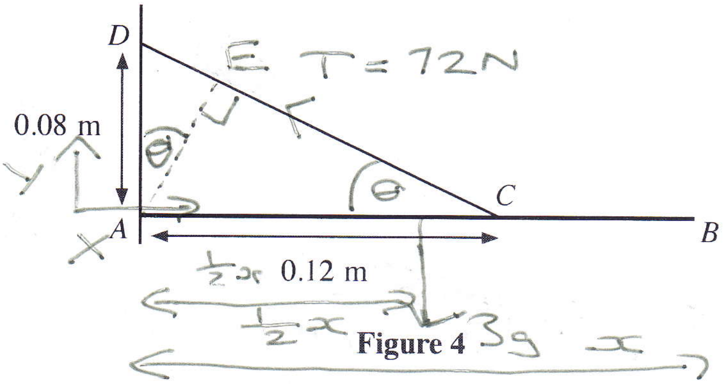


Figure 4

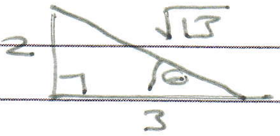
A uniform rod *AB* of mass 3 kg is freely hinged at *A* to a vertical wall. The beam is held in equilibrium by a rope that is attached to a point *C* on the beam, where *AC* = 0.12 m, as shown in Figure 4. The rope is attached to the point *D* on the wall vertically above *A*, where *AD* = 0.08 m.

The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 72 N.

(a) Find the length of *AB*. **(5)**

(b) Find the magnitude of the resultant reaction of the hinge on the beam at *A*. **(5)**

a) $\tan\theta = \frac{0.08}{0.12} = \frac{2}{3}$



$\sin\theta = \frac{2}{\sqrt{13}}$

$\cos\theta = \frac{3}{\sqrt{13}}$

$AE = 0.08\cos\theta$

$= 0.08 \times \frac{3}{\sqrt{13}} = 0.066564...\ m$

$m(A)\quad 0.066564 \times 72 = \frac{1}{2}x \times 3g$ ①

$AB = x = \frac{0.066564 \times 72 \times 2}{3g}$

$= 0.32602$

$AB = 0.33\ m\ (2sf)$

Question 11 continued

b) R (↑) $Y + 72\sin\theta = 3g$

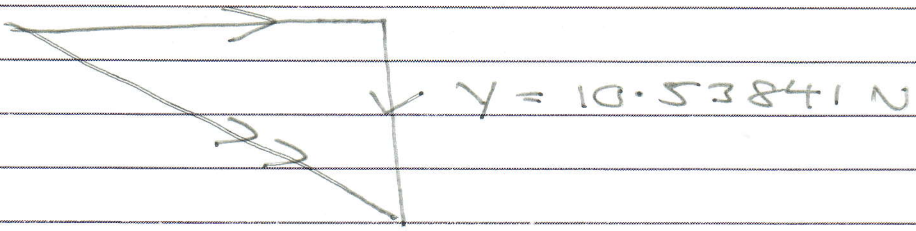
$Y = 3\times 9.8 - 72\times\frac{2}{\sqrt{13}}$

$Y = -10.53841$ N

R (→) $X = 72\cos\theta$

$X = 72\times\frac{3}{\sqrt{13}} = 59.90762$N

$X = 59.90762$



Resultant = $\sqrt{X^2 + Y^2}$

$= \sqrt{59.90762^2 + 10.53841^2}$

$= 60.827$

$= 61$ N (2sf)

TOTAL 10 MARKS

12

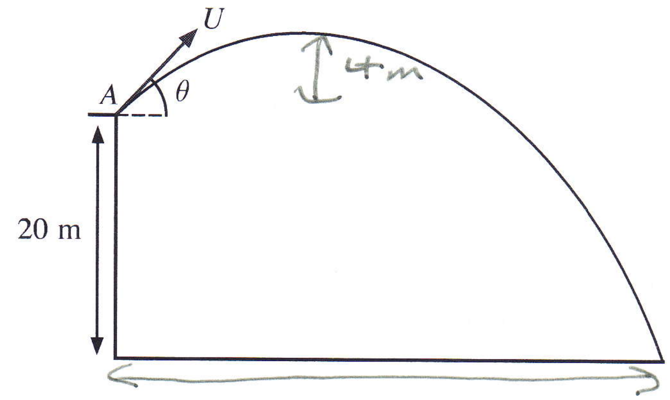


Figure 5

A ball is projected from a point A on the edge of a cliff at U m s^{-1}. The point A is at a height of 20 m above the ground, as shown in Figure 5. The angle of projection of the stone is θ, where $\tan\theta = \frac{3}{4}$. The maximum height reached by the ball above the ground is 24 m.

(a) Find U. **(3)**

At the instant the ball hits the ground, the magnitude of its velocity is V m s^{-1}.

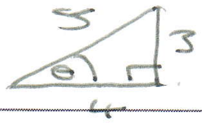
(b) Find V. **(5)**

One second after the ball is projected from the cliff, a dog standing on the ground starts to run in an attempt to catch the ball.

The height of the dog should be modelled as negligible.

(c) Calculate the minimum speed the dog must run to catch the ball at the instant it hits the ground. Show all of your working. **(4)**

a) $\tan\theta = \frac{3}{4}$



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$s = 4m$ vertical motion

↑+ve $u = U\sin\theta$

$v = 0$

$a = -g$

t

$v^2 = u^2 + 2as$

$0 = u^2\sin^2\theta + 2\times -9.8\times 4$

$0 = u^2 \times \left(\frac{3}{5}\right)^2 - 78.4$

$78.4 = 0.36\,u^2$

$u^2 = 217.7$

$u = 14.757$

$u = 14.8\ ms^{-1}$ (3sf)

Question ~~10~~ 12 continued

b) hits ground when $s = -20$

vertical motion

↑ +ve

$s = -24$

$u = 14.75729 \times \frac{3}{5}$

$v = ?$

$a = -g$

t

$$v^2 = u^2 + 2as$$

$$v^2 = \left(14.75729 \times \frac{3}{5}\right)^2 + 2 \times -9.8 \times -20$$

$$v^2 = 470.3999$$

$v = 21.688705 \text{ ms}^{-1}$ (vertical)

horizontal velocity

$$v\cos\theta = 14.75729 \times \frac{4}{5}$$

$$= 11.805832 \text{ ms}^{-1}$$

Magnitude $v = \sqrt{21.688705^2 + 11.805832^2}$

$= 24.693$

$= 25 \text{ ms}^{-1}$ (2sf)

c) Time to hit ground

$$v = u + at$$

$$\frac{v - u}{a} = t \Rightarrow t = \frac{-21.688705 - \left(14.75729 \times \frac{3}{5}\right)}{-9.8}$$

+ve ↑

$s = -20$

$u = 14.75729 \times \frac{3}{5}$

$v = -21.688705$

$a = -g$

t

$t = 3.11664$ seconds to hit floor

TOTAL 9 MARKS

Distance for dog to travel

$(\longrightarrow)$ motion of ball

$$x = u\cos\theta \times t$$

$$= 14.75729 \times \frac{4}{5} \times 3.11664$$

$$= 36.79452 \text{ m}$$

Time of dog running $= 2.11664$ s

$$\text{speed} = \frac{36.79452}{2.11664} = 17.38346 \text{ ms}^{-1}$$

$$= 17.4 \text{ ms}^{-1} \text{ (3sf)}$$