Surname	
Other Names	
Candidate Signature	

YORKSHIRE MATHS TUTOR

Centre Number						Candidate Number				

Examiner Comments	Total Marks
Yorkshire Maths Tutor	

MATHEMATICS

A LEVEL PAPER 1

CM

Bronze Set A (Edexcel Version)

Time allowed: 2 hours

Instructions to candidates:

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer ALL of the questions.
- You must write your answer for each question in the spaces provided.
- You may use a calculator.

Information to candidates:

- Full marks may only be obtained for answers to ALL of the questions.
- The marks for individual questions and parts of the questions are shown in round brackets.
- There are 13 questions in this question paper. The total mark for this paper is 100.

Advice to candidates:

- You should ensure your answers to parts of the question are clearly labelled.
- You should show sufficient working to make your workings clear to the Examiner.
- Answers without working may not gain full credit.

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1 $f(x) = x^3 - 3x + 2$

(a) Show that $(x + 2)$ is a factor of $f(x)$. **(2)**

(b) Express $f(x)$ as a product of linear factors. **(3)**

(c) Hence, or otherwise, simplify $\dfrac{3x^2 + 2x - 8}{x^3 - 3x + 2}$. **(3)**

a) $f(-2) = (-2)^3 - 3(-2) + 2 = 0$

As $f(-2) = 0$ then $x+2$ is a factor

b)

$$x+2 \overline{)\, x^3 + 0x^2 - 3x + 2}$$

$x^2 - 2x + 1$

$-x^3 + 2x^2$ ↓

$-2x^2 - 3x$ ↓

$-(-2x^2 - 4x)$

$x + 2$

$-(x + 2)$

0

$x^2 - 2x + 1 = (x - 1)(x - 1)$

$f(x) = (x+2)(x-1)(x-1)$

c) $\dfrac{3x^2 + 2x - 8}{x^3 - 3x + 2} = \dfrac{(3x-4)(x+2)}{(x+2)(x-1)(x-1)}$

$= \dfrac{3x - 4}{(x-1)^2}$

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2 (a) Given that $|2x-1|<1$, find the range of possible values of x. **(2)**

(b) Solve the equation

$$3|2^x-1|=2^x$$

giving your answers to three significant figures. **(4)**

a) $2x-1<1$ or $-(2x-1)<1$

$2x<2$ $\quad -2x+1<1$

$x<1$ $\quad 1-1<2x$

$0<2x$

$x>0$

$0<x<1$

b) Squaring both sides

$9(2^x-1)^2=2^{2x}$

$9(2^{2x}-2\times2^x+1)=2^{2x}$

$9(2^{2x}-2^{x+1}+1)=2^{2x}$

$9(2^{2x}-2^x\times2+1)=2^{2x}$

$9(2^{2x})-18\times2^x+9=2^{2x}$

$9(2^{2x})-(2^{2x})-18(2^x)+9=0$

$8(2^{2x})-18(2^x)+9=0$

$(4(2^x)-3)(2(2^x)-3)=0$

Either $4(2^x)-3=0$ or $2(2^x)-3=0$

$2^x=\frac{3}{4}$ $\quad 2^x=\frac{3}{2}$

$\ln 2^x=\ln\frac{3}{4}$ $\quad \ln 2^x=\ln\frac{3}{2}$

$x\ln2=\ln\frac{3}{4}$ $\quad x\ln2=\ln\frac{3}{2}$

$x=\frac{\ln\frac{3}{4}}{\ln2}$ $\quad x=\frac{\ln\frac{3}{2}}{\ln2}$

$x=-0.415$ $\quad x=0.585$

(3sf)

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3 The circle C has the equation

$$x^2 + 4y^2 + 8x = 12y - 3x^2 + k$$

where k is a constant.

(a) Find the coordinates of the centre of C. **(3)**

(b) Find the range of possible values of k. **(2)**

(c) Explain why the equation

$$x^2 + 2y^2 = 4$$

is not the equation of a circle. **(1)**

a) $x^2 + 8x + 3x^2 + 4y^2 - 12y - k = 0$

$4x^2 + 8x + 4y^2 - 12y - k = 0$

Completing the square

x terms

$4(x^2 + 2x) = 4((x+1)^2 - 1) = 4(x+1)^2 - 4$

y terms

$4(y^2 - 3y) = 4\left((y - \frac{3}{2})^2 - (\frac{3}{2})^2\right)$

$= 4(y - \frac{3}{2})^2 - 9$

Centre is $x = -1$, $y = \frac{3}{2}$

b) $4((x+1)^2 - 4 + 4(y - \frac{3}{2})^2 - 9 - k = 0$

$4((x+1)^2 + 4(y - \frac{3}{2})^2 = 4 + 9 + k$

divide through by 4

$(x+1)^2 + (y - \frac{3}{2})^2 = \frac{13+k}{4}$

for circle to have a +ve value for radius

$\frac{13+k}{4} > 0$

$\Rightarrow k > -13$

c) Coefficients of x^2 and y^2 are different so it is the equation of an ellipse

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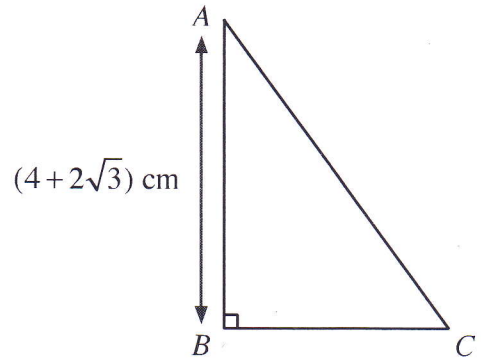

Figure 1

Figure 1 above shows a right-angled triangle ABC.

$AB = (4+2\sqrt{3})$ cm and the area of the triangle ABC is $(8+5\sqrt{3})$ cm^2.

Showing all of your working clearly, find the exact length of the hypotenuse of the triangle.

Give your answer in the form $(a+b\sqrt{3})^{\frac{1}{2}}$, where a and b are constants. **(6)**

Area $\Delta = \frac{1}{2} \times$ base $\times$ height

$(8+5\sqrt{3}) = \frac{1}{2} \times BC \times (4+2\sqrt{3})$

$BC = \frac{2(8+5\sqrt{3})}{4+2\sqrt{3}} = \frac{(16+10\sqrt{3}) \times (4-2\sqrt{3})}{(4+2\sqrt{3})(4-2\sqrt{3})}$

$BC = \frac{64-32\sqrt{3}+40\sqrt{3}-60}{16-12} = \frac{4+8\sqrt{3}}{4} = 1+2\sqrt{3}$

Using Pythagoras $AC^2 = AB^2 + BC^2$

$AC^2 = (4+2\sqrt{3})^2 + (1+2\sqrt{3})^2$

$AC^2 = 16+16\sqrt{3}+12+1+4\sqrt{3}+12$

$AC^2 = 41+20\sqrt{3}$

$AC = (41+20\sqrt{3})^{\frac{1}{2}}$

$(x+h)(x+h) = x^2 + 2xh + h^2$

$(x+h)(x^2+2xh+h^2) = x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3$

$= x^3 + 3x^2h + 3xh^2 + h^3$

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10

5

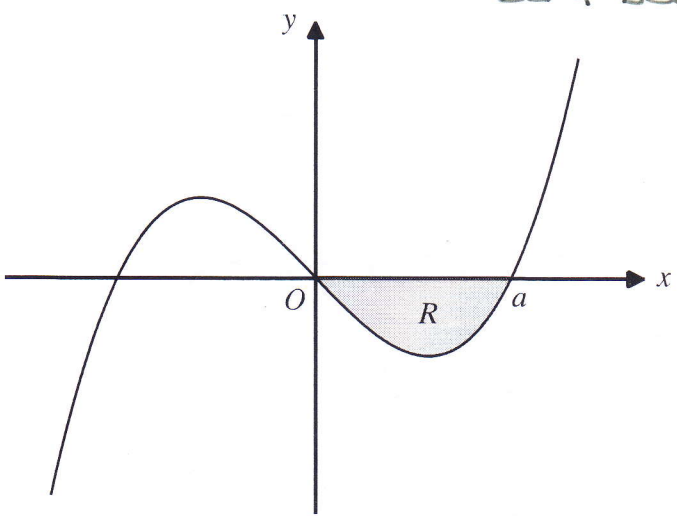

Figure 2

Figure 2 above shows a sketch of the curve C with equation $y = x^3 - 2x$.

(a) Differentiate $x^3 - 2x$ with respect to x from first principles. **(3)**

(b) Hence, find the coordinates of the stationary points on the curve C. **(3)**

The region R, shown shaded in Figure 2, is bounded by the curve C, the x-axis and the lines $x = 0$ and $x = a$, where a is a positive constant. The curve C crosses the x-axis at the point $(a, 0)$.

(c) Find the area of the shaded region R. **(5)**

a) $\frac{d}{dx}(x^3 - 2x) = \lim_{h \to 0} \frac{(x+h)^3 - 2(x+h) - x^3 + 2x}{h}$

$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h - x^3 + 2x}{h}$

$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 2h}{h}$

$= \lim_{h \to 0} 3x^2 + 3xh + h^2 - 2$

$= 3x^2 - 2$

b) $\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 2 = 0$

$3x^2 = 2$

$x^2 = \frac{2}{3}$

$x = \sqrt{\frac{2}{3}}$ or $x = -\sqrt{\frac{2}{3}}$

$x = \sqrt{\frac{2}{3}}, y = \left(\sqrt{\frac{2}{3}}\right)^3 - 2 \times \sqrt{\frac{2}{3}} = -\frac{4\sqrt{6}}{9}$

$x = -\sqrt{\frac{2}{3}}, y = \left(-\sqrt{\frac{2}{3}}\right)^3 - 2 \times -\sqrt{\frac{2}{3}} = \frac{4\sqrt{6}}{9}$

stationary points $\left(\sqrt{\frac{2}{3}}, -\frac{4\sqrt{6}}{9}\right), \left(-\sqrt{\frac{2}{3}}, \frac{4\sqrt{6}}{9}\right)$

c) $0 = x^3 - 2x$

$0 = x(x^2 - 2)$

$x = 0$ or $x = \sqrt{2}$ or $x = -\sqrt{2}$

$$\int_0^{\sqrt{2}} (x^3 - 2x)\, dx$$

$$= \left[\frac{1}{4}x^4 - x^2\right]_0^{\sqrt{2}}$$

$$= \left(\frac{1}{4} \times (\sqrt{2})^4 - (\sqrt{2})^2\right) - (0 - 0)$$

$= (1 - 2)$

$= -1$ (area under curve shows as negative number)

Area under curve = 1 square unit

Do not write outside the box

6

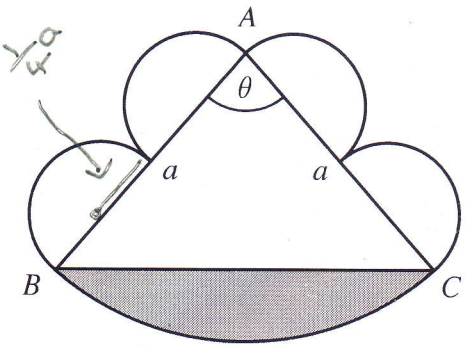

Figure 3

Figure 3 above shows a triangle ABC where $AB = AC = a$ and the angle $BAC = \theta$. Four identical semi-circles are drawn outside of the triangle ABC. A circular arc with centre A joins B to C. The area of the shaded segment is equal to the sum of the areas of the semicircles.

(a) Show that $\theta = \frac{1}{4}\pi + \sin\theta$. **(3)**

(b) Verify by calculation that θ lies between 1 and 2. **(2)**

(c) Use an iterative formula based on the equation in part (a) a suitable number of times to determine θ correct to two decimal places. Give the result of each iteration to four decimal places. **(3)**

a) Area 4 semicircles $= 2 \times \pi \times (\frac{1}{4}a)^2 = \frac{1}{8}\pi a^2$ (1)

Area sector $ABC = \frac{1}{2} \times a^2 \times \theta = \frac{1}{2}a^2\theta$ (2)

Area triangle $ABC = \frac{1}{2}a^2 \sin\theta$ (3)

Area shaded $=$ (1) $= \frac{1}{8}\pi a^2$

(1) $=$ (3) $-$ (2)

$\therefore \frac{1}{8}\pi a^2 = \frac{1}{2}a^2\theta - \frac{1}{2}a^2\sin\theta$

$\frac{1}{8}\pi = \frac{1}{2}\theta - \frac{1}{2}\sin\theta$

$\times$ through by 2

$\frac{\pi}{4} = \theta - \sin\theta$

$0 = \frac{1}{4}\pi + \sin\theta$ as required

b) $f(\theta) = \theta - \frac{1}{4}\pi - \sin\theta$

$f(1) = 1 - \frac{1}{4}\pi - \sin(1) = -0.6268691$

$f(2) = 2 - \frac{1}{4}\pi - \sin(2) = 0.3053$

As there is a change of sign, θ must be between 1 and 2

6c) $\theta_{n+1} = \frac{\pi}{4} + \sin\theta_n$

let $\theta_1 = 1$

$\theta_2 = 1.62686$

$\theta_3 = 1.7838$

$\theta_4 = 1.7627$

$\theta_5 = 1.767$

$\theta_6 = 1.7662$

} both = 1.77 to 2dp so stop

$\therefore \quad \theta = 1.77 \ (2dp)$

Do not write outside the box

7 Let n be any positive integer that is not a square number.

Prove, by contradiction, that the square root of n is irrational. **(4)**

Assume square root of n is rational

$\sqrt{n} = \frac{a}{b}$ where a and b have no common factors other than 1 (coprime) and $b \neq 0$

$n = \frac{a^2}{b^2}$

If p is a prime factor of b, then p must be a prime factor of a^2 and therefore of a.

But a and b have no other common factors other than 1, so b cannot have any other factors $\therefore b = 1$

Then $\sqrt{n} = a$, which is a contradiction as n is not a square number. So n must be irrational.

Do not write outside the box

8 (i) Find, in terms of e, the equation of the normal to the curve $y = x^3e^x$ at the point $x = 1$. **(5)**

(ii) Given that $y = \dfrac{2}{\sqrt{1+(1-x)^2}}$, find $\dfrac{dy}{dx}$. **(4)**

(i) $y = x^3 e^x$

$u = x^3 \qquad v = e^x$

$\dfrac{du}{dx} = 3x^2 \qquad \dfrac{dv}{dx} = e^x$

$\dfrac{dy}{dx} = x^3e^x + 3x^2e^x$

at $x = 1$ $\dfrac{dy}{dx} = e + 3e = 4e$

$\therefore$ gradient of normal $= \dfrac{-1}{4e}$

at $x = 1$, $y = e$ $\quad (1, e)$

$y - y_1 = m(x - x_1)$

$y - e = \dfrac{-1}{4e}(x - 1)$

(ii) $y = 2\left(1 + (1-x)^2\right)^{-\frac{1}{2}}$

$y = 2\left(1 + 1 - 2x + x^2\right)^{-\frac{1}{2}}$

$y = 2\left(2 - 2x + x^2\right)^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = -\left(2 - 2x + x^2\right)^{-\frac{3}{2}} \times (-2 + 2x)$

$= \dfrac{2 - 2x}{\left(2 - 2x + x^2\right)^{\frac{3}{2}}} = \dfrac{2(1-x)}{\left(1 + (1-x)^2\right)^{\frac{3}{2}}}$

Do not write outside the box

9 A mass, m kg, of a substance decreases with time t years.

The mass of the substance can be modelled by the equation $m = ab^t$, where a and b are constants.

(a) Show that the graph of $\ln(m)$ against t is a straight line. **(2)**

The graph of $\ln(m)$ against t passes through the point $(0, 3.91)$. It takes 5 years for the mass of the substance to decrease by half.

(b) Show that a is approximately 50. **(1)**

(c) Find, to three significant figures, the value of b. **(2)**

After 8 years, the substance is modified to increase its mass by 20 kg. The mass of the substance then decreases according to the model

$$m = P + 40e^{-0.32t}$$

where P is a constant.

(d) Find the value of P. **(3)**

a) $m = ab^t$

$\ln m = \ln ab^t$

$\ln m = \ln a + \ln b^t$

$\ln m = \ln a + t \ln b$

So graph of $\ln m$ against t is a straight line

b) at $t = 0$, $\ln(m) = 3.91$ ↑ y-intercept

$\therefore \ln a = 3.91$

$a = e^{3.91} = 49.898$

$a \approx 50$ as required

c) at $t = 5$, mass $= 49.898 \div 2$

$= 24.949475\,99$

Using $m = ab^t$

$24.949475\,99 = 49.898 \times b^5$

$\frac{1}{2} = b^5 \Rightarrow b = 0.870\,55$

$b = 0.871$ (3sf)

9d) after 8 years

$m = 49.898\ldots \times 0.87055^{8}$

$m = 16.46020143$

$m + 20 = 36.46020143$

$m = P + 40e^{-0.32t}$ at $t=0$

$\therefore \quad 36.46020143 = P + 40 \times e^{0}$

$P = 36.46020143 - 40$

$P = -3.53979$

$P = -3.54 \quad (3sf)$

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10 $f(x) = 12\cos(x) - 4\sin(x), \quad 0 \leq x \leq 2\pi$

(a) Express $f(x)$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of R as an exact value and the value of α to two decimal places. **(3)**

(b) Solve the equation $f(2x) = 4$. **(4)**

(c) Using your answer to part (a), find the maximum value of the function f and the value(s) of x at which this maximum occurs. **(3)**

a) $R\cos(x+\alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = 12 \qquad R\sin\alpha = 4$

$\tan\alpha = \frac{4}{12} \Rightarrow \alpha = 0.32175$

$\alpha = 0.32 \quad (2dp)$

$R = \sqrt{12^2+4^2} = 4\sqrt{10}$

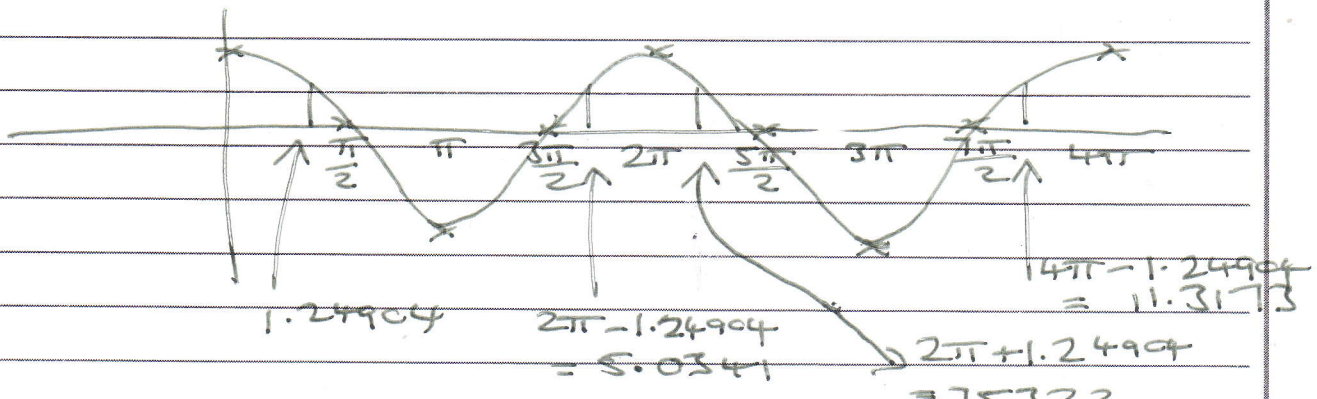
$12\cos x - 4\sin x = 4\sqrt{10}\cos(x+0.32)$

b) $4\sqrt{10}\cos(2x+0.32) = 4$

$\cos(2x+0.32) = \frac{1}{\sqrt{10}}$

$\cos^{-1}\left(\frac{1}{\sqrt{10}}\right) = 1.24904$

$0 \leq 2x \leq 4\pi$



$2x + 0.32 = 1.24904 \Rightarrow x = 0.46$

$2x + 0.32 = 5.0341 \Rightarrow x = 2.36 \quad (2dp)$

$2x + 0.32 = 7.5322 \Rightarrow x = 3.61$

$2x + 0.32 = 11.3173 \Rightarrow x = 5.50$

10 c) $4\sqrt{10}\cos(x+0.32)$

Max value is $4\sqrt{10}$

when $\cos(x+0.32) = 1$

$\cos^{-1}(1) = 0, 2\pi$

$x + 0.32 = 0 \Rightarrow x = -0.32$ (2dp)

$x + 0.32 = 2\pi \Rightarrow x = 5.96$ (2dp)

Do not write outside the box

11 A liquid flows in a right-angled corner.

The velocity of the liquid flow, $\mathbf{v}$ m s^{-1}, at a point (x, y) is given by

$$\mathbf{v} = x\mathbf{i} - y\mathbf{j}, \qquad x, y > 0$$

(a) Find the magnitude of the velocity of the liquid flow at the point $(1, 2)$. **(2)**

Streamlines are lines that are parallel to the liquid flow.

For this flow, the streamlines are the solutions to the differential equation

$$\frac{dy}{dx} = -\frac{y}{x}$$

(b) Find the general solution to this differential equation, giving y in terms of x.
Simplify your answer. **(4)**

(c) Hence, sketch the family of streamlines for this flow. **(2)**

a) at $\begin{pmatrix}1\\2\end{pmatrix}$ $\underset{\sim}{v} = \begin{pmatrix}1\\-2\end{pmatrix} = \sqrt{1^2+(-2)^2}$

$= \sqrt{5}$ ms^{-1}

b) $\frac{dy}{dx} = -\frac{y}{x}$

$\int \frac{1}{y}\,dy = -\int \frac{1}{x}\,dx$

$\ln y = -\ln x + c$

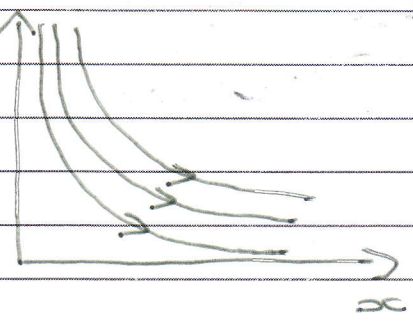
$\ln y + \ln x = c$

$\ln(yx) = c$

$yx = e^c$

$y = \frac{e^c}{x}$

c)



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12 The function f is defined such that

$$f(x) = \frac{-x}{2x^2 - 5x + 3}$$

(a) Express $f(x)$ in the form $\frac{A}{x-1} + \frac{B}{2x-3}$, where A and B are constants to be found. **(3)**

(b) Hence, obtain the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. **(4)**

a) $\frac{-x}{(x-1)(2x-3)} = \frac{A}{x-1} + \frac{B}{2x-3}$

$-x = A(2x-3) + B(x-1)$

let $x = 1$ $\quad -1 = -A \Rightarrow A = 1$

$x = \frac{3}{2}$ $\quad -\frac{3}{2} = \frac{1}{2}B \Rightarrow B = -3$

$\frac{-x}{2x^2-5x+3} = \frac{1}{x-1} - \frac{3}{2x-3}$

b) $(x-1)^{-1} = -(1-x)^{-1} =$

$-\left[1 + \frac{(-1)}{1}(-x) + \frac{(-1)(-2)}{1\times 2}(-x)^2 + \frac{(-1)(-2)(-3)}{1\times 2\times 3}(-x)^3 + \ldots\right]$

(1) $= -\left[1 + x + x^2 + x^3\right] = -1 - x - x^2 - x^3 + \ldots$

$-3(2x-3)^{-1} = +3\times 3^{-1}\left(1 - \frac{2}{3}x\right)^{-1} = \left(1 - \frac{2}{3}x\right)^{-1}$

$= 1 + \frac{(-1)}{1}\left(-\frac{2}{3}x\right) + \frac{(-1)(-2)}{1\times 2}\left(-\frac{2}{3}x\right)^2 + \frac{(-1)(-2)(-3)}{1\times 2\times 3}\left(-\frac{2}{3}x\right)^3 + \ldots$

(2) $= 1 + \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \ldots$

$\therefore f(x) = (1) + (2) = -1 - x - x^2 - x^3 + 1 + \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3$

$= -\frac{1}{3}x - \frac{5}{9}x^2 - \frac{19}{27}x^3 + \ldots$

Do not write outside the box

13 (i) An arithmetic series has first term a and common difference $(2a + 1)$.

(a) Prove that the sum of the first n terms of this series, S_n, can be given by

$$S_n = \frac{n}{2}[2a + (n-1)(2a+1)] \quad \textbf{(4)}$$

(b) Given that $a = 4$, find the value of n such that $S_n = 11225$. **(3)**

(ii) Evaluate

$$\sum_{n=1}^{\infty} \left(\tfrac{1}{2}\sin x\right)^n$$

and justify the validity of your answer. **(2)**

(i) 1st 2nd 3rd n^{th}

(1) $S_n = a + a+(2a+1) + a+2(2a+1) + \ldots + a+(n-1)(2a+1)$

$+2a+1$ $+2a+1$

in reverse

(2) $S_n = (a+(n-1)(2a+1) + \ldots\ldots a+(2a+1) + a$

(1) + (2) gives

$2S_n = n \times (2a + (n-1)(2a+1))$

$\therefore S_n = \frac{n}{2}(2a + (n-1)(2a+1))$

b) $11225 = \frac{n}{2}(8 + 9(n-1))$

$2 \times 11225 = n(8 + 9n - 9)$

$22450 = n(9n - 1)$

$22450 = 9n^2 - n$

$9n^2 - n - 22450 = 0$

$n = \frac{1 \pm \sqrt{1^2 + 4 \times 9 \times 22450}}{2 \times 9}$

$n = 50$ or $n = -\frac{449}{9}$ (impossible)

$\therefore n = 50$

13(ii)

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\sin x\right)^n$$

$$= \left(\frac{1}{2}\sin x\right)^1 + \left(\frac{1}{2}\sin x\right)^2 + \ldots$$

$\times \frac{1}{2}\sin x$

Geometric with $a = \frac{1}{2}\sin x$

$r = \frac{1}{2}\sin x$

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}\sin x}{1-\frac{1}{2}\sin x}$$

$$= \frac{\frac{1}{2}\sin x}{\frac{1}{2}(2-\sin x)} = \frac{\sin x}{2-\sin x}$$

valid for $1 - \frac{1}{2}\sin x \neq 0$

$\therefore \left|\frac{1}{2}\sin x\right| < 1$

max value of $\sin x = 1$

$\therefore \left|\frac{1}{2}\sin x\right|$ will be < 1 and is valid