YORKSHIRE MATHS TUTOR

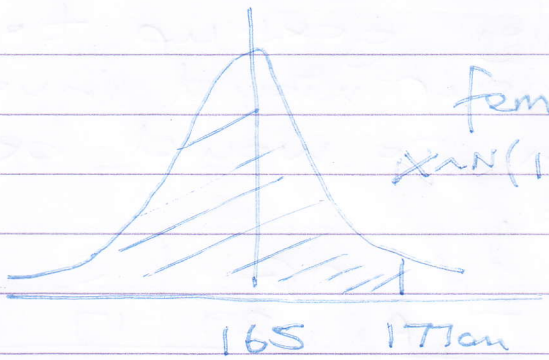
PAPER H
A Level Statistics & Mechanics

1. females

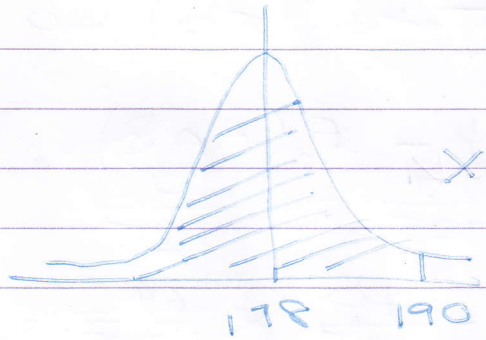
mean = μ = 165cm σ = 9cm

$X \sim N(165, 9^2)$

males $X \sim N(178, 10^2)$



females Normal CD
$X \sim N(165, 9^2)$ Lower 0
Upper 177
σ = 9
μ = 165
p = 0.9087
at 90th percentile



males Normal CD
$X \sim N(178, 10^2)$ Lower 0
Upper 190
σ = 10
μ = 178
p = 0.8849
At 88th percentile

Female relatively taller.

2. $x = t$ $y = \log_{10} C$ use $C = ab^t$

$\log_{10} C = \log_{10} a + t \log_{10} b$

given $y = 1.89 - 0.0131x$

$\log_{10} a = 1.89$

$a = 10^{1.89} = 77.6247$

$\log_{10} b = -0.0131$

$b = 10^{-0.0131} = 0.9703$

$C = 77.6 \times 0.970^t$ (3sf)

PAPER 1+

2 (continued)

b) b is the proportional rate at which the temperature changes per minute

c) The number of minutes in the table only goes up to 25, so an hour would involve extrapolation so would be unreliable.

3. a) $P(G) = \frac{50}{125}$

	G	S	B	Total
Im A	29	17	18	64
Im C	21	23	17	61
Total	50	40	35	125

b) $P([A \cap S]') = 1 - P(A \cap S)$

$1 - \frac{17}{125} = \frac{108}{125}$

c) $P(S) \times P(A) = \frac{40}{125} \times \frac{64}{125} = \frac{512}{3125}$

$P(S \cap A) = \frac{17}{125}$

As $P(S \cap A) \neq P(S) \times P(A)$

S and A are not statistically independent.

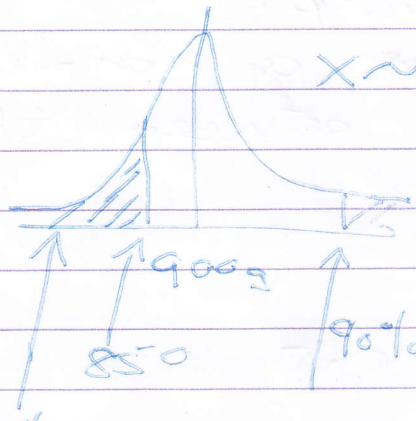
d) B and C are not mutually exclusive Being in team C does not exclude you from winning a Bronze medal.

e) $\frac{\frac{30}{100} \times 50 + \frac{60}{100} \times 40 + \frac{40}{100} \times 35}{125} =$

$\frac{15 + 24 + 14}{125} = \frac{53}{125}$

PAPER H

4. a) $\mu = 900g$ $\sigma = 100g$



$X \sim N(900, 100^2)$

$P(X < 850) = 0.30$ AS

Normal (D)
Lower 0
Upper 850
$\sigma = 100$
$\mu = 900$

b) Lowest 10% = 771.844

Inverse Normal (D) = 772g Min

Area = 0.1
$\sigma = 100$
$\mu = 900$

Upper 10% (Area = 0.9) = 1028.15

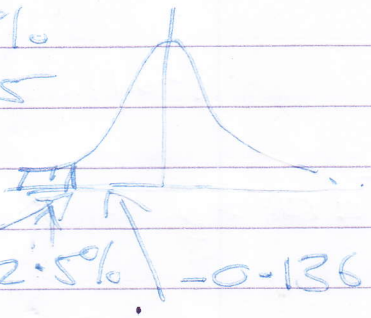
Max = 1028g

5a) PMCC = −0.136 (3 sf)

$H_0 : \rho = 0$ $H_1 : \rho < 0$

Significance level = 2.5%
= 0.025

From PMCC table
sample level = 10, 0.025
$\rho < -0.6319$



as PMCC (−0.136) > −0.6319
test statistic not in critical Region
no evidence to reject H_0
∴ No evidence to suggest weight of chickens and average weight of eggs is negatively correlated.

PAPER 1H

5b) There could be a very weak linear relationship.
There could be a non-linear relationship between the variables.

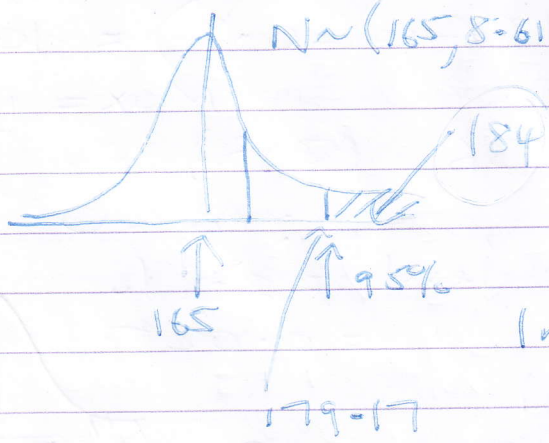
6a) n is large
p is close to 0.5

b) mean = np
variance = $np(1-p)$

c) There would be not batteries to sell if they tested them all

d) $H_0 : p = 0.55$ $H_1 : p > 0.55$

e) $N \sim (165, 8.61684^2)$



mean = $300 \times 0.55 = 165$

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{165(1-0.5)}$

$= 8.61684$

Inverse Normal
Area = 0.95
$\sigma = 8.61684$
$\mu = 165$

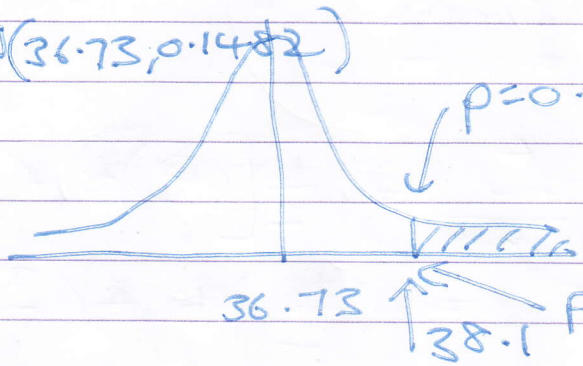
∴ Test statistic is in critical region so reject H_0
There is evidence to support manufacturers claim.

Paper 1+

7. a) $\mu = 36.73$ $\sigma^2 = 0.1482$

$\therefore \sigma = \sqrt{0.1482} = 0.384$

$X \sim N(36.73, 0.1482)$



Normal CD
Lower < 0
Upper = 38.1
$\sigma = 0.3849$
$p = 0.9998$ $\mu = 36.73$

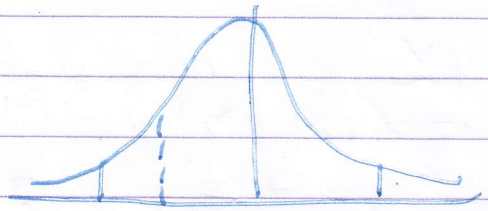
$P(X > 38.1) = 1 - 0.9998 = 0.000186$

b) Call doctor as in critical zone, unlikely that temperature should be so high.

8a) A test statistic is a statistic calculated from sample data in order to test a hypothesis about a population.

b) PMCC < -0.258 p-value = 0.001

$H_0: \rho = 0$ $H_1: \rho \neq 0$



Significance level in each tail = 0.025 for 5% significance level

NOT TOLD SAMPLE SIZE?

c)
- Similar correlations for both seasons
- Lower temperature in Autumn
- More spread for daily mean pressure in Autumn

PAPER H
Section B Mechanics

9. $\underset{\sim}{F} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} t + \begin{pmatrix} 6 \\ -12 \end{pmatrix} t^2$, $m = 2\,kg$

a) $\underset{\sim}{F} = m\underset{\sim}{a}$

$$\underset{\sim}{a} = \frac{\underset{\sim}{F}}{m} = \frac{1}{2}\left[\begin{pmatrix} 8 \\ 4 \end{pmatrix} t + \begin{pmatrix} 6 \\ -12 \end{pmatrix} t^2\right]$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} t + \begin{pmatrix} 3 \\ -6 \end{pmatrix} t^2 \; ms^{-2}$$

b) $\underset{\sim}{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} t + \begin{pmatrix} 3 \\ -6 \end{pmatrix} t^2$

integrate to get $\underset{\sim}{v}$

$$\underset{\sim}{v} = \frac{1}{2}\begin{pmatrix} 4 \\ 2 \end{pmatrix} t^2 + \frac{1}{3}\begin{pmatrix} 3 \\ -6 \end{pmatrix} t^3 + c = \begin{pmatrix} 2 \\ 1 \end{pmatrix} t^2 + \begin{pmatrix} 1 \\ -2 \end{pmatrix} t^3$$

$\therefore$ at $t = 0$ $\underset{\sim}{v} = 0$ $\therefore c = 0$

Integrate again to get $\underset{\sim}{s}$

$$\underset{\sim}{s} = \frac{1}{3}\begin{pmatrix} 2 \\ 1 \end{pmatrix} t^3 + \frac{1}{4}\begin{pmatrix} 1 \\ -2 \end{pmatrix} t^4 + c$$

at $t = 0$, $\underset{\sim}{s} = 0$ $\therefore c = 0$

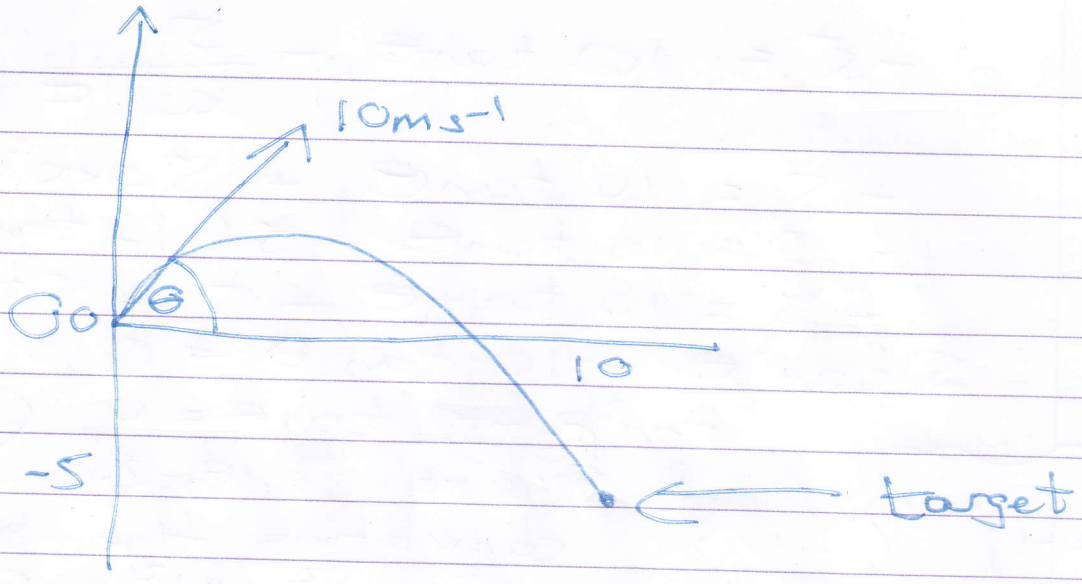
$$\underset{\sim}{s} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} t^3 + \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{2} \end{pmatrix} t^4$$

c) sub $t = 1$ in $\underset{\sim}{v}$

$$\underset{\sim}{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \times 1 + \begin{pmatrix} 1 \\ -2 \end{pmatrix} \times 1$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} ms^{-1}$$

10 a)



Horizontal velocity (→) constant

$V = 10\cos\theta \ ms^{-1}$

Vertical motion (↑) +ve to target

$s = -5$

$u = 10\sin\theta$

$v = ?$

$a = -10$

$t = ?$

Horizontal motion

time to get to target

$t = \frac{10}{10\cos\theta}$

$= \frac{1}{\cos\theta}$

Using vertical motion (↑)

$s = ut + \frac{1}{2}at^2$

$-5 = 10\sin\theta \times \frac{1}{\cos\theta} + \frac{1}{2} \times -10 \times \left(\frac{1}{\cos\theta}\right)^2$

$-5 = \frac{10\sin\theta}{\cos\theta} - \frac{5}{\cos^2\theta}$

PAPER H

$$-5 = 10\tan\theta - \frac{5}{\cos^2\theta}$$

$$-5 = 10\tan\theta - 5\sec^2\theta$$
$$-5 = 10\tan\theta - 5(1+\tan^2\theta)$$
$$-5 = 10\tan\theta - 5 - 5\tan^2\theta$$
$$5\tan^2\theta - 10\tan\theta = 0$$
$$\tan^2\theta - 2\tan\theta = 0 \quad (1)$$

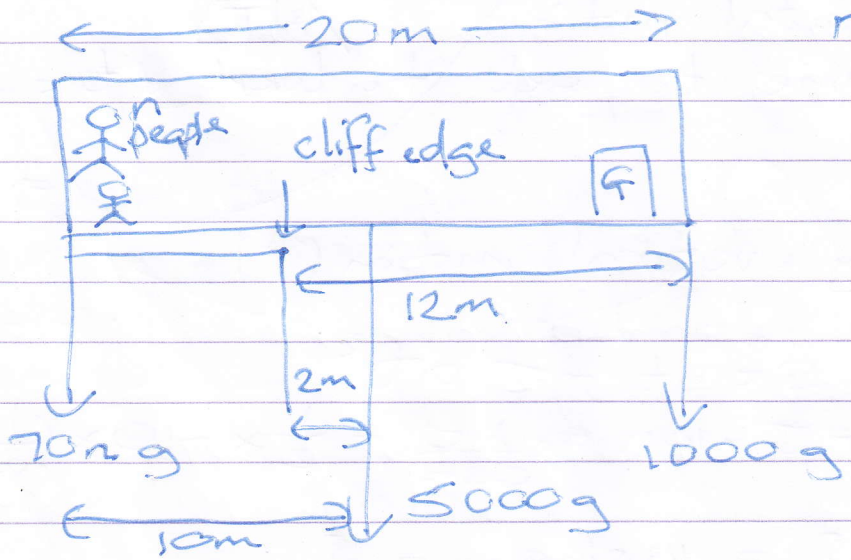
$(\tan\theta - 1)^2$
$= \tan^2\theta - 2\tan\theta + 1$

completing the square
$(\tan\theta - 1)^2 - (1)^2 = 0$
$(\tan\theta - 1)^2 = 1$
as required

b) Using (1)

$$\tan\theta(\tan\theta - 2) = 0$$
$$\tan\theta = 0 \quad \text{or} \quad \tan\theta = 2$$
$$\theta = 0 \quad \text{or} \quad \theta = 63.434^\circ$$

11.



n is number of people

PAPER H

11 a) m (cliff edge) (+ve) $= -8 \times 70ng + 2 \times 5000g + 12 \times 1000g$

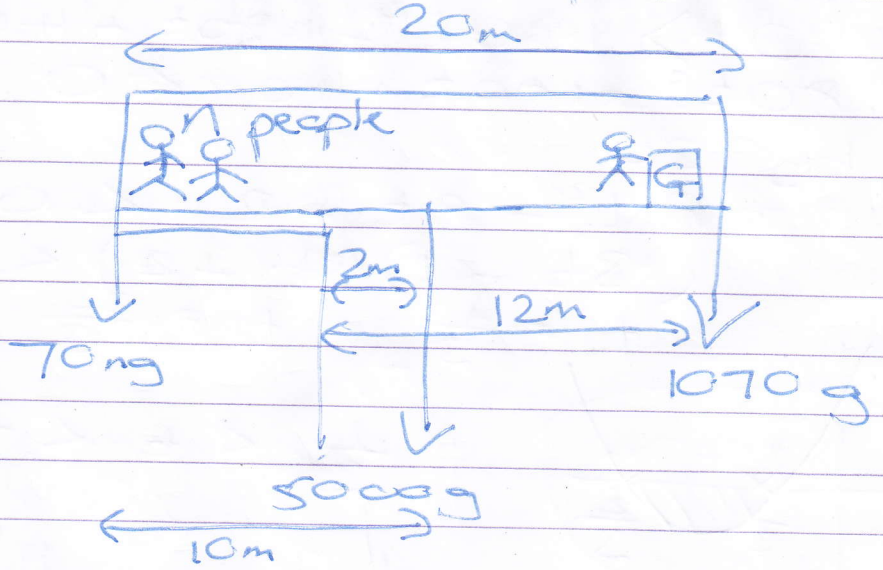
$= -560ng + 22000g$ Nm clockwise

b) $560ng = 22000g$

$n = \frac{22000}{560} = 39.285$

$n = 40$ people

c)



m (Cliff edge) (+ve)

$0 = -8 \times 70ng + 2 \times 5000g + 12 \times 1070g$

$560ng = 22840g$

$n = \frac{22840}{560} = 40.78$ round up $n = 41$

Including person who retrieves the gold

42 people

PAPER H

12.a) $a = 180 - 360t$

Integrate to get v

$$v = 180t - \frac{360t^2}{2}$$

$$v = 180t - 180t^2$$

b) $180t - 180t^2 > 40$

$0 > 180t^2 - 180t + 40$

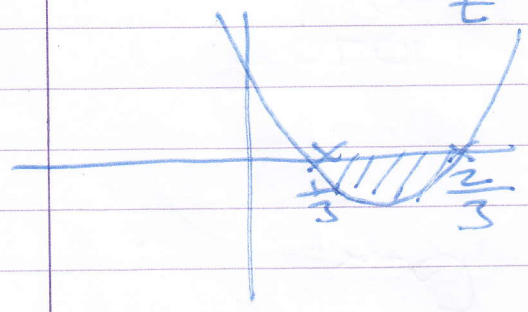
$180t^2 - 180t + 40 < 0$

$\div$ by 20

$9t^2 - 9t + 2 < 0$

$(3t - 1)(3t - 2) < 0$

$t = \frac{1}{3}$ $t = \frac{2}{3}$



$\frac{1}{3} < t < \frac{2}{3}$

$\frac{1}{3}$ ↓ 20 minutes

$\frac{2}{3}$ ↓ 40 minutes

c) To find average speed, need distance travelled

Integrate v to get s

$$s = \frac{180t^2}{2} - \frac{180t^3}{3} = 90t^2 - 60t^3$$

distance in 1 hour ($t = 1$)

$s = 90 - 60 = 30$ km

Average speed = 30 km/h