4.	Attempts $V = \pi \int x^2 e^{2x} \mathrm{d}x$	M1
	$= \pi \left[\frac{x^2 e^{2x}}{2} - \int x e^{2x} dx \right]$ (M1 needs parts in the correct direction)	M1 A1
	$= \pi \left[\frac{x^2 e^{2x}}{2} - \left(\frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right) \right]$ (M1 needs second application of parts)	M1 A1√
	M1A1√ refers to candidates $\int x e^{2x} \mathrm{d}x$, but dependent on prev. M1	
	$= \pi \left[\frac{x^2 e^{2x}}{2} - \left(\frac{x e^{2x}}{2} - \frac{e^{2x}}{4} \right) \right]$	A1 cao
	Substitutes limits 3 and 1 and subtracts to give… [dep. on second and third Ms]	dM1
	$= \pi \left[\frac{13}{4} e^6 - \frac{1}{4} e^2 \right]$ or any correct exact equivalent.	A1
	[Omission of π loses first and last marks only]	**[8]**

Jun 07

Question Number	Scheme		Marks
Aliter 2. (a) **Way 2**	Volume $= \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(3+6x)^2} dx$	**Use** of $V = \pi \int y^2\, dx$. Can be implied. Ignore limits.	B1
	$= (\pi) \int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2}\, dx$	Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and π	M1
	$= (\pi)\left[\frac{(3+6x)^{-1}}{(-1)(6)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$	Integrating to give $\pm p(3+6x)^{-1}$ $-\frac{1}{6}(3+6x)^{-1}$	M1 A1
	$= (\pi)\left[-\frac{1}{6}(3+6x)^{-1}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$		
	$= (\pi)\left[\left(\frac{-1}{6(6)}\right) - \left(\frac{-1}{6(\frac{3}{2})}\right)\right]$		
	$= (\pi)\left[-\frac{1}{36} - (-\frac{1}{9})\right]$		
	$= \frac{\pi}{12}$	Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef
			[5]

Note: π is not needed for the middle three marks of question 2(a).

Jan 07

Question Number	Scheme		Marks
2. (a)	Volume $= \pi \underline{\int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx} = \frac{\pi}{9}\int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(1+2x)^2} dx$	**Use** of $V = \pi \underline{\int y^2\, dx}$. Can be implied. Ignore limits.	B1
	$= \left(\frac{\pi}{9}\right)\int_{-\frac{1}{4}}^{\frac{1}{2}} (1+2x)^{-2}\, dx$	Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and $\frac{\pi}{9}$	M1
	$= \left(\frac{\pi}{9}\right)\left[\frac{(1+2x)^{-1}}{(-1)(2)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$	Integrating to give $\underline{\pm p(1+2x)^{-1}}$ $\underline{-\frac{1}{2}(1+2x)^{-1}}$	M1 A1
	$= \left(\frac{\pi}{9}\right)\left[\ \underline{-\tfrac{1}{2}(1+2x)^{-1}}\ \right]_{-\frac{1}{4}}^{\frac{1}{2}}$		
	$= \left(\frac{\pi}{9}\right)\left[\left(\frac{-1}{2(2)}\right) - \left(\frac{-1}{2(\frac{1}{2})}\right)\right]$		
	$= \left(\frac{\pi}{9}\right)\left[-\tfrac{1}{4} - (-1)\right]$		
	$= \frac{\pi}{12}$	Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef
			[5]
(b)	From Fig.1, AB $= \frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$ units		
	As $\frac{3}{4}$ units $\equiv$ 3cm		
	then scale factor $k = \frac{3}{\left(\frac{3}{4}\right)} = 4$.		
	Hence Volume of paperweight $= (4)^3\left(\frac{\pi}{12}\right)$	$(4)^3 \times$ (their answer to part (a))	M1
	$V = \underline{\frac{16\pi}{3}}\ \text{cm}^3 = 16.75516...\ \text{cm}^3$	$\underline{\frac{16\pi}{3}}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef	A1
			[2]
			7 marks

Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks of question 2(a).

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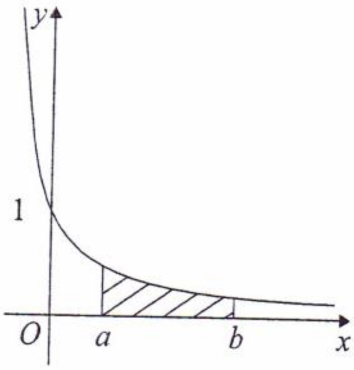

Figure 2

The curve shown in Figure 2 has equation $y = \dfrac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(5)

$$\text{Volume} = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_a^b \frac{1}{(2x+1)^2} \, dx$$

$$= \pi \left[-\frac{1}{2} \frac{1}{(2x+1)} \right]_a^b$$

$$= \pi \left(\frac{-1}{2(2b+1)} + \frac{1}{2(2a+1)} \right)$$

$$= \frac{\pi}{2} \left(\frac{-(2a+1) + (2b+1)}{(2a+1)(2b+1)} \right)$$

$$= \pi \left(\frac{(b-a)}{(2a+1)(2b+1)} \right)$$

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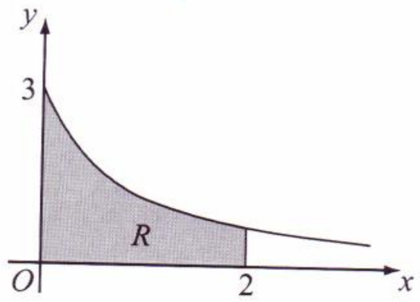

Figure 1

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R.

(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)

a) $y = 3(1+4x)^{-1/2}$

$\text{Area} = 3\int_0^2 (1+4x)^{-1/2}\,dx$

$= 3\left[\frac{2}{4}(1+4x)^{1/2}\right]_0^2$

$= 3\left(3/_2 - 1/_2\right) = 3$

(b) $V = \pi\int y^2\,dx$

$= \pi\int_0^2 \left(\frac{3}{\sqrt{1+4x}}\right)^2 dx$

$= \pi\int_0^2 \frac{9}{1+4x}\,dx$

$\ln 1 = 0$

$= \pi\left[\frac{9}{4}\ln|1+4x|\right]_0^2 = \pi\left[\frac{9}{4}\ln 9 - 0\right] = \frac{9}{4}\pi\ln 9$

8. (*a*) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,dx.$$

(7)

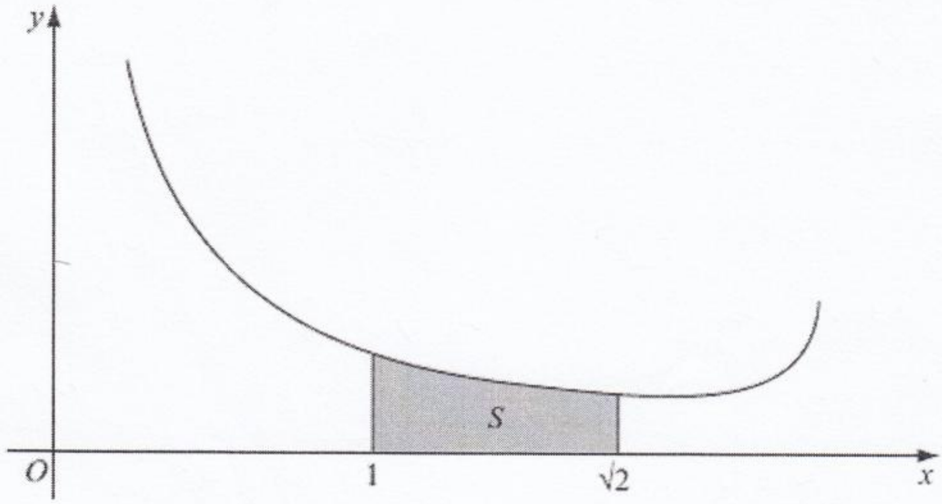

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region S, shown in Figure 3, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Using your answer to part (*a*), find the exact volume of the solid of revolution formed.

(3)

TOTAL FOR PAPER: 75 MARKS

END

8a) $\frac{dx}{du} = -2\sin u$

$dx = -2\sin u \, du$

Limits

x	$u = \cos^{-1}\left(\frac{x}{2}\right)$
1	$\pi/3$
$\sqrt{2}$	$\pi/4$

$$I = \int_{\pi/3}^{\pi/4} \frac{1}{4\cos^2 u\sqrt{4-4\cos^2 u}} (-2\sin u)\, du$$

'−' will swap limits

$$= \int_{\pi/4}^{\pi/3} \frac{2\sin u}{4\cos^2 u\sqrt{4(1-\cos^2 u)}}\, du$$

$1-\cos^2 u = \sin^2 u$

$$= \int_{\pi/4}^{\pi/3} \frac{2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\, du$$

$$= \int_{\pi/4}^{\pi/3} \frac{\cancel{2\sin u}}{4\cos^2 u\,\cancel{(2\sin u)}}\, du$$

$$= \int_{\pi/4}^{\pi/3} \frac{1}{4\cos^2 u}\, du$$

$$= \int_{\pi/4}^{\pi/3} \frac{1}{4}\sec^2 u\, du$$

$\int \sec^2 u \, du = \tan u$

$$= \left[\frac{1}{4}\tan u\right]_{\pi/4}^{\pi/3}$$

$$= \frac{\sqrt{3}-1}{4}.$$

(b)

$$V = \pi \int y^2 \, dx$$

$$= \pi \int_1^{\sqrt{2}} \left(\frac{4}{x(4-x^2)^{1/4}} \right)^2 dx$$

$$= 16\pi \int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} \, dx$$

original integral (a)

$$= 16\pi \left(\frac{\sqrt{3}-1}{4} \right)$$

C4 Jan 2012

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4.

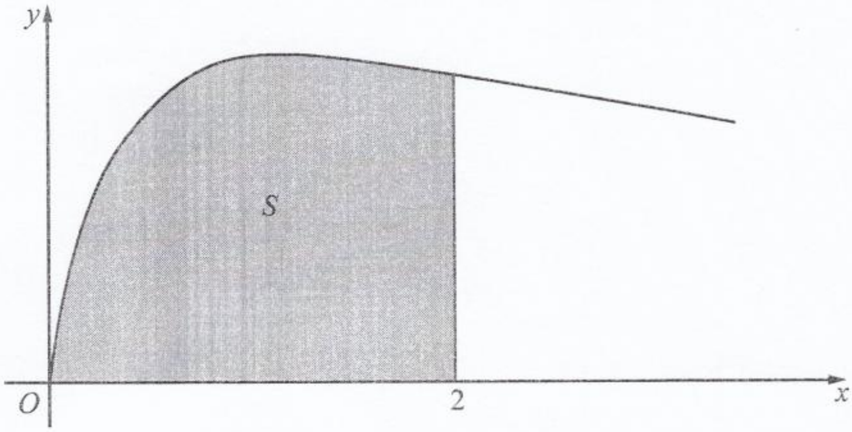

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geqslant 0$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

$$V = \pi \int y^2 \, dx$$

$$= \pi \int_0^2 \left(\sqrt{\frac{2x}{3x^2+4}}\right)^2 dx$$

$$= \pi \int_0^2 \frac{2x}{3x^2+4} \, dx$$

$$= \pi \left[\frac{1}{3} \ln(3x^2+4) \right]_0^2$$

$$= \pi \left[\frac{1}{3} \ln 16 - \frac{1}{3} \ln 4 \right]$$

$$V = \pi \frac{1}{3} \ln 4$$

June 06

edexcel

Question Number	Scheme		Marks
3. (a)	Area Shaded $= \int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)\,dx$		
	$= \left[\dfrac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{2\pi}$	Integrating $3\sin\left(\frac{x}{2}\right)$ to give $k\cos\left(\frac{x}{2}\right)$ with $k \neq 1$. Ignore limits.	M1
	$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$	$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$	A1 oe.
	$= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$	$\underline{12}$	A1 cao
	(Answer of 12 with no working scores M0A0A0.)		[3]
(b)	Volume $= \underline{\pi \int_0^{2\pi} \left(3\sin\left(\frac{x}{2}\right)\right)^2}\,dx = 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right)\,dx$	**Use** of $\underline{V = \pi \int y^2}\,dx$. Can be implied. Ignore limits.	M1
	$\left[\text{NB: } \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$ $\left[\text{NB: } \underline{\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right) \text{ gives } \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}}\right]$	Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ or the Double Angle Formula for $\sin^2 x$	M1 *
	$\therefore$ Volume $= 9(\pi) \int_0^{2\pi} \left(\dfrac{1-\cos x}{2}\right) dx$	Correct expression for Volume Ignore limits and π.	A1
	$= \dfrac{9(\pi)}{2} \int_0^{2\pi} \underline{(1-\cos x)}\,dx$		
	$= \dfrac{9(\pi)}{2}\underline{[x - \sin x]}_0^{2\pi}$	$\underline{\text{Integrating to give } \pm ax \pm b\sin x}$; $\underline{\text{Correct integration } k - k\cos x \to kx - k\sin x}$	depM1 *; A1
	$= \dfrac{9\pi}{2}[(2\pi - 0) - (0 - 0)]$		
	$= \dfrac{9\pi}{2}(2\pi) = \underline{9\pi^2}$ or $\underline{88.8264...}$	Use of limits to give either $9\pi^2$ or awrt 88.8 Solution must be completely correct. No flukes allowed.	A1 **cso** [6]
			9 marks

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8. (a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, d\theta$.

(2)

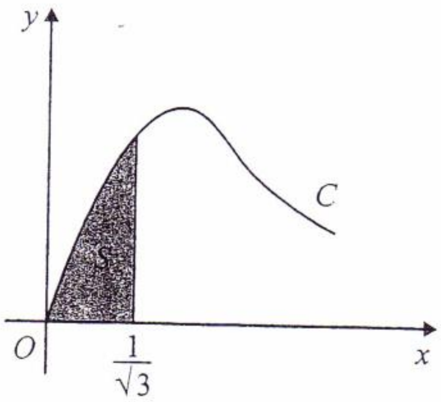

Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \quad y = 2\sin 2\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$$

where k is a constant.

(5)

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

a) $\int \sin^2\theta \, d\theta = \frac{1}{2}\int 1 - \cos 2\theta \, d\theta$

$= \frac{\theta}{2} - \frac{\sin 2\theta}{4} + c$

H34265A02628

Question 8 continued

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Volume of Revolution $= \pi \int y^2 \frac{dx}{d\theta} \, d\theta$

$x = \tan\theta \qquad y = 2\sin 2\theta$

$\frac{dx}{d\theta} = \sec^2\theta \qquad y^2 = 4\sin^2 2\theta$

$$\text{Volume} = \pi \int 4\sin^2 2\theta \, . \, \sec^2\theta \, d\theta$$

$$= \pi \int 4 \times \frac{(2\sin\theta\cos\theta)^2}{\cos^2\theta} \, d\theta$$

$$= 16\pi \int \sin^2\theta \, d\theta \qquad x = 0 \qquad x = \frac{1}{\sqrt{3}}$$

$\tan\theta = 0 \qquad \tan\theta = 1/\sqrt{3}$

$\theta = 0 \qquad \theta = \pi/6$

$$= 16\pi \int_0^{\pi/6} \sin^2\theta \, d\theta$$

(c) $V = 16\pi \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/6}$

$$= 16\pi \left[\pi/12 - \frac{\sqrt{3}}{8} \right]$$

$$= \frac{4}{3}\pi^2 - 2\pi\sqrt{3} \qquad p = 4/3$$

$q = -2$

Q8

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

C4 June 2011

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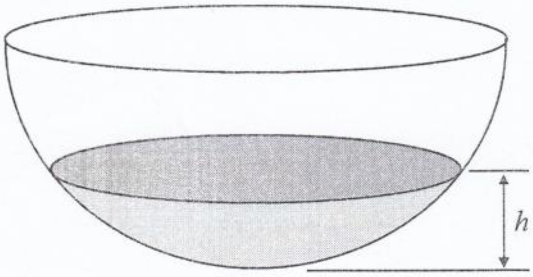

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume $V \text{m}^3$ is given by

$$V = \frac{1}{12}\pi h^2(3-4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$ **(4)**

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^3s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$ **(2)**

a) $V = \frac{1}{4}\pi h^2 - \frac{1}{3}\pi h^3$

$\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$

$(h = 0.1)$

$\frac{dV}{dh} = 0.05\pi - 0.01\pi$

$= 0.04\pi$

b)

P 3 8 1 6 0 A 0 6 2 4

C4 June 2011

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Question 3 continued

(b) $\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt}$

$\dfrac{dV}{dt} = \dfrac{\pi}{800}$ (given)

$\dfrac{dh}{dV} = \dfrac{1}{\frac{dV}{dh}}$

$\dfrac{dh}{dV} = \dfrac{1}{\frac{1}{2}\pi h - \pi h^2}$

$\dfrac{dh}{dt} = \dfrac{1}{\frac{1}{2}\pi h - \pi h^2} \times \dfrac{\pi}{800}$

$h = 0.1$

$\dfrac{dh}{dt} = \dfrac{1}{32}$

Q3

(Total 6 marks)

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6.

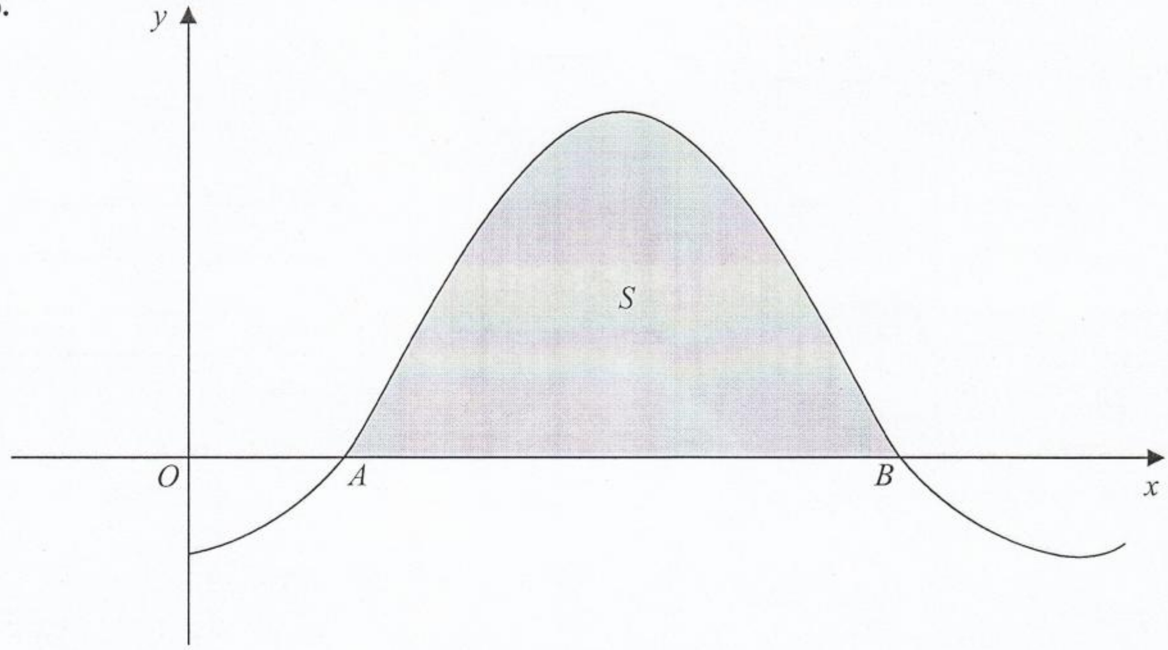


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(a) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B. **(3)**

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(b) Find, by integration, the exact value of the volume of the solid generated. **(6)**

a) $y=0 \qquad 0 = 1 - 2\cos x$

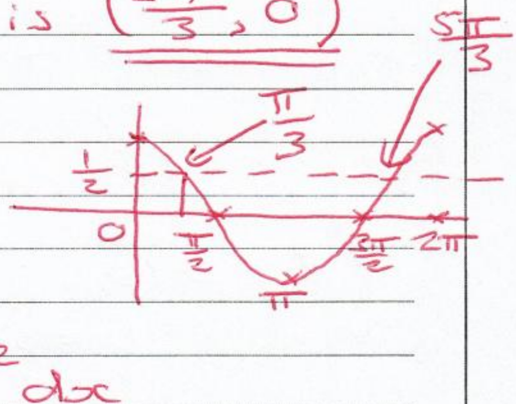
$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

$\underline{\underline{A \text{ is } \left(\frac{\pi}{3}, 0\right)}}$ and $\underline{\underline{B \text{ is } \left(\frac{5\pi}{3}, 0\right)}}$

b) Volume of revolution

$$= \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} y^2 \, dx$$



$$= \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 \, dx$$

C4 Jan 2013

6 b continued

$$= \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 4\cos x + 4\cos^2 x)\, dx$$

$$\boxed{\begin{aligned} \cos 2x &= 2\cos^2 x - 1 \\ 2\cos^2 x &= \cos 2x + 1 \\ \cos^2 x &= \tfrac{1}{2}\cos 2x + \tfrac{1}{2} \end{aligned}}$$

$$= \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \left(1 - 4\cos x + 4\left(\tfrac{1}{2}\cos 2x + \tfrac{1}{2}\right)\right) dx$$

$$= \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 4\cos x + 2\cos 2x + 2)\, dx$$

$$= \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (3 - 4\cos x + 2\cos 2x)\, dx$$

$$= \pi \left[3x - 4\sin x + \sin 2x\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$$

$$= \pi \left[\left(5\pi - 4\sin\frac{5\pi}{3} + \sin\frac{10\pi}{3}\right) - \left(\pi - 4\sin\frac{\pi}{3} + \sin\frac{2\pi}{3}\right)\right]$$

$$= \pi \left[\left(5\pi + 4\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) - \left(\pi - 4\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)\right)$$

$$= \pi\left(3\sqrt{3} + 4\pi\right)$$

C4 June 2013

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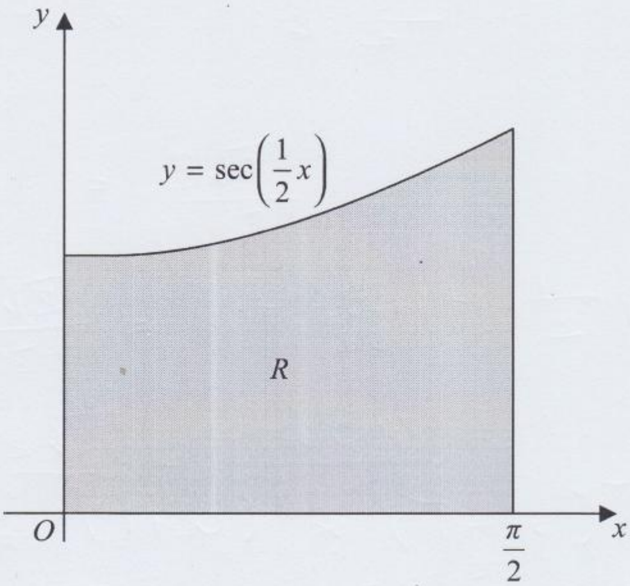

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276	1.154701	1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places.

(1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.

(3)

Region R is rotated through 2π radians about the x-axis.

(c) Use calculus to find the exact volume of the solid formed.

(4)

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3b)

$$\text{Area} = \frac{1}{2} \times \frac{\pi}{6}\left[1 + 1.414214 + 2(1.035276 + 1.154701)\right]$$

$$\approx 1.778709$$

$$\approx \underline{\underline{1.7787}} \quad (4\text{dp})$$

c)

$$\text{volume} = \pi \int_0^{\frac{\pi}{2}} \sec^2\left(\tfrac{1}{2}x\right)\, dx$$

$$= \pi \left[2\tan\left(\tfrac{1}{2}x\right)\right]_0^{\frac{\pi}{2}}$$

$$= \pi\left(2\tan\left(\tfrac{\pi}{4}\right) - 0\right)$$

$$= \underline{\underline{2\pi}}$$