6. [*In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})$ m. Find

(a) the speed of A,

(2)

(b) the direction in which A is moving, giving your answer as a bearing.

(3)

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

(c) show that A and B will collide at a point P and find the position vector of P.

(5)

Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(d) find the distance of B from P when $t = 7$ s.

(6)

JAW 2006

JAN 2007

Leave blank

3. A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

(a) the acceleration of P in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(b) the magnitude of $\mathbf{F}$,

(4)

(c) the velocity of P at time $t = 6$ s.

(3)

JAN 2008

Leave blank

6. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of P,

(2)

(b) the direction of motion of P, giving your answer as a bearing.

(3)

At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}.

(c) Find the values of u and v.

(5)

(d) Find the total time taken for P to move from A to a position which is due south of A.

(3)

JAN 2009

Leave blank

1. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the value of u.

(5)

JAN 2009

Leave blank

6. Two forces, $(4\mathbf{i} - 5\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P of mass m kg. The resultant of the two forces is **R**. Given that **R** acts in a direction which is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) find the angle between **R** and the vector **j**,

(3)

(b) show that $2p + q + 3 = 0$.

(4)

Given also that $q = 1$ and that P moves with an acceleration of magnitude $8\sqrt{5}$ m s^{-2},

(c) find the value of m.

(7)

JAN 2010

Leave blank

7. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s} = (3t + 9)\,\mathbf{i} + (4t - 6)\,\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

Leave blank

4. A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. The velocity of P is $(2\mathbf{i}-5\mathbf{j})$ m s^{-1} at time $t=0$, and $(7\mathbf{i}+10\mathbf{j})$ m s^{-1} at time $t=5$ s.

Find

(a) the speed of P at $t=0$,

(2)

(b) the vector $\mathbf{F}$ in the form $a\mathbf{i}+b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to $\mathbf{i}$.

(4)

M1 Jan 2012

Leave blank

3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j}) \text{ N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j}) \text{ N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j}) \text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$,

(2)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

M1 Jan 2012

Leave blank

7. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A boat P is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i}+8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

JUNE 2006

Leave blank

7. [*In this question the unit vectors* **i** *and* **j** *are due east and north respectively.*]

A ship S is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j})$ km h^{-1}. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i} + 5\mathbf{j})$ km. Find

(a) the speed of S,

(2)

(b) the bearing on which S is moving.

(2)

The ship is heading directly towards a submerged rock R. A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

(c) Find the position vector of R.

(2)

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of 5 km h^{-1}. Assuming that S continues to move with this new constant velocity, find

(d) an expression for the position vector of the ship t hours after 1400,

(4)

(e) the time when S will be due east of R,

(2)

(f) the distance of S from R at the time 1600.

(3)

JUNE 2007

Leave blank

7. A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3\mathbf{i} - 4\mathbf{j})$ km with respect to a fixed origin O. At 1430 on the same day, B is at the point with position vector $(8\mathbf{i} + 11\mathbf{j})$ km.

(a) Find the velocity of B, giving your answer in the form $p\mathbf{i} + q\mathbf{j}$.

(3)

At time t hours after noon, the position vector of B is $\mathbf{b}$ km.

(b) Find, in terms of t, an expression for $\mathbf{b}$.

(3)

Another boat C is also moving with constant velocity. The position vector of C, $\mathbf{c}$ km, at time t hours after noon, is given by

$$\mathbf{c} = (-9\mathbf{i} + 20\mathbf{j}) + t(6\mathbf{i} + \lambda\mathbf{j}),$$

where λ is a constant. Given that C intercepts B,

(c) find the value of λ,

(5)

(d) show that, before C intercepts B, the boats are moving with the same speed.

(3)

JUNE 2009

Leave blank

2. A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

(a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$.

(2)

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

(b) find the value of p.

(4)

JUNE 2009

Leave blank

8. *[In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.]*

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(a) Find the speed of H.

(2)

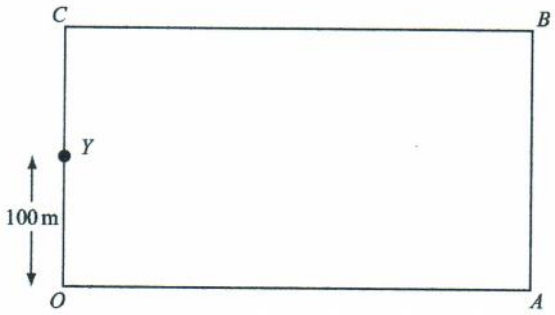

Figure 3

A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in Figure 3. At twelve noon hiker H is at the point Y with position vector 100 **j** m, relative to the fixed origin O.

(b) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker K is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(c) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = \left[(9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}\right] \text{ metres.}$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

MAY 2008

Leave blank

3. A particle P of mass 0.4 kg moves under the action of a single constant force **F** newtons. The acceleration of P is $(6\mathbf{i} + 8\mathbf{j})$ m s^{-2} . Find

(a) the angle between the acceleration and **i**,

(2)

(b) the magnitude of **F**.

(3)

At time t seconds the velocity of P is **v** m s^{-1} . Given that when $t = 0$, $\mathbf{v} = 9\mathbf{i} - 10\mathbf{j}$,

(c) find the velocity of P when $t = 5$.

(3)

M1 May 2010

Leave blank

1. A particle P is moving with constant velocity $(-3\mathbf{i}+2\mathbf{j})$ m s^{-1}. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i}-7\mathbf{j})$ m. Find the distance of P from the origin at time $t = 2$ s.

(5)

M1 May 2011

Leave blank

7. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i}-3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i}+4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i}+\mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,

(ii) $\mathbf{q}$,

(iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

(i) Q is due north of P,

(ii) Q is north-west of P.

(4)

M1 May 2012

Leave blank

6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(a) Find the direction in which S is moving, giving your answer as a bearing.

(3)

At time t hours after noon, the position vector of S is **s** km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

(b) Write down **s** in terms of t.

(2)

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(c) Find the distance of S from B when $t = 3$

(4)

(d) Find the distance of S from B when S is due north of B.

(4)

Leave blank

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by $\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

M1 MAY 2013

Leave blank

7. *[In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively.]*

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

(i) parallel to **j**,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)