4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)

a) As l_1 and l_2 are perpendicular ∴ dot product = 0

$\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} = 0 \qquad -2q + 2 - 8 = 0$

$\underline{q = -3}$

(b) Intersect when

$$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$$

$11 - 2\lambda = -5 - 3\mu$ ①

$2 + \lambda = 11 + 2\mu$ ②

Solve simultaneously $\lambda = 5$ $\mu = -2$ Substitute into $17 - 4\lambda = p + 2\mu$ ③

$\underline{p = 1}$

(c) $\underline{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\underline{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$

Intersect at $\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or write $(1, 7, -3)$

(d) Let $\vec{OX} = \underline{i} + 7\underline{j} - 3\underline{k}$ be point of intersection.

Vector $\vec{AX} = \vec{OX} - \vec{OA}$

$= \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$

$\vec{AX} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$

$\vec{OB} = \vec{OA} + \vec{AB}$ Vector $\vec{AB}$ must be twice $\vec{AX}$

(X is midpoint of A and B)

$= \vec{OA} + 2\vec{AX}$

$= \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$

$\vec{OB} = \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $-7\underline{i} + 11\underline{j} - 19\underline{k}$

4. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(*a*) Write down the coordinates of A.

(1)

(*b*) Find the value of $\cos\theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(*c*) Find the coordinates of X.

(1)

(*d*) Find the vector $\overrightarrow{AX}$.

(2)

(*e*) Hence, or otherwise, show that $\left|\overrightarrow{AX}\right| = 4\sqrt{26}$.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(*f*) find the length of AY, giving your answer to 3 significant figures.

(3)

4 a) $A\ (-6, 4, -1)$

(b) $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = 12 + 4 + 3 = \sqrt{4^2 + 1^2 + 3^2}\sqrt{3^2 + (-4)^2 + 1^2} \cos\theta$

$$\cos\theta = \frac{19}{26}$$

(c) $X : (10, 0, 11)$

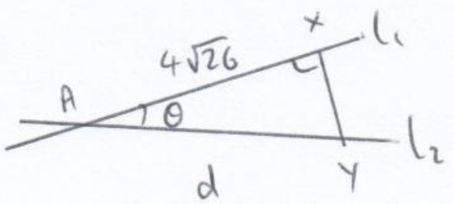
(d) $\overrightarrow{AX} = \begin{pmatrix} 10 \\ 0 \\ 11 \end{pmatrix} - \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$

$$= \begin{pmatrix} 16 \\ -4 \\ 12 \end{pmatrix}$$

(e) $|\overrightarrow{AX}| = \sqrt{16^2 + (-4)^2 + 12^2}$

$$= \sqrt{416} = 4\sqrt{26}$$

(f)



$$\frac{|\overrightarrow{AX}|}{d} = \cos\theta$$

$$d = \frac{4\sqrt{26}}{19/26} = \underline{27.9}$$

4. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(*a*) Find $\overrightarrow{AB}$.

(2)

(*b*) Find a vector equation of l.

(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant.

Given that AC is perpendicular to l, find

(*c*) the value of p,

(4)

(*d*) the distance AC.

(2)

---

4 a) $\overrightarrow{AB} = -2\underline{i} + 2\underline{j} - \underline{k} - (\underline{i} - 3\underline{j} + 2\underline{k})$

$= -3\underline{i} + 5\underline{j} - 3\underline{k}$

(b) $\underline{r} = \underline{i} - 3\underline{j} + 2\underline{k} + \lambda(-3\underline{i} + 5\underline{j} - 3\underline{k})$

(c) $\overrightarrow{AC} = 2\underline{i} + p\underline{j} - 4\underline{k} - (\underline{i} - 3\underline{j} + 2\underline{k})$

$= \underline{i} + (p+3)\underline{j} - 6\underline{k}$

$\overrightarrow{AC} \cdot \overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$

$-3 + 5p + 15 + 18 = 0$

$\underline{p = -6}$

(d) $AC^2 = (2-1)^2 + (-6+3)^2 + (-4-2)^2$

$AC = \sqrt{46}$

C4 Jan 2012

Leave blank

7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

(c) Show that the size of the angle BAD is 109°, to the nearest degree.

(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.

(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.

(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.

(2)

a) $\overrightarrow{OA} = 2\underline{i} - \underline{j} + 5\underline{k}$ $\quad \overrightarrow{OB} = 5\underline{i} + 2\underline{j} + 10\underline{k}$

$\overrightarrow{AB} = (5\underline{i} + 2\underline{j} + 10\underline{k}) - (2\underline{i} - \underline{j} + 5\underline{k})$

$= 3\underline{i} + 3\underline{j} + 5\underline{k}$

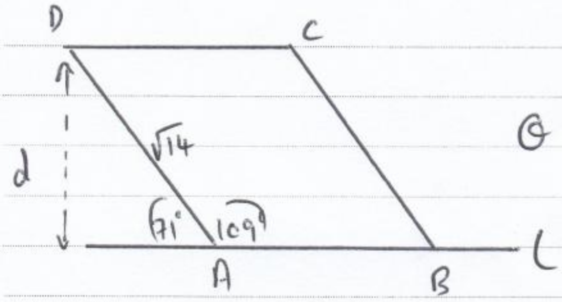
(b) $\underline{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ $\quad$ or $\quad$ $\underline{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$

P 4 0 0 8 5 A 0 2 0 2 8

C4 Jan 2012

Leave blank

Question 7 continued



$\theta = B\hat{A}D$

d is shortest distance C to l

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

$\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{|\overrightarrow{AB}|\,|\overrightarrow{AD}|}$

$\cos\theta = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3^2) + (3)^2 + (5)^2} \cdot \sqrt{(-3)^2 + 2^2 + (-1)^2}}$

$\cos\theta = \dfrac{-8}{\sqrt{43}\sqrt{14}}$

$\theta = 109°$

P 4 0 0 8 5 A 0 2 1 2 8

Turn over

C4 Jan 2012

Leave blank

Question 7 continued

(d) $\vec{OC} = \vec{OD} + \vec{DC}$

$= \vec{OD} + \vec{AB}$

$= (-\underline{i} - \underline{j} + 4\underline{k}) + (3\underline{i} + 3\underline{j} + 5\underline{k})$

$= 2\underline{i} + 4\underline{j} + 9\underline{k}.$

(or use $\vec{OC} = \vec{OB} + \vec{BC}$)

(e) Area $ABCD = \left(\frac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right) \times 2$

$= 23.2$

(f) $\frac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.2$

$d = \sqrt{14}\sin 71$

$d = 3.54$

Leave blank

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection. **(6)**

(b) Show that l_1 and l_2 are perpendicular to each other. **(2)**

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that A lies on l_1. **(1)**

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B. **(3)**

a) Lines meet where

$$\begin{pmatrix}-9\\0\\10\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\1\\17\end{pmatrix} + \mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$$

$$-9 + 2\lambda = 3 + 3\mu \quad ①$$

$$\lambda = 1 - \mu \quad ②$$

$$10 - \lambda = 17 + 5\mu \quad ③$$

① $- 2\times$②

$$-9 = 1 + 5\mu$$

$$\mu = -2$$

sub. into ②

$$\lambda = 3$$

$$\underline{r} = \begin{pmatrix}-9\\0\\10\end{pmatrix} + 3\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}-3\\3\\7\end{pmatrix} \quad \text{or} \quad \underline{r} = -3\underline{i} + 3\underline{j} + 7\underline{k}$$

H30427A01628

Question 6 continued

finally check subs. $\mu = -2,\ \lambda = 3$

into eqn ③

$$10 - 3 = 17 + 5(-2)$$

$$7 = 7 \checkmark.$$

b) $\underline{d}_1 = 2\underline{i} + \underline{j} - \underline{k} \quad \underline{d}_2 = 3\underline{i} - \underline{j} + 5\underline{k}$

$$d_1 . d_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} . \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$$

$$= (2 \times 3) + (1 \times -1) + (-1 \times 5) = 0$$

$\Rightarrow l_1$ perpendicular to l_2.

c)

Equating: $\cancel{-9\lambda}$ $\quad -9 + 2\lambda = 5$

$$\lambda = 7$$

$$\underline{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$$

$= \overrightarrow{OA}$ hence A lies on l_1.

Leave blank

Question 6 continued

d)

$\overrightarrow{OX} = -3\underline{i} + 3\underline{j} + 7\underline{k}$

$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix}-3\\3\\7\end{pmatrix} - \begin{pmatrix}5\\7\\3\end{pmatrix} = \begin{pmatrix}-8\\-4\\4\end{pmatrix}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \overrightarrow{OA} + 2\overrightarrow{AX}$

$\overrightarrow{OB} = \begin{pmatrix}5\\7\\3\end{pmatrix} + 2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$

$= \begin{pmatrix}-11\\-1\\11\end{pmatrix}$ or $-11\underline{i} - \underline{j} + 11\underline{k}$

7. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(a) Find a vector equation for the line l. **(3)**

(b) Find $|\overrightarrow{CB}|$. **(2)**

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place. **(3)**

(d) Find the shortest distance from the point C to the line l. **(3)**

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(e) find the area of the triangle CXB, giving your answer to 3 significant figures. **(3)**

a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

$\underline{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ $\quad$ or $\quad r = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

(b) $\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$

$= \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}$

$|\overrightarrow{CB}| = \sqrt{1^2 + 5^2 + (-10)^2}$

$= \sqrt{126}$

H34265A02228

Leave blank

(c) $\cos\theta = \dfrac{\vec{CB}\cdot\vec{AB}}{|CB||AB|}$

$\vec{CB}\cdot\vec{AB} = \begin{pmatrix}1\\5\\-10\end{pmatrix}\begin{pmatrix}2\\1\\-2\end{pmatrix}$

$= 2+5+20$

$= 27.$

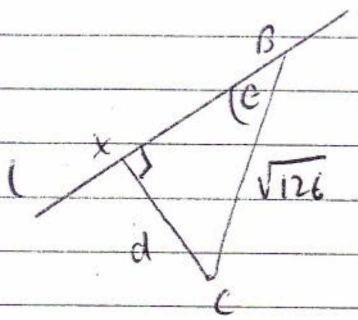
$= \dfrac{27}{\sqrt{126}\sqrt{9}}$

$|AB| = \sqrt{2^2+1^2+2^2}$

$= \sqrt{9}$

$\theta = 36.7^\circ$

(d)



$\sin\theta = \dfrac{d}{\sqrt{126}}$

$d = \sqrt{126}\ \sin(36.7^\circ)$

$d = 6.7$ ✗

(e) Area $= \frac{1}{2} ab\sin C$

$= \frac{1}{2} \cdot 3\sqrt{5} \cdot \sqrt{126}\ \sin(90-36.7^\circ)$

$= 30.2$

✗ $\cos\theta = \dfrac{3}{\sqrt{14}}$

or $(BX)^2 = (BC)^2 - d^2$

$\cos^2\theta = \dfrac{9}{14}$

$= 126 - 45$

$\sin^2\theta = 1 - 9/14$

$= 81$

$= \dfrac{5}{14}$

$BX = 9$

Area $= \frac{1}{2} \times BX \times d$

$\sin\theta = \dfrac{\sqrt{70}}{14}$

Area $= \dfrac{1}{2} \times 9 \times 3\sqrt{5}$

$d = \dfrac{\sqrt{70}}{14} \times \sqrt{126}$

$= 30.2$

$d = 3\sqrt{5}$

H 3 4 2 6 5 A 0 2 4 2 8

7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(*a*) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(*b*) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(*c*) Hence, or otherwise, find the area of the triangle ABC.

(5)

---

7. a) j components $3 + 2\lambda = 9$

$\lambda = 3 \qquad (\mu = 1)$

$C : (5, 9, -1)$

(b) $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 5 + 2 = \sqrt{6}\sqrt{29} \cos(A\hat{C}B)$

$A\hat{C}B = 57.95^\circ$

(c) $A\ (2, 3, -4) \quad B(-5, 9, -5)$

$\overrightarrow{AC} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \quad \overrightarrow{BC} = \begin{pmatrix} 10 \\ 0 \\ 4 \end{pmatrix}$

$AC^2 = 3^2 + 6^2 + 3^2 \Rightarrow AC = 3\sqrt{6}$

$BC^2 = 10^2 + 4^2 \Rightarrow BC = 2\sqrt{29}$

$\Delta ABC = \frac{1}{2} AC \times BC \sin A\hat{C}B$

$= \frac{1}{2} 3\sqrt{6} \cdot 2\sqrt{29} \sin A\hat{C}B = 33.5^\circ$

C4 June 2011

Leave blank

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1:\ \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2:\ \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.
(6)

(b) Find, to the nearest $0.1°$, the acute angle between l_1 and l_2.
(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1.
(1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.
(4)

a) $6 - \lambda = -5 + 2\mu$

$-3 + 2\lambda = 15 - 3\mu$

$-2 + 3\lambda = 3 + \mu$

Solving $\lambda = 3$

$\mu = 4$

$\underline{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ or $\underline{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + 4\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$

$k:\ \underline{r} = 7$ $\qquad k:\ \underline{r} = 7$

as L.H.S = R.H.S. lines intersect.

P 3 8 1 6 0 A 0 1 4 2 4

C4 June 2011

Leave blank

Question 6 continued

(b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}2\\-3\\1\end{pmatrix} = -2-6+3 = \sqrt{14}\sqrt{14}\cos\theta$

$(\theta = 110.92°)$

Acute Angle $= 69.1°$

(c) $\underset{\sim}{r} = \begin{pmatrix}6\\-3\\-2\end{pmatrix} + 1\begin{pmatrix}-1\\2\\3\end{pmatrix} = \begin{pmatrix}5\\-1\\1\end{pmatrix} \Rightarrow B$ lies on l_1

(d) Let d be shortest distance from B to l_2

$\vec{AB} = \begin{pmatrix}5\\-1\\1\end{pmatrix} - \begin{pmatrix}3\\3\\7\end{pmatrix} = \begin{pmatrix}2\\-4\\-6\end{pmatrix}$

$|\vec{AB}| = \sqrt{2^2+(-4)^2+(-6)^2}$

$= \sqrt{56}$

$\dfrac{d}{\sqrt{56}} = \sin\theta$

$d = \sqrt{56}\sin 69.1°$

$d \simeq 6.99.$

P38160A01524

Turn over

Leave blank

8. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

(6)

a) $\underline{a} = \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix}$ $\quad \underline{b} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix}$

$$\overrightarrow{AB} = \underline{b} - \underline{a}$$

$$= \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

b) $\begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10-2\lambda \\ 2+\lambda \\ 3+\lambda \end{pmatrix}$

c) $C = \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix}$

$$\overrightarrow{CP} = \begin{pmatrix} 10-2\lambda \\ 2+\lambda \\ 3+\lambda \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 7-2\lambda \\ -10+\lambda \\ \lambda \end{pmatrix}$$

C4 June 2012

Leave blank

Question 8 continued

$\vec{CP}$ is perpendicular to l

$$\Rightarrow \begin{pmatrix} 7-2\lambda \\ -10+\lambda \\ \lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$-14 + 4\lambda - 10 + \lambda + \lambda = 0$$

$$6\lambda = 24$$

$$\lambda = 4$$

$$\therefore \quad p = \begin{pmatrix} 10-8 \\ 2+4 \\ 3+4 \end{pmatrix}$$

$$p = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$$

P41484A02932

C4 Jan 2013

Leave blank

7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection. **(5)**

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place. **(3)**

Given that the point A has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1,

(c) find the exact coordinates of P. **(6)**

a) $l_1 \quad \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$

$l_2 \quad \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$\underset{\sim}{i}$ components $9 + \lambda = 2 + 2\mu$ ①

$\underset{\sim}{j}$ components $13 + 4\lambda = -1 + \mu$ ②

$\underset{\sim}{k}$ components $-3 - 2\lambda = 1 + \mu$ ③

② − ③ gives $16 + 6\lambda = -2$

$6\lambda = -2 - 16$

$6\lambda = -18$

$\lambda = -3$

in ① gives $9 - 3 = 2 + 2\mu$

$6 - 2 = 2\mu$

$4 = 2\mu$

$\mu = 2$

Using l_1 position of intersection is

$\begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9-3 \\ 13-12 \\ -3+6 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$

Position of intersection is $6\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$

C4 Jan 2013

7 b) $\cos\theta = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}| . |\underset{\sim}{b}|}$ where $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are directional vectors of l_1 and l_2

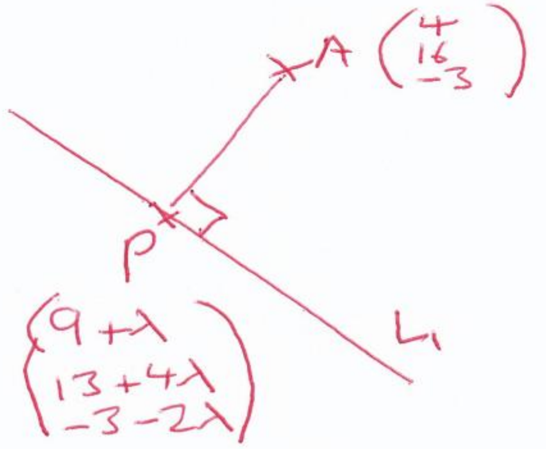
$$\cos\theta = \frac{\begin{pmatrix}1\\4\\-2\end{pmatrix} \cdot \begin{pmatrix}2\\1\\1\end{pmatrix}}{\sqrt{1^2+4^2+(-2)^2} \cdot \sqrt{2^2+1^2+1^2}} = \frac{2+4-2}{\sqrt{21}\sqrt{6}}$$

$$\cos\theta = \frac{4}{\sqrt{21}\sqrt{6}}$$

$$\theta = 69.123897^\circ$$

$$\underline{\underline{\theta = 69.1^\circ}} \quad (1dp)$$

7 c)



$$\overrightarrow{AP} = \underset{\sim}{p} - \underset{\sim}{a}$$

$$= \begin{pmatrix}9+\lambda\\13+4\lambda\\-3-2\lambda\end{pmatrix} - \begin{pmatrix}4\\16\\-3\end{pmatrix}$$

$$\overrightarrow{AP} = \begin{pmatrix}5+\lambda\\-3+4\lambda\\-2\lambda\end{pmatrix}$$

If $\overrightarrow{AP}$ is perpendicular to l_1 their dot product is $\underline{\underline{\text{zero}}}$

$$\begin{pmatrix}5+\lambda\\-3+4\lambda\\-2\lambda\end{pmatrix} \cdot \begin{pmatrix}1\\4\\-2\end{pmatrix} = 5+\lambda-12+16\lambda+4\lambda=0$$

direction vector of $\overrightarrow{AP}$

direction vector of l_1

$$21\lambda - 7 = 0$$

$$21\lambda = 7$$

$$\lambda = \frac{7}{21} = \frac{1}{3}$$

Coordinates of P are $\begin{pmatrix}9+\frac{1}{3}\\13+4\times\frac{1}{3}\\-3-2\times\frac{1}{3}\end{pmatrix}$

$$P \text{ is } \begin{pmatrix}\frac{28}{3}\\\frac{43}{3}\\-\frac{11}{}\end{pmatrix}$$

Leave blank

8. With respect to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O, where p is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to l,

(a) find the value of p.

(4)

Given also that B is a point on l such that $\angle BPA = 45°$,

(b) find the coordinates of the two possible positions of B.

(5)

a) $\underset{\sim}{a} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$ $\quad \underset{\sim}{p} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix}$

direction of L is $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

$\overrightarrow{PA}$ is perpendicular to l so its dot product will be zero

$\overrightarrow{PA} = \underline{a} - \underline{p} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} = \begin{pmatrix} 3+p \\ -2 \\ 4p \end{pmatrix}$

If L is perpendicular to $\overrightarrow{PA}$

$\begin{pmatrix} 3+p \\ -2 \\ 4p \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = 0$

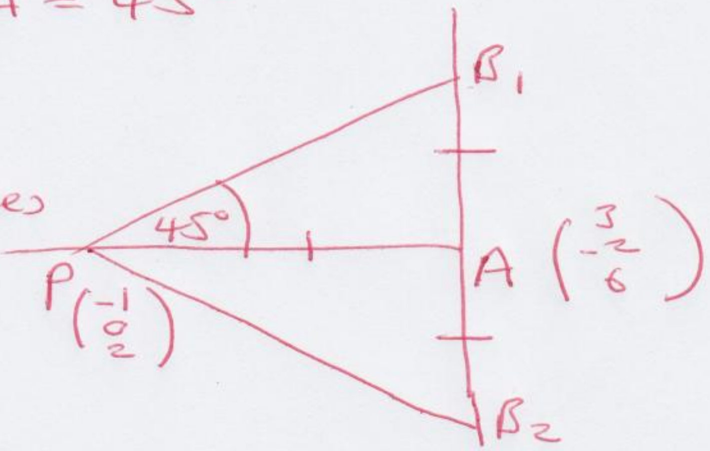
$6 + 2p - 4 - 4p = 0$

$2 = 2p$

$\underline{\underline{p = 1}}$

C4 June 2013

8b) If angle BPA = 45°
then triangles
APB_1 and APB_2
will be isosceles
and length
$AP = AB_1 = AB_2$



P is $\begin{pmatrix}-p\\0\\2p\end{pmatrix} = \begin{pmatrix}-1\\0\\2\end{pmatrix}$

length $AP = \sqrt{(3--1)^2+(-2)^2+(6-2)^2} = \sqrt{16+4+16}$
$= \sqrt{36} = 6$

so length $AB_1 = 6$

B_1 is point $\begin{pmatrix}13+2\lambda\\8+2\lambda\\1-\lambda\end{pmatrix}$

$\overrightarrow{AB_1} = \begin{pmatrix}13+2\lambda\\8+2\lambda\\1-\lambda\end{pmatrix} - \begin{pmatrix}3\\-2\\6\end{pmatrix} = \begin{pmatrix}10+2\lambda\\10+2\lambda\\-5-\lambda\end{pmatrix}$

$|\overrightarrow{AB}| = 6$ so

$\sqrt{(10+2\lambda)^2+(10+2\lambda)^2+(-5-\lambda)^2} = 6$

square both sides

$100+40\lambda+4\lambda^2+100+40\lambda+4\lambda^2+25+10\lambda+\lambda^2 = 36$

$9\lambda^2+90\lambda+225-36 = 0$

$9\lambda^2+90\lambda+189 = 0$ $(\div$ by $9)$

$\lambda^2+10\lambda+21 = 0$

$(\lambda+7)(\lambda+3) = 0$

$\lambda = -7$ or $\lambda = -3$

B_1 is $\begin{pmatrix}13+2\times-7\\8+2\times-7\\1--7\end{pmatrix} = \begin{pmatrix}-1\\-6\\8\end{pmatrix}$

B_2 is $\begin{pmatrix}13+2\times-3\\8+2\times-3\\1--3\end{pmatrix} = \begin{pmatrix}7\\2\\4\end{pmatrix}$