6. *[In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively.]*

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})$ m. Find

(a) the speed of A, **(2)**

(b) the direction in which A is moving, giving your answer as a bearing. **(3)**

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

(c) show that A and B will collide at a point P and find the position vector of P. **(5)**

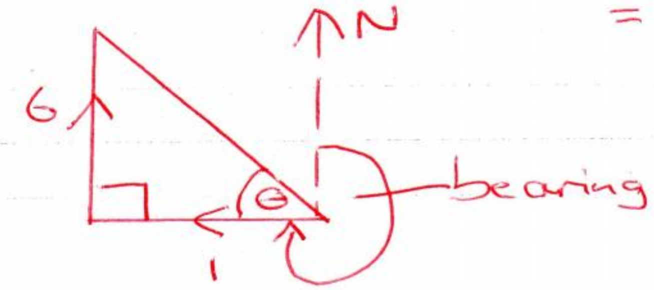
Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(d) find the distance of B from P when $t = 7$ s. **(6)**

a) speed $= \sqrt{(1)^2 + (6)^2} = 6.0827625$

$= 6.08$ ms^{-1} (3 sf)

b)



$\tan\theta = \frac{6}{1}$

$\theta = \tan^{-1} 6$

$\theta = 80.537678°$

Bearing $= 270 + \theta$

$= 350.53768$

$= 351°$ (3 sf)

Question 6 continued

c) Position vector of A at time t

is $(2\underset{\sim}{i} - 10\underset{\sim}{j}) + t(-\underset{\sim}{i} + 6\underset{\sim}{j})$ ①

Position vector of B at time t

is $(-26\underset{\sim}{i} + 4\underset{\sim}{j}) + t(3\underset{\sim}{i} + 4\underset{\sim}{j})$ ②

If they collide, $\underset{\sim}{i}$ components same

and $\underset{\sim}{j}$ components same

From ① + ② $\underset{\sim}{i}$ components $2 - t = -26 + 3t$

$\therefore 4t = 28$

$t = 4$ seconds

Position vector of A at $t = 7$ is

$2\underset{\sim}{i} - 10\underset{\sim}{j} + 7(-\underset{\sim}{i} + 6\underset{\sim}{j})$

$= -5\underset{\sim}{i} + 32\underset{\sim}{j}$

So position P of collision is $-5\underset{\sim}{i} + 32\underset{\sim}{j}$ m

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Question 6 continued

6 d) if direction is $3\underset{\sim}{i} + 4\underset{\sim}{j}$

and speed is $8 ms^{-1}$

Speed for $3\underset{\sim}{i} + 4\underset{\sim}{j}$

$= \sqrt{3^2 + 4^2} = 5 ms^{-1}$

so we need to $\times$ direction vector by $\frac{8}{5}$ to get speed $8 ms^{-1}$

new velocity of B is $\frac{8}{5}(3\underset{\sim}{i} + 4\underset{\sim}{j})$

Position vector of B after 7 seconds

$= (-26\underset{\sim}{i} + 4\underset{\sim}{j}) + \frac{8}{5}(3\underset{\sim}{i} + 4\underset{\sim}{j}) \times 7$

$= -26\underset{\sim}{i} + 33.6\underset{\sim}{i} + 4\underset{\sim}{j} + 44.8\underset{\sim}{j}$

$\underset{\sim}{b} = 7.6\underset{\sim}{i} + 48.8\underset{\sim}{j}$

Distance of B from P

$= \underset{\sim}{b} - \underset{\sim}{p} = (7.6\underset{\sim}{i} + 48.8\underset{\sim}{j})$

$- (-5\underset{\sim}{i} + 32\underset{\sim}{j})$

$= 12.6\underset{\sim}{i} + 16.8\underset{\sim}{j}$

To get distance from B to P

$= \sqrt{(12.6^2) + (16.8)^2}$

$= \underline{\underline{21 m}}$

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3. A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

(a) the acceleration of P in terms of **i** and **j**,

(2)

(b) the magnitude of **F**,

(4)

(c) the velocity of P at time $t = 6$ s.

(3)

a) $\text{acceleration} = \dfrac{\text{end velocity} - \text{start velocity}}{\text{time}}$

$\text{acceleration} = \dfrac{(15\underset{\sim}{i} - 4\underset{\sim}{j}) - (3\underset{\sim}{i} + 2\underset{\sim}{j})}{4}$

$\therefore \text{acceleration} = \dfrac{12\underset{\sim}{i} - 6\underset{\sim}{j}}{4} = 3\underset{\sim}{i} - 1.5\underset{\sim}{j} \text{ ms}^{-2}$

b) $F = ma$

$F = 2 \times (3\underset{\sim}{i} - 1.5\underset{\sim}{j})$

$F = 6\underset{\sim}{i} - 3\underset{\sim}{j} \text{ N}$

$|\underset{\sim}{F}| = \sqrt{6^2 + (-3)^2} = \sqrt{36 + 9}$

$= \sqrt{45} = 6.7082039$

$|\underset{\sim}{F}| = 6.71 \text{ N} \ (3\text{sf})$

c) $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$\underset{\sim}{v} = ?, \ \underset{\sim}{u} = 3\underset{\sim}{i} + 2\underset{\sim}{j} \text{ ms}^{-1}, \ \underset{\sim}{a} = 3\underset{\sim}{i} - 1.5\underset{\sim}{j} \text{ ms}^{-2}$

$\underset{\sim}{t} = 6$

$\underset{\sim}{v}_6 = (3\underset{\sim}{i} + 2\underset{\sim}{j}) + (3\underset{\sim}{i} - 1.5\underset{\sim}{j}) \times 6$

$\underset{\sim}{v}_6 = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 18\underset{\sim}{i} - 9\underset{\sim}{j}$

$\underset{\sim}{v}_6 = 21\underset{\sim}{i} - 7\underset{\sim}{j} \text{ ms}^{-1}$

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6. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of P,

(2)

(b) the direction of motion of P, giving your answer as a bearing.

(3)

At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}.

(c) Find the values of u and v.

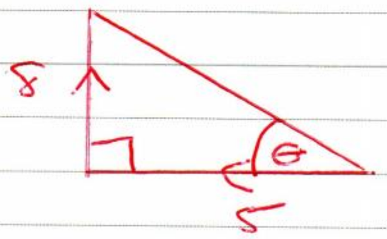
(5)

(d) Find the total time taken for P to move from A to a position which is due south of A.

(3)

a) speed $= \sqrt{5^2 + 8^2} = 9.4339811$
$= 9.43 \text{ms}^{-1}$ (3sf)

b)



$\tan\theta = \frac{8}{5}$

$\theta = \tan^{-1}\frac{8}{5}$

$\theta = 57.994617°$

Direction of P is $270° + 57.994617$
$= 327.99462$
$= 328°$ (3sf)

6c) At $t=0$ position is $(7\mathbf{i}-10\mathbf{j})$

velocity for time $t=0$ to $t=3$ is $(-5\mathbf{i}+8\mathbf{j})\ \text{ms}^{-1}$

At $t=3$, position vector is

$$7\mathbf{i}-10\mathbf{j}+3(-5\mathbf{i}+8\mathbf{j})$$

$$=7\mathbf{i}-15\mathbf{i}-10\mathbf{j}+24\mathbf{j}$$

$$=-8\mathbf{i}+14\mathbf{j}$$

For next 4 seconds, velocity is $(u\mathbf{i}+v\mathbf{j})$

So position vector after this time is

$$-8\mathbf{i}+14\mathbf{j}+4(u\mathbf{i}+v\mathbf{j})=0$$

(zero as after 4 seconds it passes through origin)

$\therefore\ -8\mathbf{i}+4u\mathbf{i}=0$ (for $\mathbf{i}$ component)

$4u=8$

$u=2$

and $14\mathbf{j}+4v\mathbf{j}=0$ (for $\mathbf{j}$ component)

$4v=-14$

$v=-3.5$

d) Position vector "t" seconds after changing course is

$$-8\mathbf{i}+14\mathbf{j}+t(2\mathbf{i}-3.5\mathbf{j})$$

For a position due South, set $\mathbf{i}$ components to $7\mathbf{i}$ ($(7\mathbf{i}-10\mathbf{j})$ is position vector A)

$$-8\mathbf{i}+2t\mathbf{i}=7\mathbf{i}$$

$2t\mathbf{i}=7\mathbf{i}+8\mathbf{i}$

$2t=15\ \therefore t=7.5$

$\therefore$ Total time $=3+7.5$

$=\underline{10.5\text{ seconds}}$

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1. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the value of u. **(5)**

$a = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ m s^{-2}

$u = u$ m s^{-1}

$v = -6\underset{\sim}{i} + \underset{\sim}{j}$ m s^{-1}

$t = 3$ seconds

$v = u + at$

$-6\underset{\sim}{i} + \underset{\sim}{j} = u + 3(2\underset{\sim}{i} - 5\underset{\sim}{j})$

$-6\underset{\sim}{i} + \underset{\sim}{j} = u + 6\underset{\sim}{i} - 15\underset{\sim}{j}$

$u = -12\underset{\sim}{i} + 16\underset{\sim}{j}$

$\Rightarrow u = \sqrt{(-12)^2 + (16)^2}$

$u = 20$ m s^{-1}

Leave blank

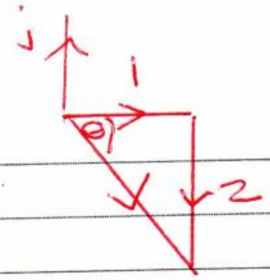
6. Two forces, $(4\mathbf{i} - 5\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P of mass m kg. The resultant of the two forces is $\mathbf{R}$. Given that $\mathbf{R}$ acts in a direction which is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) find the angle between $\mathbf{R}$ and the vector $\mathbf{j}$, **(3)**

(b) show that $2p + q + 3 = 0$. **(4)**

Given also that $q = 1$ and that P moves with an acceleration of magnitude $8\sqrt{5}$ m s^{-2},

(c) find the value of m. **(7)**



a) vector $\underset{\sim}{i} - 2\underset{\sim}{j}$

$\tan\theta = \frac{2}{1}$

$\theta = \tan^{-1} 2$

$\theta = 63.434949$

$\therefore$ angle between R and vector $\underset{\sim}{j}$ is

$90 + \theta = 153.4°$ (1dp)

b) $(4\underset{\sim}{i} - 5\underset{\sim}{j}) + (p\underset{\sim}{i} + q\underset{\sim}{j})$

$= (4+p)\underset{\sim}{i} + (q-5)\underset{\sim}{j}$

as this is parallel to $(\underset{\sim}{i} - 2\underset{\sim}{j})$ the $\underset{\sim}{j}$ component is -2 times the $\underset{\sim}{i}$ component

so $(q-5) = -2(4+p)$

$q - 5 = -8 - 2p$

$\therefore 2p + q + 3 = 0$ as required

6c) given that $q = 1$

then $2p + q + 3 = 0$

$2p + 1 + 3 = 0$

$2p = -4$

$p = -2$

so $R = (4+p)\underset{\sim}{i} + (q-5)\underset{\sim}{j}$

$= (4-2)\underset{\sim}{i} + (1-5)\underset{\sim}{j}$

$R = 2\underset{\sim}{i} - 4\underset{\sim}{j}$

Magnitude of R is $\sqrt{2^2 + (-4)^2}$

$= \sqrt{20}$ N

Force = mass × acceleration

$\therefore \sqrt{20} = m \times 8\sqrt{5}$

$m = \dfrac{\sqrt{20}}{8\sqrt{5}} = \dfrac{\cancel{\sqrt{5}}^{1} \times \sqrt{4}}{8 \times \cancel{\sqrt{5}}_{1}} = \dfrac{2}{8} = \dfrac{1}{4}$

Value of $m = \dfrac{1}{4}$

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7. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s} = (3t + 9)\,\mathbf{i} + (4t - 6)\,\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

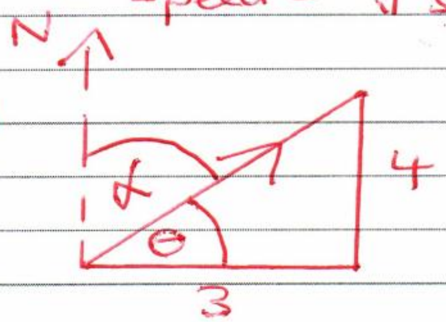
(6)

a) $v = \dfrac{(21\underset{\sim}{i} + 10\underset{\sim}{j}) - (9\underset{\sim}{i} - 6\underset{\sim}{j})}{4}$

$= \dfrac{12\underset{\sim}{i} + 16\underset{\sim}{j}}{4} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

Speed $= \sqrt{3^2 + 4^2} = 5 \text{ km h}^{-1}$

b)



$v = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\frac{4}{3}$

$\theta = 53.1°$

Direction is angle $\alpha = 90 - \theta$

$= 90 - 53.1°$

$= 36.9°$

$= 037°$ (bearing to 2 sf)

7c) $\underset{\sim}{S} = 9\underset{\sim}{i} - 6\underset{\sim}{j} + t(3\underset{\sim}{i} + 4\underset{\sim}{j})$

start position   velocity

$\underset{\sim}{s} = 9\underset{\sim}{i} + 3t\underset{\sim}{i} - 6\underset{\sim}{j} + 4t\underset{\sim}{j}$

$\underset{\sim}{s} = (3t+9)\underset{\sim}{i} + (4t-6)\underset{\sim}{j}$

as required

7d) Position vector of S relative to L

$\underset{\sim}{s} = (3T+9)\underset{\sim}{i} + (4T-6)\underset{\sim}{j}$

and

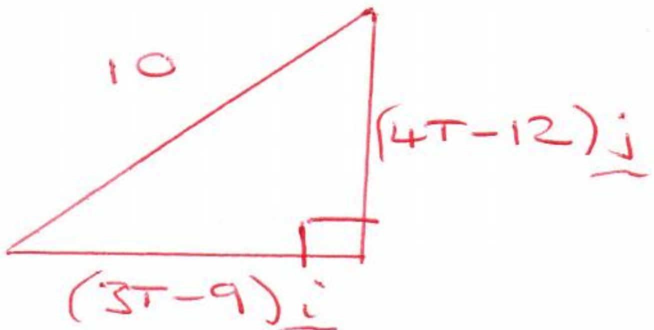
$\underset{\sim}{L} = 18\underset{\sim}{i} + 6\underset{\sim}{j}$

$\underset{\sim}{s} - \underset{\sim}{L} = (3T+9)\underset{\sim}{i} + (4T-6)\underset{\sim}{j} - (18\underset{\sim}{i} + 6\underset{\sim}{j})$

$= 3T\underset{\sim}{i} + 9\underset{\sim}{i} + 4T\underset{\sim}{j} - 6\underset{\sim}{j} - 18\underset{\sim}{i} - 6\underset{\sim}{j}$

$= (3T-9)\underset{\sim}{i} + (4T-12)\underset{\sim}{j}$

Diagram shows the
$\underset{\sim}{i}$ and $\underset{\sim}{j}$ vectors
10km apart



By Pythagoras

$(3T-9)^2 + (4T-12)^2 = 10^2$

$9T^2 - 54T + 81 + 16T^2 - 96T + 144 = 100$

$25T^2 - 150T + 125 = 0$

$25(T^2 - 6T + 5) = 0$

$25(T-5)(T-1) = 0$

Either $T = 1$ or $T = 5$ hours

M1 JAN 2011

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4. A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. The velocity of P is $(2\mathbf{i}-5\mathbf{j})$ m s^{-1} at time $t=0$, and $(7\mathbf{i}+10\mathbf{j})$ m s^{-1} at time $t=5$ s.

$\underset{\sim}{u}$ $\underset{\sim}{v}$

Find

(a) the speed of P at $t=0$,

(2)

(b) the vector **F** in the form $a\mathbf{i}+b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to **i**.

(4)

b) $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$7\underset{\sim}{i} + 10\underset{\sim}{j} = 2\underset{\sim}{i} - 5\underset{\sim}{j} + \underset{\sim}{a} \times 5$

$7\underset{\sim}{i} - 2\underset{\sim}{i} + 10\underset{\sim}{j} + 5\underset{\sim}{j} = 5\underset{\sim}{a}$

$5\underset{\sim}{i} + 15\underset{\sim}{j} = 5\underset{\sim}{a}$

$\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{F} = m\underset{\sim}{a}$

$\underset{\sim}{F} = 2(\underset{\sim}{i} + 3\underset{\sim}{j}) = 2\underset{\sim}{i} + 6\underset{\sim}{j}$ N

a) speed $= \sqrt{2^2 + (-5)^2} = \sqrt{4+25}$

$= \sqrt{29}$ ms^{-1}

c) $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$\underset{\sim}{v} = (2\underset{\sim}{i} - 5\underset{\sim}{j}) + (\underset{\sim}{i} + 3\underset{\sim}{j})t$

When moving parallel to $\underset{\sim}{i}$

$\underset{\sim}{j}$ components are zero

so $-5 + 3t = 0$

$3t = 5$

$t = \frac{5}{3}$ seconds

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3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\text{ N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\text{ N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q. **(3)**

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$, **(2)**

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$. **(3)**

a) If in equilibrium, sum of forces is 0

so $F_1 + F_2 + F_3 = 0$

$\underset{\sim}{i}$ components $7 + 5 + p = 0$

$p = -12$

$\underset{\sim}{j}$ components $-9 + 6 + q = 0$

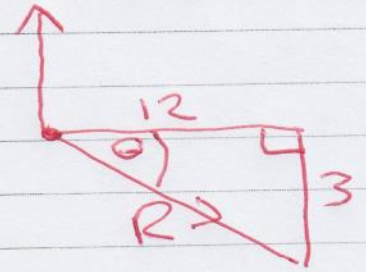
$q = 3$

b) $F_1 + F_2 = 7\underset{\sim}{i} - 9\underset{\sim}{j} + 5\underset{\sim}{i} + 6\underset{\sim}{j}$

$= 12\underset{\sim}{i} - 3\underset{\sim}{j}$

Magnitude $R = \sqrt{12^2 + 3^2} = \sqrt{153}$

$= 12.4\text{N}$ (3sf)

c) 

$\theta = \tan^{-1}\left(\frac{3}{12}\right) = 14.0362$

$= 14°$ (to nearest degr

Angle R makes with j = 90 + 14

= 104° (nearest degree)

M1 Jan 2012

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7. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A boat P is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i}+8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

a) Speed $= \sqrt{4^2+8^2} = 8.9442719$
$= 8.94$ km h^{-1} (3sf)

b) $\underset{\sim}{p} = 2\underset{\sim}{i} - 8\underset{\sim}{j} + t(-4\underset{\sim}{i} + 8\underset{\sim}{j})$
$= (2-4t)\underset{\sim}{i} + (8t-8)\underset{\sim}{j}$

c) when due west of Q, $\underset{\sim}{j}$ components will be equal.

for $\underset{\sim}{q} = (18-6t)\underset{\sim}{i} + (12-8t)\underset{\sim}{j}$

so $8t - 8 = 12 - 8t$
$16t = 20$
$t = \frac{20}{16} = 1.25$

d) when $t = 1.25$, i component of $q = 18 - 6\times1.25 = 10.5$
i component of $p = 2 - 4\times1.25 = -3$
So distance between P and Q $= 10.5 - -3 = 13.5$ km

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7. [*In this question the unit vectors* **i** *and* **j** *are due east and north respectively.*]

A ship S is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j})\ \text{km h}^{-1}$. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i} + 5\mathbf{j})$ km. Find

(a) the speed of S,
(2)

(b) the bearing on which S is moving.
(2)

The ship is heading directly towards a submerged rock R. A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

(c) Find the position vector of R.
(2)

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of $5\ \text{km h}^{-1}$. Assuming that S continues to move with this new constant velocity, find

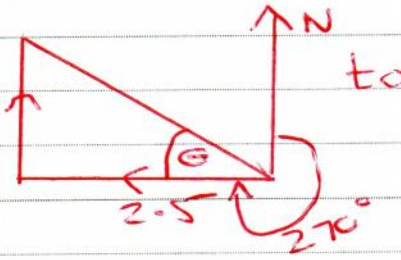
(d) an expression for the position vector of the ship t hours after 1400,
(4)

(e) the time when S will be due east of R,
(2)

(f) the distance of S from R at the time 1600.
(3)

a) Speed = $\sqrt{(2.5)^2 + 6^2} = 6.5\ \text{kmh}^{-1}$

b)



$\tan\theta = \frac{6}{2.5}$

$\theta = \tan^{-1}\frac{6}{2.5}$

$\theta = 67.380135°$

Bearing on which S is moving

$= 270 - 67.380135$

$= 337.38$

$= 337°$ (3sf)

7 c) Position vector of R is

vector S + (3 × constant velocity vector)

$$= 16\underset{\sim}{i} + 5\underset{\sim}{j} + 3(-2.5\underset{\sim}{i} + 6\underset{\sim}{j})$$

$$= 16\underset{\sim}{i} - 7.5\underset{\sim}{i} + 5\underset{\sim}{j} + 18\underset{\sim}{j}$$

$$= 8.5\underset{\sim}{i} + 23\underset{\sim}{j}$$

d) at 1400

position is $16\underset{\sim}{i} + 5\underset{\sim}{j} + 2(-2.5\underset{\sim}{i} + 6\underset{\sim}{j})$

$$= 16\underset{\sim}{i} - 5\underset{\sim}{i} + 5\underset{\sim}{j} + 12\underset{\sim}{j}$$

$$= 11\underset{\sim}{i} + 17\underset{\sim}{j}$$

At 1400, velocity is constant $5\underset{\sim}{j}$ km h^{-1}

At this velocity, at 1500 position would be

$$11\underset{\sim}{i} + 17\underset{\sim}{j} + (5\underset{\sim}{j})$$

at 1600 position would be

$$11\underset{\sim}{i} + 17\underset{\sim}{j} + (2 \times 5\underset{\sim}{j})$$

at 1400 + t hours would be

$$11\underset{\sim}{i} + 17\underset{\sim}{j} + (t \times 5\underset{\sim}{j})$$

$$= 11\underset{\sim}{i} + (17 + 5t)\underset{\sim}{j}$$

e) position R is $8.5\underset{\sim}{i} + 23\underset{\sim}{j}$

For S to be due East of R, j components must be equal

$$\therefore \quad 17 + 5t = 23$$

$$5t = 23 - 17$$

$$t = \frac{6}{5}$$

Time is $1400 + 1\frac{1}{5}$ hours = 1512 hours

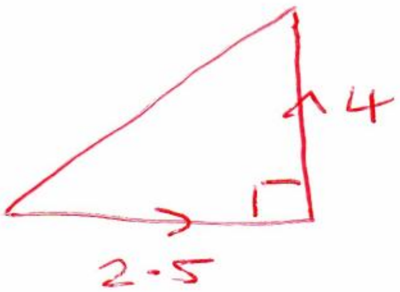
7f) At 1600, let position be $\underset{\sim}{s}$

$$\underset{\sim}{s} = 11\underset{\sim}{i} + 17\underset{\sim}{j} + (2 \times 5\underset{\sim}{j})$$
$$= 11\underset{\sim}{i} + 27\underset{\sim}{j}$$

& position $\underset{\sim}{r} = 8.5\underset{\sim}{i} + 23\underset{\sim}{j}$

$$\underset{\sim}{s} - \underset{\sim}{r} = (11\underset{\sim}{i} + 27\underset{\sim}{j}) - (8.5\underset{\sim}{i} + 23\underset{\sim}{j})$$
$$= 2.5\underset{\sim}{i} + 4\underset{\sim}{j}$$

$$\text{Distance} = \sqrt{(2.5)^2 + 4^2}$$
$$= 4.7169906$$
$$= 4.72 \text{ km} \quad (3 \text{ sf})$$



7 a) continued

$$\text{velocity} = \frac{5\underset{\sim}{i} + 15\underset{\sim}{j}}{2\cdot 5} = 2\underset{\sim}{i} + 6\underset{\sim}{j}$$

This means, in <u>one</u> hour, the boat moves $2\underset{\sim}{i} + 6\underset{\sim}{j}$

$v_B = 2\underset{\sim}{i} + 6\underset{\sim}{j}$

b) $\underset{\sim}{r}_t = \underset{\sim}{b} = \underset{\sim}{r}_0 + t v_B$

after 1 hour, position would be

$\underset{\sim}{r}_0 + 1 \times \underset{\sim}{v}_B$

2 hours $\underset{\sim}{r}_0 + 2\underset{\sim}{v}_B$ etc

$\underset{\sim}{b} = \underset{\sim}{r}_0 + t\underset{\sim}{v}_B$

$\therefore \underset{\sim}{b} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + t(2\underset{\sim}{i} + 6\underset{\sim}{j})$

$\therefore \underset{\sim}{b} = (3 + 2t)\underset{\sim}{i} + (6t - 4)\underset{\sim}{j}$

c) For boats C and B to intercept, their $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components need to be identical at point of intersection.

$\underset{\sim}{c} = (-9\underset{\sim}{i} + 20\underset{\sim}{j}) + t(6\underset{\sim}{i} + \lambda\underset{\sim}{j})$

$\underset{\sim}{c} = (6t - 9)\underset{\sim}{i} + (20 + \lambda t)\underset{\sim}{j}$

So if we compare $\underset{\sim}{i}$ components of B and C

$3 + 2t = 6t - 9$

$\therefore 12 = 4t$ → after 3 hours boats will intercept

$\therefore t = 3$

Compare $\underset{\sim}{j}$ components

$6t - 4 = 20 + \lambda t$ (but $t = 3$)

$18 - 4 = 20 + 3\lambda$

$\therefore 14 - 20 = 3\lambda$

$-6 = 3\lambda$ $\therefore \lambda = -2$

7. A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3\mathbf{i} - 4\mathbf{j})$ km with respect to a fixed origin O. At 1430 on the same day, B is at the point with position vector $(8\mathbf{i} + 11\mathbf{j})$ km.

(a) Find the velocity of B, giving your answer in the form $p\mathbf{i} + q\mathbf{j}$.

(3)

At time t hours after noon, the position vector of B is $\mathbf{b}$ km.

(b) Find, in terms of t, an expression for $\mathbf{b}$.

(3)

Another boat C is also moving with constant velocity. The position vector of C, $\mathbf{c}$ km, at time t hours after noon, is given by

$$\mathbf{c} = (-9\mathbf{i} + 20\mathbf{j}) + t(6\mathbf{i} + \lambda\mathbf{j}),$$

where λ is a constant. Given that C intercepts B,

(c) find the value of λ,

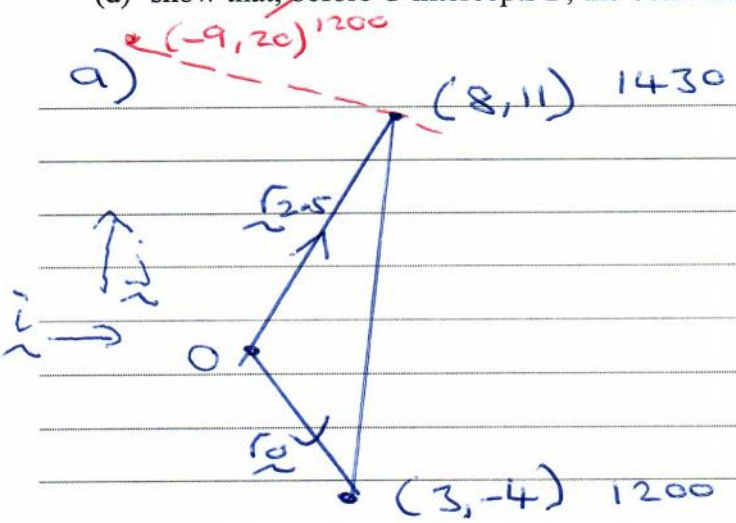
(5)

boat C

(d) show that, before C intercepts B, the boats are moving with the same speed.

(3)

a)



at $t=0$, $\underset{\sim}{r}_0 = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ Start position at noon

$\underset{\sim}{r}_{2.5} = 8\underset{\sim}{i} + 11\underset{\sim}{j}$ position at 1430

velocity = $\dfrac{\text{change in position}}{\text{time taken}}$

a) $\underset{\sim}{v} = \dfrac{\underset{\sim}{r}_{2.5} - \underset{\sim}{r}_0}{t}$

$= \dfrac{8\underset{\sim}{i} + 11\underset{\sim}{j} - (3\underset{\sim}{i} - 4\underset{\sim}{j})}{2.5}$

7 d) $v_c = 6\underset{\sim}{i} - 2\underset{\sim}{j}$

this was λ

the velocity comes from the "t" part of the position vector

$$\underset{\sim}{r} = \underbrace{(-9\underset{\sim}{i} + 20\underset{\sim}{j})}_{\text{start position}} + t\underbrace{(6\underset{\sim}{i} + 2\underset{\sim}{j})}_{\text{velocity}}$$

Find magnitude of $\underset{\sim}{v}_B$ and $\underset{\sim}{v}_c$ and show they are the same

$$|\underset{\sim}{v}_B| = \sqrt{2^2+6^2} = \sqrt{40}$$

$$|\underset{\sim}{v}_c| = \sqrt{6^2+(-2)^2} = \sqrt{40}$$

$\therefore$ speeds are the same

2. A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

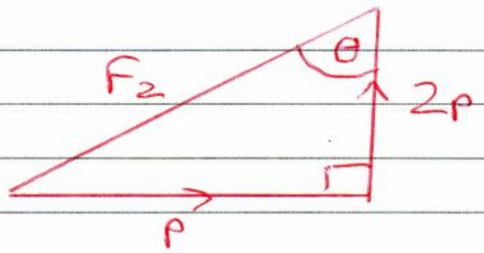
(a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$.

(2)

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

(b) find the value of p.

(4)



a) Angle between F_2 and $\underset{\sim}{j}$ from diagram is θ

$\tan\theta = \frac{p}{2p} = \frac{1}{2}$

$\theta = \tan^{-1}\frac{1}{2}$

$\theta = 26.565051$

$\theta = 26.6^\circ$ (3 sf)

b) $F_1 + F_2 = (\underset{\sim}{i} - 3\underset{\sim}{j}) + (p\underset{\sim}{i} + 2p\underset{\sim}{j})$

$= (1+p)\underset{\sim}{i} + (-3+2p)\underset{\sim}{j}$

R is parallel to $\underset{\sim}{i}$

$\therefore (-3+2p) = 0$

$\therefore 2p = 3$

$p = \frac{3}{2}$

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(a) Find the speed of H.

(2)

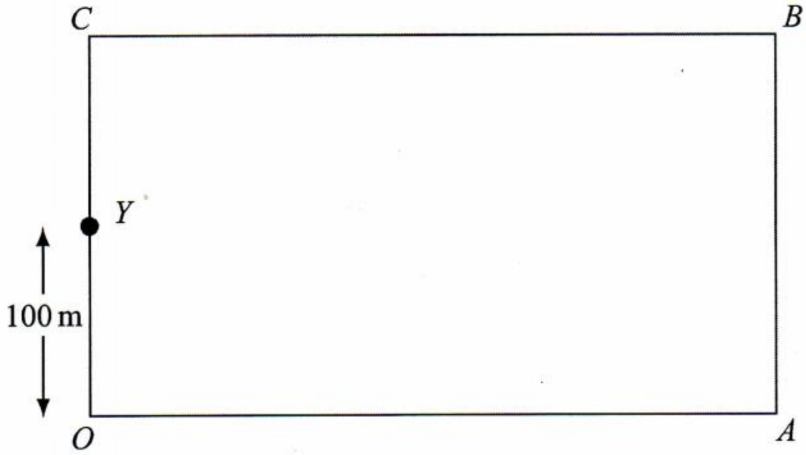


Figure 3

A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in Figure 3. At twelve noon hiker H is at the point Y with position vector $100\,\mathbf{j}$ m, relative to the fixed origin O.

(b) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker K is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(c) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = \left[(9 - 0.45t)\,\mathbf{i} + (2.7t - 54)\,\mathbf{j}\right] \text{ metres.}$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

a) speed = $\sqrt{(1\cdot2)^2 + (0\cdot9)^2} = 1\cdot5 \text{ m s}^{-1}$

8b) Position at noon is $100\underset{\sim}{j}$

Position vector t seconds after noon for Hiker H

$\underset{\sim}{H} = 100j + t(1.2\underset{\sim}{i} - 0.9\underset{\sim}{j})$ m

c) Hiker K

$\underset{\sim}{K} = 9i + 46\underset{\sim}{j} + t(0.75\underset{\sim}{i} + 1.8\underset{\sim}{j})$

For $\overrightarrow{HK}$ take vector $\underset{\sim}{H}$ from $\underset{\sim}{K}$

$\underset{\sim}{K} - \underset{\sim}{H} =$

$(9\underset{\sim}{i} + 46\underset{\sim}{j} + t(0.75\underset{\sim}{i} + 1.8\underset{\sim}{j})$

$- (100\underset{\sim}{j} + t(1.2\underset{\sim}{i} - 0.9\underset{\sim}{j}))$

$= (9\underset{\sim}{i} + 0.75t\underset{\sim}{i} + 46j + 1.8t\underset{\sim}{j})$

$- (\quad 1.2t\underset{\sim}{i} + 100\underset{\sim}{j} - 0.9t\underset{\sim}{j})$

$= 9\underset{\sim}{i} - 0.45t\underset{\sim}{i} - 54\underset{\sim}{j} + 2.7t\underset{\sim}{j}$

$= (9 - 0.45t)\underset{\sim}{i} + (2.7t - 54)\underset{\sim}{j}$ m

as requested

d) Hikers meet when $\overrightarrow{HK} = 0$

$\therefore 9 - 0.45t = 0$ and $2.7t - 54 = 0$

$t = \frac{9}{0.45}$ $\qquad t = \frac{54}{2.7}$

$t = 20$ seconds $\qquad t = 20$ seconds

Point of intersection is

$\underset{\sim}{H} = 100\underset{\sim}{j} + 20(1.2\underset{\sim}{i} - 0.9\underset{\sim}{j})$

$= 100\underset{\sim}{j} + 24\underset{\sim}{i} - 18j$

$= 24\underset{\sim}{i} + 82\underset{\sim}{j}$

3. A particle P of mass 0.4 kg moves under the action of a single constant force **F** newtons. The acceleration of P is $(6\mathbf{i} + 8\mathbf{j})$ m s^{-2} . Find

(a) the angle between the acceleration and **i**,

(2)

(b) the magnitude of **F**.

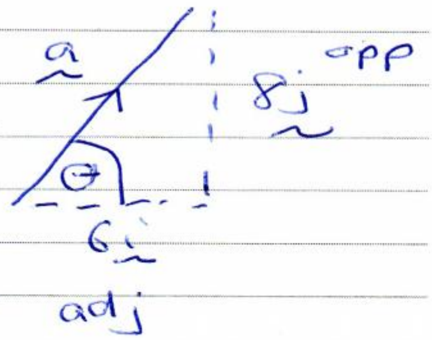
(3)

At time t seconds the velocity of P is **v** m s^{-1} . Given that when $t = 0$, $\mathbf{v} = 9\mathbf{i} - 10\mathbf{j}$,

(c) find the velocity of P when $t = 5$.

(3)

a) $\underset{\sim}{a} = 6\underset{\sim}{i} + 8\underset{\sim}{j}$



$\therefore \tan\theta = \frac{8}{6}$

$\theta = \tan^{-1}\frac{8}{6}$

$= 53.1301\ldots$

$= 53.1°$ (3sf)

b) $\underset{\sim}{F} = m\underset{\sim}{a}$

$= 0.4(6\underset{\sim}{i} + 8\underset{\sim}{j})$

$= 2.4\underset{\sim}{i} + 3.2\underset{\sim}{j}$

$|\underset{\sim}{F}| = \sqrt{2.4^2 + 3.2^2}$

$= 4$ N

Leave blank

Question 3 continued

c) $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

where $\underset{\sim}{u} = 9\underset{\sim}{i} - 10j$

$\underset{\sim}{a} = \underset{\sim}{6i} + 8\underset{\sim}{j}$

$\underset{\sim}{v} = 9\underset{\sim}{i} - 10\underset{\sim}{j} + 5(6\underset{\sim}{i} + 8j)$

$\underset{\sim}{v} = 9\underset{\sim}{i} - 10\underset{\sim}{j} + 30\underset{\sim}{i} + 40\underset{\sim}{j}$

$\underset{\sim}{v} = 39\underset{\sim}{i} + 30\underset{\sim}{j} \quad ms^{-1}$

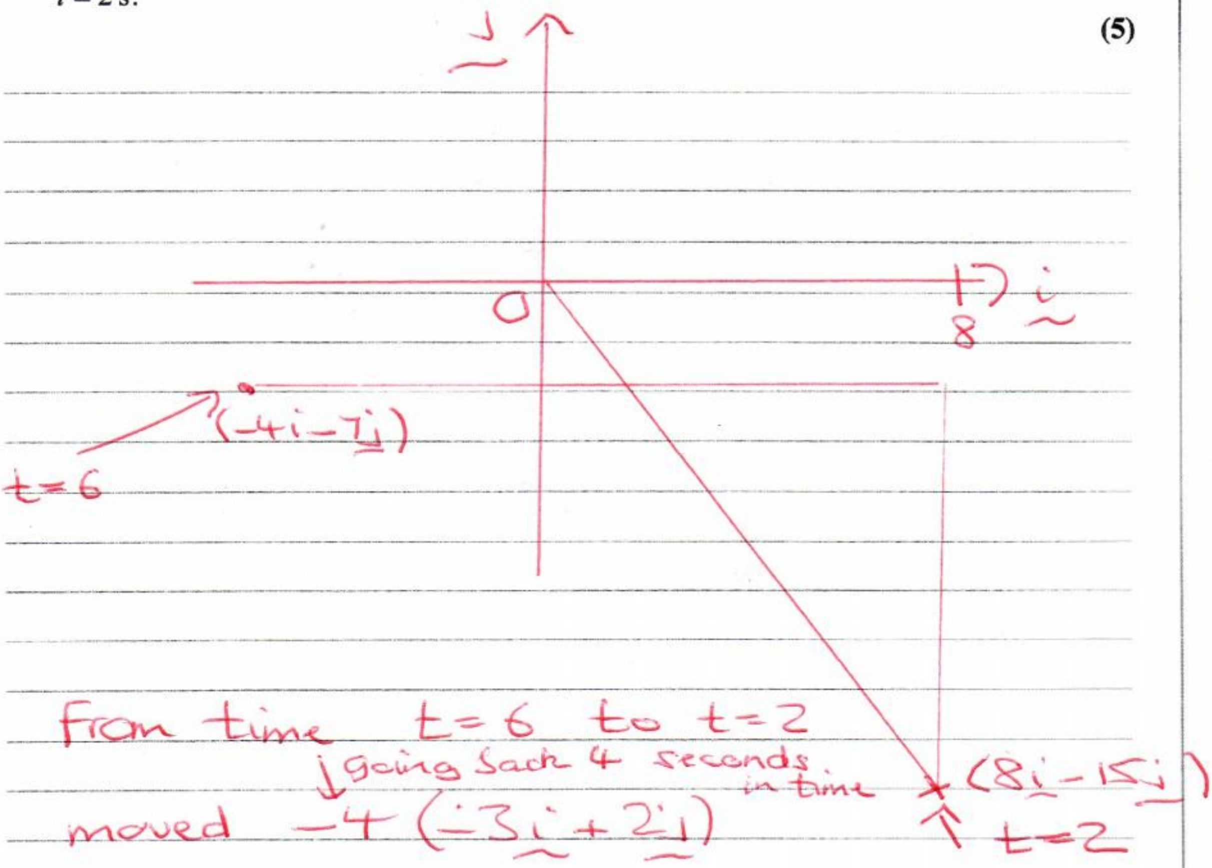
Q3

(Total 8 marks)

May 2010

1. A particle P is moving with constant velocity $(-3\mathbf{i}+2\mathbf{j})\ \mathrm{m\,s^{-1}}$. At time $t=6$ s P is at the point with position vector $(-4\mathbf{i}-7\mathbf{j})$ m. Find the distance of P from the origin at time $t=2$ s.

(5)



From time $t=6$ to $t=2$

going back 4 seconds in time

moved $-4(-3i+2j)$

$(8i-15j)$

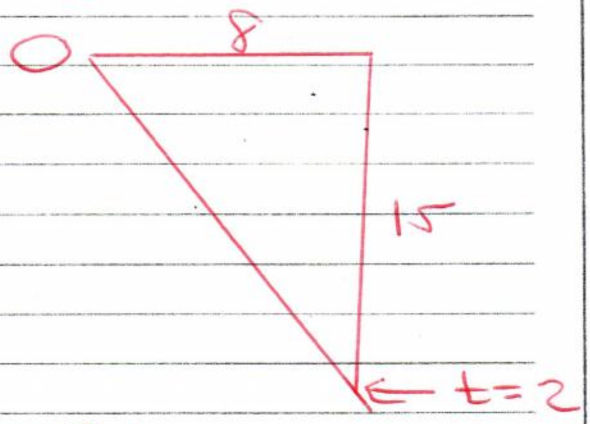
$t=2$

$= 12i - 8j$ units

So position vector will be

$(-4i+12i \ -7j-8j)$

$= 8i-15j$



Distance to origin

$= \sqrt{8^2+15^2}$

$= 17$ m

at time $t=2$s

N 3 5 3 9 0 A 0 2 2 8

May 2011

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7. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,

(ii) $\mathbf{q}$,

(iii) $\overrightarrow{PQ}$.

(5)

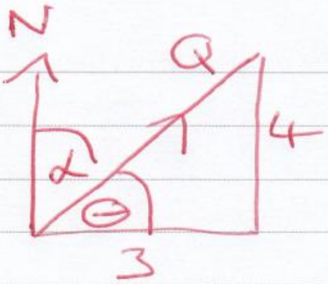
(c) Find the time when

(i) Q is due north of P,

(ii) Q is north-west of P.

(4)

a)



$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\frac{4}{3}$

$\theta = 53 \cdot 13°$

Bearing of Q is $\alpha = 90 - \theta$

$= 36 \cdot 869$

$= 037°$ (nearest degree)

May 2011

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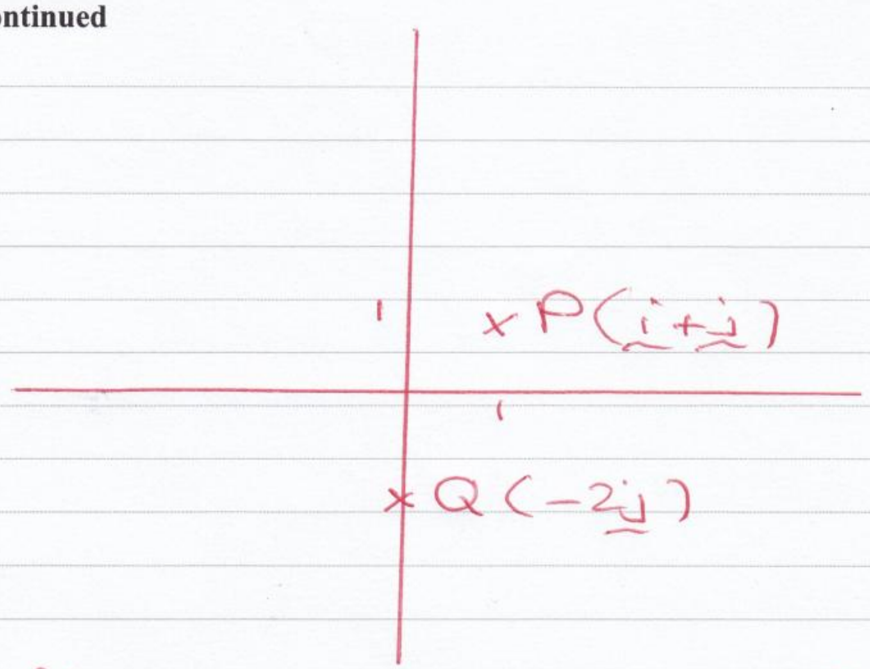
Question 7 continued



b) after t hours, position of P is

(i) $\underset{\sim}{r}_P = \underset{\sim}{p} = \underset{\sim}{r}_0 + \underset{\sim}{v}t$

$\underset{\sim}{p} = (\underset{\sim}{i}+\underset{\sim}{j}) + t(2\underset{\sim}{i} - 3\underset{\sim}{j})$

(ii) $\underset{\sim}{q} = -2\underset{\sim}{j} + t(3\underset{\sim}{i}+4\underset{\sim}{j})$

(iii) $\overrightarrow{PQ} = \underset{\sim}{q} - \underset{\sim}{p}$

$= -2\underset{\sim}{j} + 3t\underset{\sim}{i} + 4t\underset{\sim}{j} - \underset{\sim}{i} - \underset{\sim}{j} - 2t\underset{\sim}{i} + 3t\underset{\sim}{j}$

$= 3t\underset{\sim}{i} - 2t\underset{\sim}{i} - \underset{\sim}{i} - 2\underset{\sim}{j} + 4t\underset{\sim}{j} - \underset{\sim}{j} + 3t\underset{\sim}{j}$

$= t\underset{\sim}{i} - \underset{\sim}{i} - 3\underset{\sim}{j} + 7t\underset{\sim}{j}$

$\overrightarrow{PQ} = (t-1)\underset{\sim}{i} + (7t-3)\underset{\sim}{j}$

c)(i) Q due north of P when $\underset{\sim}{i}$ components of $\overrightarrow{PQ} = 0$

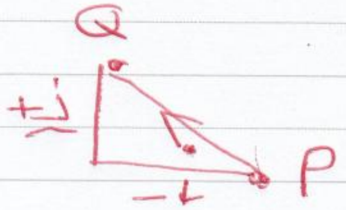
$\therefore t = 1$

Due north at 3pm

May 2011

Question 7 continued

(ii) North west



when $\underset{\sim}{j}$ component $\div$ $\underset{\sim}{i}$ component

equals -1 (gradient $= -1$)

$$\frac{7t-3}{t-1} = -1$$

$$7t - 3 = -1(t-1)$$
$$7t - 3 = -t + 1$$
$$8t = 4$$
$$t = 0.5 \text{ hours}$$

North-west at 2.30 pm

Leave blank

Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

M1 MAY 2012

Leave blank

6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(a) Find the direction in which S is moving, giving your answer as a bearing. **(3)**

At time t hours after noon, the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

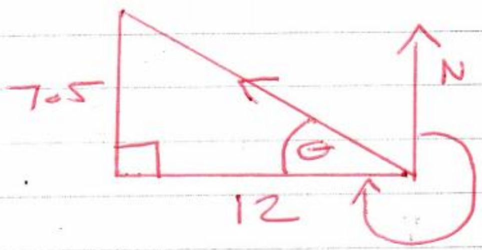
(b) Write down $\mathbf{s}$ in terms of t. **(2)**

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(c) Find the distance of S from B when $t = 3$ **(4)**

(d) Find the distance of S from B when S is due north of B. **(4)**

a)



$\tan\theta = \frac{7.5}{12}$

$\theta = \tan^{-1}\frac{7.5}{12}$

$\theta = 32.005383°$

Bearing is $270 + 32.005383$
$= 302.00538$
$= 302°$ (3sf)

b) $\underset{\sim}{s} = \underset{\sim}{r_0} + \underset{\sim}{v}t$

$\underset{\sim}{s} = (40\underset{\sim}{i} - 6\underset{\sim}{j}) + t(-12\underset{\sim}{i} + 7.5\underset{\sim}{j})$

c) When $t = 3$,

$\underset{\sim}{s} = (40 - 12t)\underset{\sim}{i} + (7.5t - 6)\underset{\sim}{j}$
$= (40 - 12\times3)\underset{\sim}{i} + (7.5\times3 - 6)\underset{\sim}{j}$
$= 4\underset{\sim}{i} + 16.5\underset{\sim}{j}$

$\overrightarrow{BS} = \underset{\sim}{s} - \underset{\sim}{b} = (4\underset{\sim}{i} + 16.5\underset{\sim}{j}) - (7\underset{\sim}{i} + 12.5\underset{\sim}{j})$
$= -3\underset{\sim}{i} + 4\underset{\sim}{j}$

Distance $BS = \sqrt{3^2 + 4^2} = 5$ km

M1 MAY 2012

Q 6 d) When S is due north of B

$\underset{\sim}{i}$ components will be equal

So $40 - 12t = 7$

$40 - 7 = 12t$

$33 = 12t$

$t = \frac{33}{12} = 2.75$ hours

When $t = 2.75$

$\underset{\sim}{s} = (40 - 12 \times 2.75)\underset{\sim}{i} + (7.5 \times 2.75 - 6)\underset{\sim}{j}$

$= 7\underset{\sim}{i} + 14.625\underset{\sim}{j}$

$\overrightarrow{SB} = \underset{\sim}{s} - \underset{\sim}{b} = (7\underset{\sim}{i} + 14.625\underset{\sim}{j}) - (7\underset{\sim}{i} + 12.5\underset{\sim}{j})$

$= 2.125\underset{\sim}{j}$

Distance when due north is

2.125 km

M1 JAN 2013

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6. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by $\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

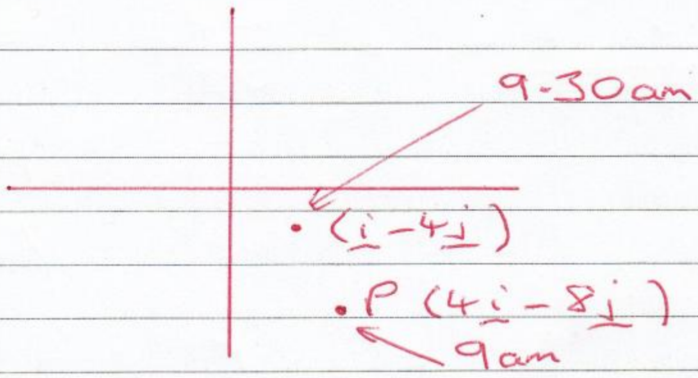
(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

a)



Distance travelled 9 am → 9.30 am

$= \sqrt{(1-4)^2 + (8-4)^2} = \sqrt{9+16} = \sqrt{25}$

$= 5$ km

Speed $= \dfrac{\text{dist}}{\text{time}} = \dfrac{5}{0.5} = 10$ km h^{-1}

b) $\underset{\sim}{r} = \underset{\sim}{r}_0 + \underset{\sim}{v}t$

$\underset{\sim}{v} = \dfrac{(\underset{\sim}{i} - 4\underset{\sim}{j}) - (4\underset{\sim}{i} - 8\underset{\sim}{j})}{0.5} = \dfrac{-3\underset{\sim}{i} + 4\underset{\sim}{j}}{0.5}$

$\underset{\sim}{v} = -6\underset{\sim}{i} + 8\underset{\sim}{j}$

$\underset{\sim}{r} = (4\underset{\sim}{i} - 8\underset{\sim}{j}) + t(-6\underset{\sim}{i} + 8\underset{\sim}{j})$

$\underset{\sim}{r} = 4\underset{\sim}{i} - 8\underset{\sim}{j} - 6t\underset{\sim}{i} + 8t\underset{\sim}{j}$

$\underset{\sim}{r} = (4 - 6t)\underset{\sim}{i} + (8t - 8)\underset{\sim}{j}$ as required

M1 JAN 2013

6c) At 10am, lighthouse due west of ship ($\underset{\sim}{j}$ components equal)

10am $t = 1$, $\underset{\sim}{r} = (4-6)\underset{\sim}{i} + (8-8)\underset{\sim}{j}$

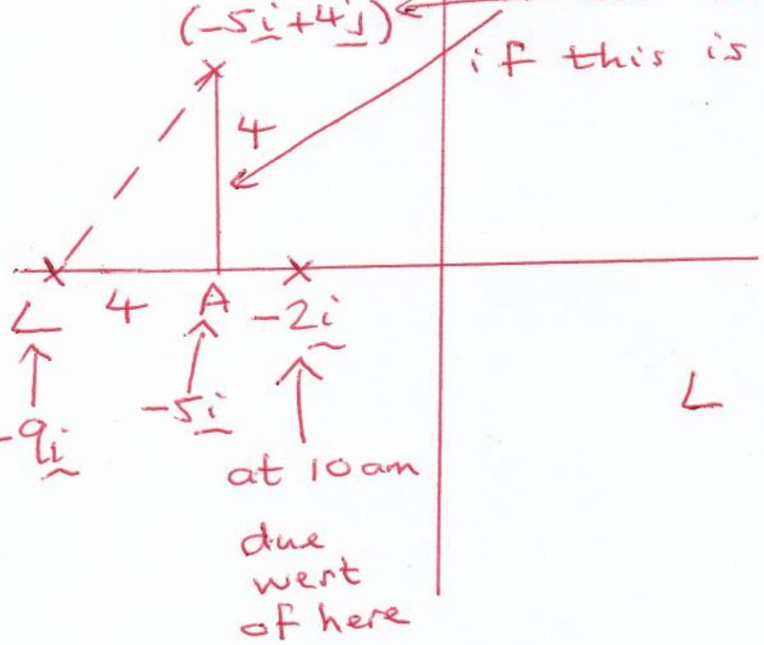
$\underset{\sim}{r} = -2\underset{\sim}{i}$

10·30am $t = 1·5$, $\underset{\sim}{r} = (4 - 6\times1·5)\underset{\sim}{i} + (8\times1·5 - 8)\underset{\sim}{j}$

$\underset{\sim}{r} = -5\underset{\sim}{i} + 4\underset{\sim}{j}$

at 10·30am Lighthouse SW

if this is 4 units, AL = 4 units for SW



Using diagram

L is at $\underline{\underline{-9\underset{\sim}{i}}}$

M1 MAY 2013

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7. *[In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively.]*

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

(i) parallel to **j**,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

a) $\underset{\sim}{v} = (1-2t)\underset{\sim}{i} + (3t-3)\underset{\sim}{j}$

at $t=0$

$\underset{\sim}{v} = \underset{\sim}{i} - 3\underset{\sim}{j}$

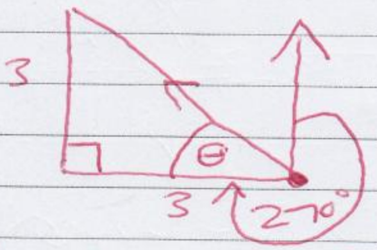
speed $= \sqrt{1^2+3^2} = 3.1622777$

$= \underline{\underline{3.16 \text{ ms}^{-1}}}$ (3sf)

b) at $t = 2$

$\underset{\sim}{v} = (1-2\times2)\underset{\sim}{i} + (3\times2-3)\underset{\sim}{j}$

$\underset{\sim}{v} = -3\underset{\sim}{i} + 3\underset{\sim}{j}$



$\tan\theta = \frac{3}{3}$

$\theta = \tan^{-1} 1$

$\theta = 45°$

Bearing $= 270 + 45$

$= \underline{\underline{315°}}$

P 4 1 8 2 8 A 0 2 2 2 8

M1 MAY 2013

7c) (i) parallel to $\underline{j}$, then $\underline{i}$ component 0

$$1 - 2t = 0$$
$$t = \frac{1}{2}$$

(ii) parallel to $-\underline{i} - 3\underline{j}$

$$\begin{pmatrix} 1 - 2t \\ 3t - 3 \end{pmatrix} = \mu \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

$1 - 2t = -\mu$ (1) $\rightarrow$ $\mu = 2t - 1$

$3t - 3 = -3\mu$ (2)

sub in (2) and solve

$$3t - 3 = -3(2t - 1)$$
$$3t - 3 = -6t + 3$$
$$9t = 6$$
$$t = \frac{6}{9} = \frac{2}{3}$$