**3.** $$y = \sqrt{(10x - x^2)}.$$

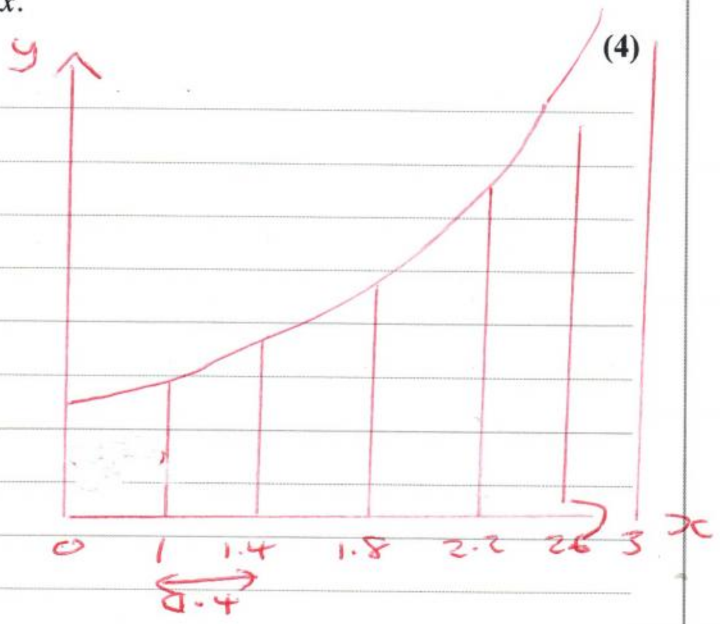
(a) Complete the table below, giving the values of $y$ to 2 decimal places.

| $x$ | 1 | 1.4 | 1.8 | 2.2 | 2.6 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 3 | 3.47 | 3.84 | 4.14 | 4.39 | 4.58 |

**(2)**

(b) Use the trapezium rule, with all the values of $y$ from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\,dx$.

**(4)**




Trapezium rule

$$\int_a^b f(x)\,dx \approx \frac{1}{2}\text{width}\left[\text{1st height} + 2(\text{sum middle heights}) + \text{last height}\right]$$

width $\rightarrow$ 0.4

$$\int_1^3 \sqrt{(10x - x^2)}\,dx \approx \frac{1}{2} \times 0.4\left[3 + 2(3.47 + 3.84 + 4.14 + 4.39) + 4.58\right]$$

$$\approx 0.2\left[3 + 4.58 + 2(15.84)\right]$$

$$\approx 7.852$$

C2 JAN 2011

Leave blank

**6.** $$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of $y$ to 2 decimal places.

| $x$ | 2 | 2.25 | 2.5 | 2.75 | 3 |
|---|---|---|---|---|---|
| $y$ | 0.5 | 0.38 | 0.30 | 0.24 | 0.2 |

**(2)**

(b) Use the trapezium rule, with all the values of $y$ from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\, dx$.

**(4)**

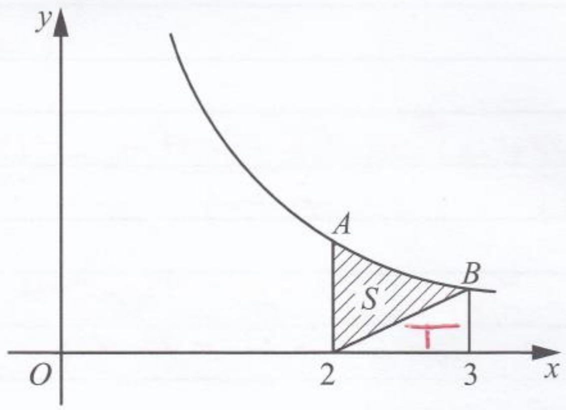



**Figure 2**

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points $A$ and $B$ on the curve, $x = 2$ and $x = 3$ respectively.

The region $S$ is bounded by the curve, the straight line through $B$ and (2, 0), and the line through $A$ parallel to the $y$-axis. The region $S$ is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of $S$.

**(3)**

b) $\int_2^3 \frac{5}{3x^2-2}\, dx$

$= \frac{1}{2} \times 0.25 \left[ 0.5 + 0.2 + 2(0.38 + 0.30 + 0.24) \right]$

$= 0.3175$

$= 0.32$ (to 2dp)

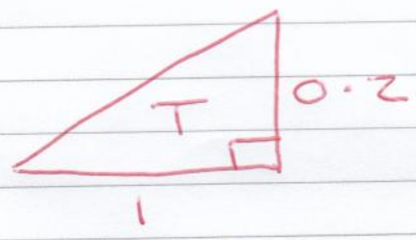


H 3 5 4 0 3 A 0 1 2 2 8

Leave blank

**Question 6 continued**

c) Area of Triangle T to find first




Area $= \frac{1}{2} \times 1 \times 0{\cdot}2 = 0{\cdot}1$

Area $S = 0{\cdot}3175 - 0{\cdot}1$
$= 0{\cdot}2175$
$= 0{\cdot}22$ (2dp)

C2 Jan 2012

Leave blank

**6.**

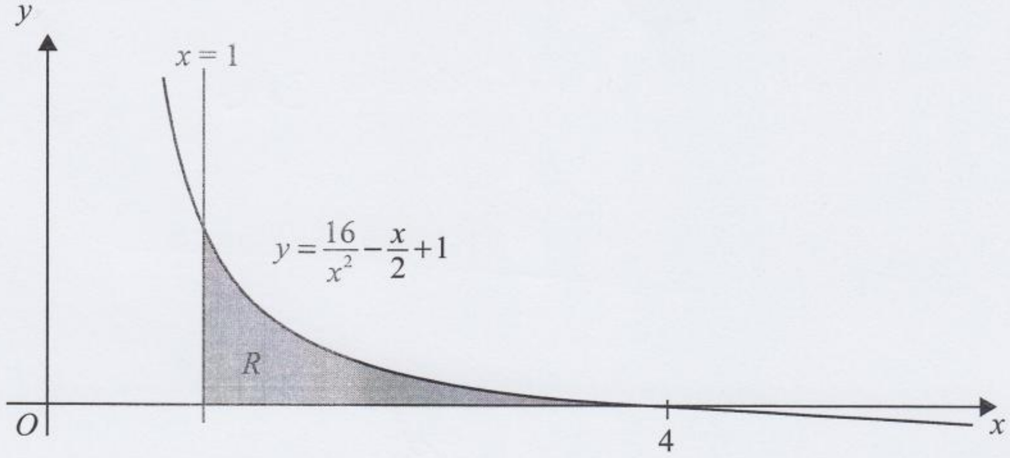



**Figure 1**

Figure 1 shows the graph of the curve with equation

$$y=\frac{16}{x^2}-\frac{x}{2}+1, \qquad x>0$$

The finite region $R$, bounded by the lines $x=1$, the $x$-axis and the curve, is shown shaded in Figure 1. The curve crosses the $x$-axis at the point $(4, 0)$.

(a) Complete the table with the values of $y$ corresponding to $x=2$ and 2.5

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 16.5 | 7.361 | 4 | 2.31 | 1.278 | 0.556 | 0 |

**(2)**

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of $R$, giving your answer to 2 decimal places.

**(4)**

(c) Use integration to find the exact value for the area of $R$.

**(5)**

b) $\text{Area} = \frac{1}{2}\times 0.5\times\left[16.5+0+2(7.361+4+2.31+1.278+0.556)\right]$

$= 11.8775$

$= 11.88$ (2dp)

C2 Jan 2012

6c) $y = 16x^{-2} - \frac{x}{2} + 1$

$\int \left(16x^{-2} - \frac{x}{2} + 1\right) dx$

$= \left[-16x^{-1} - \frac{x^2}{4} + x\right]_1^4$

$= \left(-\frac{16}{4} - \frac{4^2}{4} + 4\right) - \left(-\frac{16}{1} - \frac{1}{4} + 1\right)$

$= (-4 - 4 + 4) - (-16 - \frac{1}{4} + 1)$

$= -4 - (-15 \cdot 25)$

$= 11 \cdot 25$

Leave blank

any real number

**5.** (a) In the space provided, sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the $y$-axis.

**(2)**

(b) Complete the table, giving the values of $3^x$ to 3 decimal places.

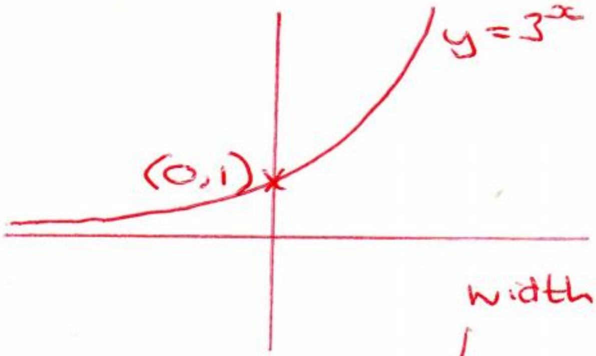
| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| $3^x$ | 1 | 1.246 | 1.552 | 1.933 | 2.408 | 3 |

**(2)**

(c) Use the trapezium rule, with all the values from your table, to find an approximation for the value of $\int_0^1 3^x \mathrm{d}x$.

**(4)**

a)




width

twice the middle heights

c) $\int_0^1 3x\,dx \approx \frac{1}{2} \times 0.2\left[1+3+2(1.246+1.552+1.933+2.408)\right]$

$= 1.8278$

$= 1.83$ (2dp)

Leave blank

**5.** The curve $C$ has equation

$$y = x\sqrt{(x^3+1)}, \qquad 0 \leqslant x \leqslant 2.$$

(a) Complete the table below, giving the values of $y$ to 3 decimal places at $x = 1$ and $x = 1.5$.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $y$ | 0 | 0.530 | 1.414 | 3.137 | 6 |

**(2)**

(b) Use the trapezium rule, with all the $y$ values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\,\mathrm{d}x$, giving your answer to 3 significant figures.

**(4)**

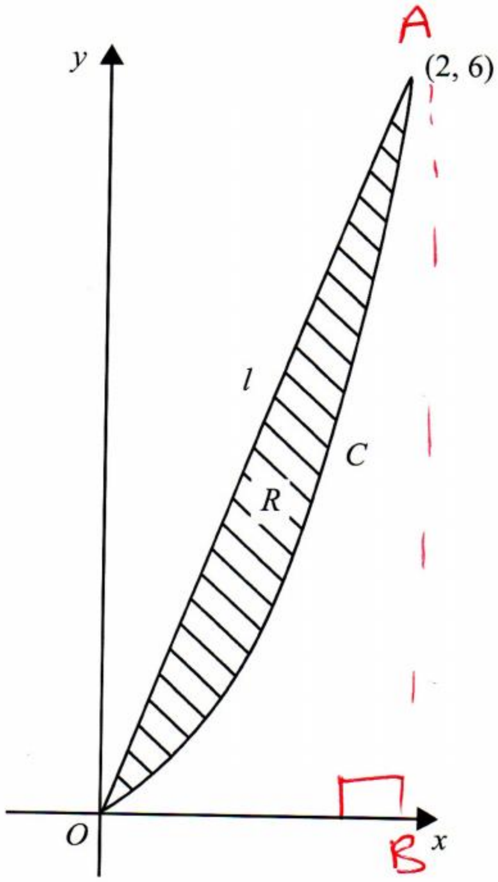



**Figure 2**

Figure 2 shows the curve $C$ with equation $y = x\sqrt{(x^3+1)}$, $0 \leqslant x \leqslant 2$, and the straight line segment $l$, which joins the origin and the point (2, 6). The finite region $R$ is bounded by $C$ and $l$.

(c) Use your answer to part (b) to find an approximation for the area of $R$, giving your answer to 3 significant figures.

**(3)**

5b) Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{1}{2}\text{width}\left[1^{st}\text{ height} + \text{last height} + 2(\text{sum middle heights})\right]$$

$$= \frac{1}{2}\times(0.5)\left[0 + 6 + 2(0.530 + 1.414 + 3.137)\right]$$

$$= 4.0405$$

$$= 4.04 \quad (3\text{ sf})$$

c) Area of triangle OBA

$$= \frac{1}{2}\times 6\times 2 = 6 \text{ unit}^2$$

Area Shaded = Area triangle OBA − Area under curve

$$= 6 - 4.0405$$

$$= 1.9595$$

$$= 1.96 \text{ unit}^2 \quad (3\text{ sf})$$

Leave blank

**2.** $$y = \sqrt{(5^x + 2)}$$

(a) Complete the table below, giving the values of $y$ to 3 decimal places.

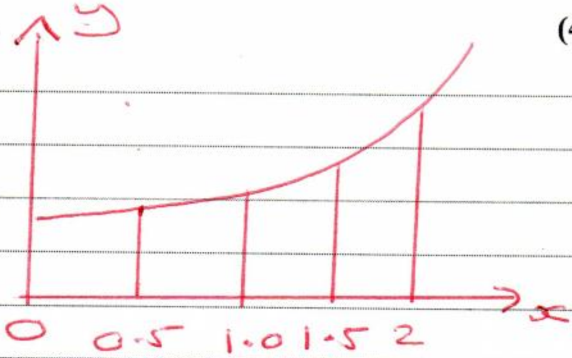
| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $y$ | 1.732 | 2.058 | 2.646 | 3.630 | 5.196 |

1st height          last height

**(2)**

(b) Use the trapezium rule, with all the values of $y$ from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x+2)}\,dx$.

**(4)**




Trapezium rule

$$\int_a^b f(x)\,dx \approx \frac{1}{2}\text{width}\left[\text{1st height} + 2(\text{sum middle heights}) + \text{last height}\right]$$

$$\int_0^2 \sqrt{(5^x+2)}\,dx \approx \frac{1}{2}(0.5)\left[1.732 + 2(2.058 + 2.646 + 3.630) + 5.196\right]$$

$$\approx 5.899$$

Leave blank

**4.** (a) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $\sqrt{(2^x+1)}$ | 1.414 | 1.554 | 1.732 | 1.957 | 2.236 | 2.580 | 3 |

**(2)**

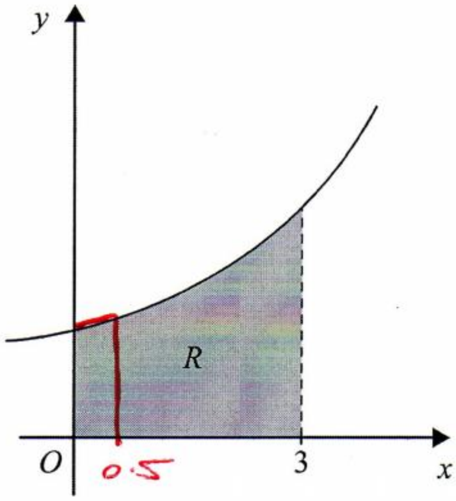



**Figure 1**

Figure 1 shows the region $R$ which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the $x$-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of $R$.

**(4)**

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of $R$.

**(2)**

b) Trapezium rule

$\int_0^3 \sqrt{(2^x+1)}\, dx = \frac{1}{2} \times \text{class width} \times$

$[\text{first} + \text{last} + 2(\text{middle terms})]$

$= \frac{1}{2} \times 0.5 [1.414 + 3 + 2(1.554 + 1.732$

$+ 1.957 + 2.236 + 2.580)]$

$= 6.133$

c) Overestimate since the trapezia lie above the curve

C2 June 2010

Leave blank

**1.** $$y = 3^x + 2x$$

(a) Complete the table below, giving the values of $y$ to 2 decimal places.

| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| $y$ | 1 | 1.65 | 2.35 | 3.13 | 4.01 | 5 |

**(2)**

(b) Use the trapezium rule, with all the values of $y$ from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\,dx$.

**(4)**

b) Trapezium Rule

$\frac{1}{2} \times 0.2 \left[1 + 5 + 2(1.65 + 2.35 + 3.13 + 4.01)\right]$

$= 2.828$

$= 2.83 \quad (2dp)$

C2 May 2012

Leave blank

**7.** $$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of $y$ to 3 decimal places.

| $x$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|
| $y$ | 1 | 1.251 | 1.494 | 1.741 | 2 |

**(2)**

(b) Use the trapezium rule with all the values of $y$ from your table to find an approximation for the value of $\int_0^1 \sqrt{(3^x + x)}\, dx$

You must show clearly how you obtained your answer.

**(4)**

$= \frac{1}{2} \times 0.25 \left[1 + 2 + 2(1.251 + 1.494 + 1.741)\right]$

$= \frac{1}{2} \times 0.25 \times [11.972]$

$= 1.4965$



P 4 0 6 8 5 A 0 2 0 2 8

C2 Jan 2013

Leave blank

**9.**

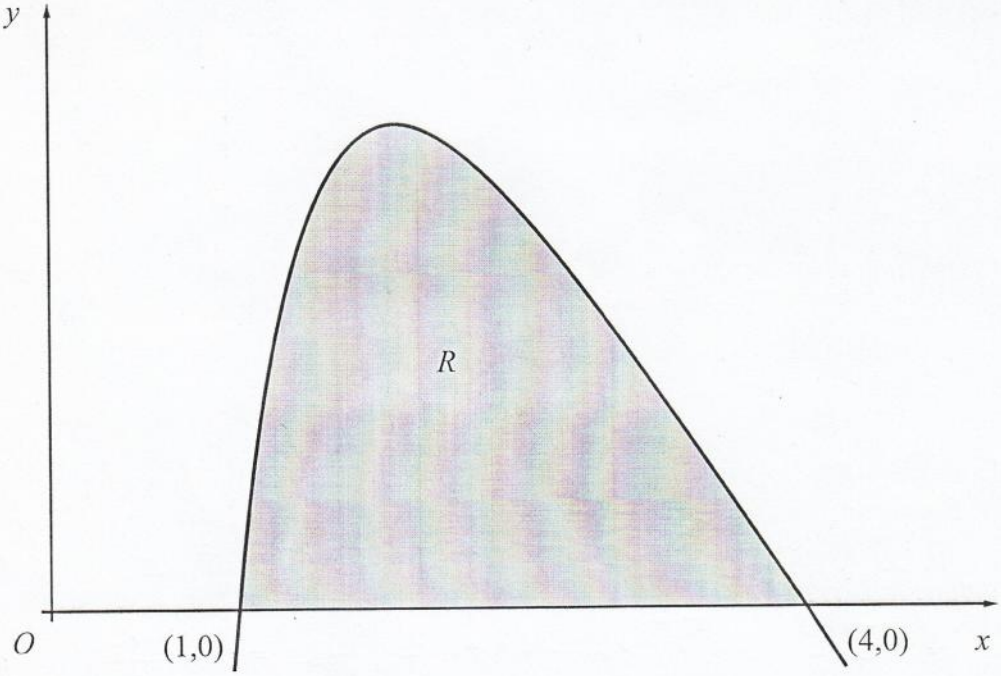



**Figure 2**

The finite region $R$, as shown in Figure 2, is bounded by the $x$-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0$$

The curve crosses the $x$-axis at the points (1, 0) and (4, 0).

(a) Complete the table below, by giving your values of $y$ to 3 decimal places.

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | 5.866 | 6.272 | 5.210 | 3.634 | 1.856 | 0 |

**(2)**

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of $R$, giving your answer to 2 decimal places.

**(4)**

(c) Use integration to find the exact value for the area of $R$.

**(6)**

b) $\text{Area} = \frac{1}{2} \times 0.5 \times [0 + 0 + 2(5.866 + 6.272 + 5.210 + 3.634 + 1.856)]$

$= 11.419$

$= \underline{\underline{11.42}}$ (2dp)

C2 Jan 2013

9 c) $\int_1^4 \left(27 - 2x - 9\sqrt{x} - \frac{16}{x^2}\right) dx$

$= \int_1^4 \left(27 - 2x - 9x^{\frac{1}{2}} - 16x^{-2}\right) dx$

$= \left[27x - x^2 - 9 \times \frac{2}{3} x^{\frac{3}{2}} - \frac{16x^{-1}}{-1}\right]_1^4$

$= \left[27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1}\right]_1^4$

$= \left(27 \times 4 - 4^2 - (6 \times 4^{\frac{3}{2}}) + \frac{16}{4}\right) - \left(27 \times 1 - 1^2 - 6 + 16\right)$

$= (108 - 16 - 48 + 4) - (27 - 1 - 6 + 16)$

$= (48) - (36)$

$= \underline{\underline{12}}$ square units

C2 MAY 2013

Leave blank

4. $$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of $y$ to 3 decimal places.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 5 | 4 | 2.5 | 1.538 | 1 | 0.690 | 0.5 |

**(1)**

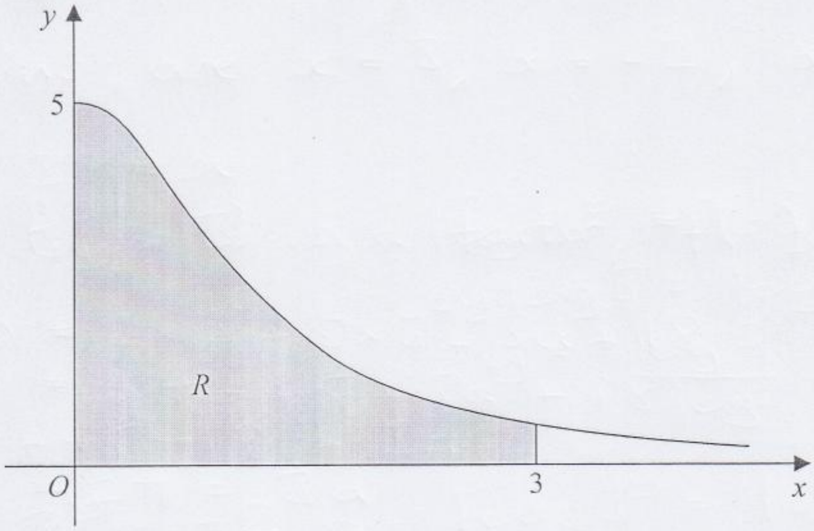



**Figure 1**

Figure 1 shows the region $R$ which is bounded by the curve with equation $y = \frac{5}{(x^2+1)}$, the $x$-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of $y$ from your table, to find an approximate value for the area of $R$.

**(4)**

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2+1)}\right) dx$$

giving your answer to 2 decimal places.

**(2)**

b) ~~Area~~ $\approx \frac{1}{2} \times \frac{1}{2} \left[5 + 0.5 + 2(4 + 2.5 + 1.538 + 1 + 0.690)\right]$

$= \underline{\underline{6.239}}$



P 4 1 8 5 9 A 0 1 0 3 2

C2 MAY 2013

c) $\int_0^3 \left(4 + \frac{5}{x^2+1}\right) dx$

$= \int_0^3 4 \, dx + \int_0^3 \frac{5}{x^2+1} \, dx$

From a) this is 6·239

$= \left[4x\right]_0^3 + 6{\cdot}239$

$= (12) - (0) + 6{\cdot}239$

$= 18{\cdot}239$

$= \underline{\underline{18{\cdot}24}}$ (2dp)