1.

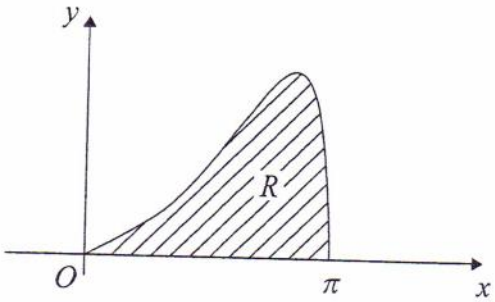

Figure 1

The curve shown in Figure 1 has equation $y = e^x\sqrt{(\sin x)}$, $0 \leqslant x \leqslant \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	1·84432	4·81048	8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(4)

Area $\approx \frac{1}{2}\left(\frac{\pi}{4}\right)\left[0 + 0 + 2\left(1.84432 + 4.81048 + 8.87207\right)\right]$

Area ≈ 12.1948

2.

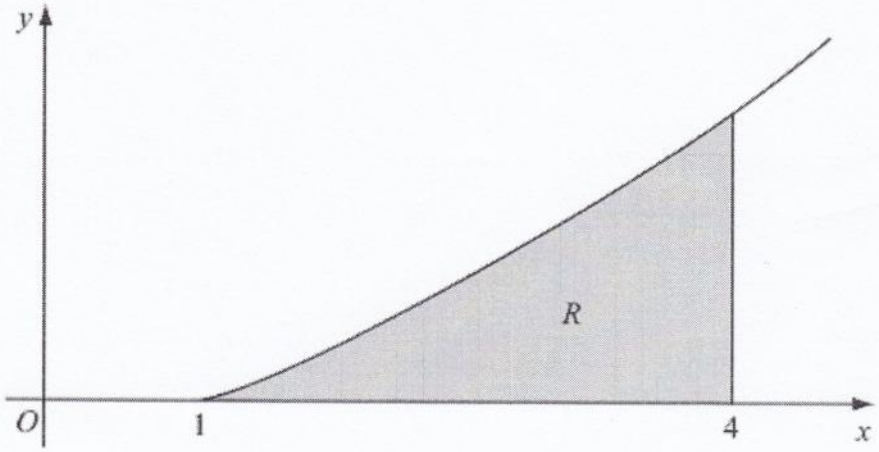

Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608	1.386	2.291	3.296	4.385	5.545

(*a*) Copy and complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) (i) Use integration by parts to find $\int x \ln x \, dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

a) $y = x\ln x$

(b) $A = \frac{1}{2} \times 0.5\left(0 + 5.545 + 2\left(0.608 + 1.386 + 2.291 + 3.296 + 4.385\right)\right)$

$A = 7.37$

c i) $\int x\ln x \, dx$

$u = \ln x \qquad \frac{du}{dx} = \frac{1}{x}$

$v = \frac{x^2}{2} \qquad \frac{dv}{dx} = x$

$I = \frac{x^2 \ln x}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx$

$= \frac{x^2 \ln x}{2} - \int \frac{x}{2} \, dx$

$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$

(ii) $\left[\frac{x^2 \ln x}{2} - \frac{x^2}{4}\right]_1^4 = (8\ln 4 - 4) - (-1/4)$

$= 8\ln 4 - \frac{15}{4}$

$= 8(2\ln 2) - \frac{15}{4}$

$= \frac{1}{4}(64\ln 2 - 15)$

($\ln 4$ need in terms of $\ln 2$
$\Rightarrow \ln 4 = 2\ln 2$)

7.

$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx.$$

(*a*) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2	0.1847	0.1745	0.1667

(2)

(*b*) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

(*c*) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I.

(8)

TOTAL FOR PAPER: 75 MARKS

END

(b) $I \approx \frac{1}{2}(1)\left[0.2 + 0.1667 + 2(0.1847 + 0.1745 + 0.1667)\right]$

≈ 0.543

(c) $x = (u-4)^2 + 1$

$\frac{dx}{du} = 2(u-4)$

$dx = 2(u-4)\,du$

Limits

x	u
2	5
5	6

$x = (u-4)^2 + 1$

$$\int_2^5 \frac{1}{4+\sqrt{x-1}}\,dx = \int_5^6 \frac{1}{u} \times 2(u-4)\,du$$

$u = \sqrt{x-1} + 4$

$$= \int_5^6 2 - \frac{8}{u}\,du$$

$$= \left[2u - 8\ln u\right]_5^6 \quad \cancel{= 2 + 8\ln\left(\tfrac{5}{6}\right)}$$

$$= 2 + 8\ln\left(\tfrac{5}{6}\right)$$

C4 Jan 2012

Leave blank

6.

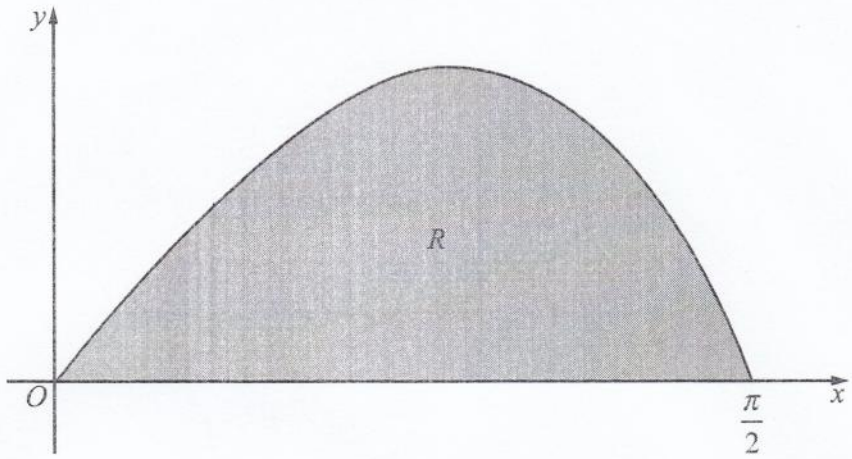

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0	0.73508	1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant.

(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

P 4 0 0 8 5 A 0 1 6 2 8

C4 Jan 2012

Leave blank

Question 6 continued

(b) Area $\approx \frac{1}{2}\left(\frac{\pi}{8}\right)\left[0+0+2(0.73508+1.17157+1.02280)\right]$

≈ 1.1504

(c)

$u = 1+\cos x$

$\frac{du}{dx} = -\sin x$

$$\int \frac{2\sin 2x}{1+\cos x}\,dx = \int \frac{2(2\sin x\cos x)}{1+\cos x}\,dx$$

$$= \int 4\,\frac{(u-1)}{u}\,(-1)\,du$$

$$= 4\int \frac{1-u}{u}\,du$$

$$= 4\int \frac{1}{u} - 1\,du$$

$$= 4\left[\ln u - u\right] + c$$

$$= 4\left[\ln(1+\cos x) - (1+\cos x)\right] + c$$

$$= 4\ln(1+\cos x) - 4\cos x + K$$

P 4 0 0 8 5 A 0 1 8 2 8

C4 Jan 2012

Leave blank

Question 6 continued

(d) $I = \left[4\ln(1+\cos x) - 4\cos x\right]_0^{\pi/2}$

$= \left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\left(\pi/2\right)\right] - \left[4\ln(1+\cos 0) - 4\cos 0\right]$

$= (4\ln 1 - 0) - (4\ln 2 - 4)$

$= 4 - 4\ln 2$

Error $= (4 - 4\ln 2) - 1.1504$

$= 0.077$

Q6

(Total 12 marks)

C4 June 2005

Question Number	Scheme	Marks
5.	(a) $\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx$ Attempting parts in the right direction	M1 A1
	$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$	A1
	$\left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_0^1 = \frac{1}{4} + \frac{1}{4} e^2$	M1 A1
		(5)
	(b) $x = 0.4 \Rightarrow y \approx 0.89022$	
	$x = 0.8 \Rightarrow y \approx 3.96243$ Both are required to 5 d.p	B1
		(1)
	(c) $I \approx \frac{1}{2} \times 0.2 \times [\ \ldots\ ]$	B1
	$\approx \ \ldots \times \left[0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243) \right]$ ft their answers to (b)	M1 A1ft
	$\approx 0.1 \times 21.67522$	
	≈ 2.168 cao	A1
		(4)
		[10]
	Note $\frac{1}{4} + \frac{1}{4} e^2 \approx 2.097 \ldots$	

June 06

edexcel

Question Number	Scheme		Marks
6. (a)			
	$\begin{array}{c\|c\|c\|c\|c\|c} x & 1 & 1.5 & 2 & 2.5 & 3 \\ \hline y & 0 & 0.5\ln 1.5 & \ln 2 & 1.5\ln 2.5 & 2\ln 3 \\ \hline \text{or } y & 0 & 0.2027325541\ldots & \ln 2 & 1.374436098\ldots & 2\ln 3 \end{array}$		
		Either $0.5\ln 1.5$ and $1.5\ln 2.5$ or awrt 0.20 and 1.37 (or mixture of decimals and ln's)	B1
			[1]
(b)(i)	$I_1 \approx \frac{1}{2}\times 1\times\underline{\{0+2(\ln 2)+2\ln 3\}}$	For structure of trapezium rule $\underline{\{\ldots\ldots\ldots\ldots\}}$;	M1;
	$=\frac{1}{2}\times 3.583518938\ldots = 1.791759\ldots = 1.792$ (4sf)	1.792	A1 cao
(ii)		Outside brackets $\frac{1}{2}\times 0.5$	B1;
	$I_2 \approx \frac{1}{2}\times 0.5\ ;\times\underline{\{0+2(0.5\ln 1.5+\ln 2+1.5\ln 2.5)+2\ln 3\}}$	For structure of trapezium rule $\underline{\{\ldots\ldots\ldots\ldots\}}$;	M1 √
	$=\frac{1}{4}\times 6.737856242\ldots = 1.684464\ldots$	awrt 1.684	A1
			[5]
(c)	With increasing ordinates, <u>the line segments at the top of the trapezia are closer to the curve.</u>	<u>Reason</u> or an appropriate diagram elaborating the correct reason.	B1
			[1]

Beware: In part (b) candidate can add up the individual trapezia:

(b)(i) $I_1 \approx \frac{1}{2}\underline{(0+\ln 2)}+\frac{1}{2}\underline{(\ln 2+\ln 3)}$

(ii) $I_2 \approx \frac{1}{2}.\frac{1}{2}\underline{(0+0.5\ln 1.5)}+\frac{1}{2}.\frac{1}{2}\underline{(0.5\ln 1.5+\ln 2)}+\frac{1}{2}.\frac{1}{2}\underline{(\ln 2+1.5\ln 2.5)}+\frac{1}{2}.\frac{1}{2}\underline{(1.5\ln 2.5+2\ln 3)}$

June 06

edexcel

Question Number	Scheme		Marks
6. (d)	$\left\{\begin{array}{l} u = \ln x \quad \Rightarrow \quad \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 \quad \Rightarrow \quad v = \frac{x^2}{2} - x \end{array}\right\}$	Use of 'integration by parts' formula in the correct direction	M1
	$I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$	Correct expression	A1
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int\left(\frac{x}{2} - 1\right) dx}$	An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ...	
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)} \quad (+c)$	... integrate; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$		
	$= \left(\frac{3}{2}\ln 3 - \frac{9}{4} + 3\right) - \left(-\frac{1}{2}\ln 1 - \frac{1}{4} + 1\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + 0 - \frac{3}{4} \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
Aliter **6.** (d) **Way 2**	$\int (x-1)\ln x\, dx = \int x\ln x\, dx - \int \ln x\, dx$		
	$\int x\ln x\, dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4} \quad (+\,c)$	Correct integration	A1
	$\int \ln x\, dx = x\ln x - \int x.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= x\ln x - x \quad (+\,c)$	Correct integration	A1
	$\therefore \int_1^3 (x-1)\ln x\, dx = \left(\frac{9}{2}\ln 3 - 2\right) - \left(3\ln 3 - 2\right) = \frac{3}{2}\ln 3$ **AG**	Substitutes limits of 3 and 1 into both integrands and subtracts. $\frac{3}{2}\ln 3$	ddM1 A1 cso
			[6]

June 06

edexcel

Question Number	Scheme		Marks
Aliter			
6. (d)	$\left\{\begin{array}{ll} u = \ln x & \Rightarrow \ \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = (x-1) & \Rightarrow \ v = \frac{(x-1)^2}{2} \end{array}\right\}$	Use of 'integration by parts' formula in the correct direction	M1
Way 3			
	$I = \frac{(x-1)^2}{2}\ln x - \int \frac{(x-1)^2}{2x}\,dx$	Correct expression	A1
	$= \frac{(x-1)^2}{2}\ln x - \int \frac{x^2-2x+1}{2x}\,dx$	Candidate multiplies out numerator to obtain three terms…	
	$= \frac{(x-1)^2}{2}\ln x - \int \underline{\left(\frac{1}{2}x - 1 + \frac{1}{2x}\right)}\,dx$	… multiplies at least one term through by $\frac{1}{x}$ and then attempts to …	
	$= \frac{(x-1)^2}{2}\ln x - \underline{\left(\frac{x^2}{4} - x + \frac{1}{2}\ln x\right)}$ (+c)	… integrate the result; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{(x-1)^2}{2}\ln x - \frac{x^2}{4} + x - \frac{1}{2}\ln x\right]_1^3$		
	$= \left(2\ln 3 - \frac{9}{4} + 3 - \frac{1}{2}\ln 3\right) - \left(0 - \frac{1}{4} + 1 - 0\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= 2\ln 3 - \frac{1}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]

Beware: $\int \frac{1}{2x}\,dx$ can also integrate to $\frac{1}{2}\ln 2x$

Beware: If you are marking using WAY 2 please make sure that you allocate the marks in the order they appear on the mark scheme. For example if a candidate only integrated lnx correctly then they would be awarded M0A0M1A1M0A0 on ePEN.

June 06

edexcel

Question Number	Scheme		Marks
Aliter **6.** (d) **Way 4**	By substitution $u = \ln x \quad \Rightarrow \quad \frac{du}{dx} = \frac{1}{x}$		
	$I = \int (e^u - 1).ue^u \, du$	Correct expression	
	$= \int u(e^{2u} - e^u)\,du$	Use of 'integration by parts' formula in the correct direction	M1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\int\left(\frac{1}{2}e^{2u} - e^u\right)} dx$	Correct expression	A1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\left(\frac{1}{4}e^{2u} - e^u\right)} \ (+c)$	Attempt <u>to integrate</u>; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u\right]_{\ln 1}^{\ln 3}$		
	$= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - \left(0 - 0 - \frac{1}{4} + 1\right)$	Substitutes limits of ln3 and ln1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
			13 marks

Leave blank

7.

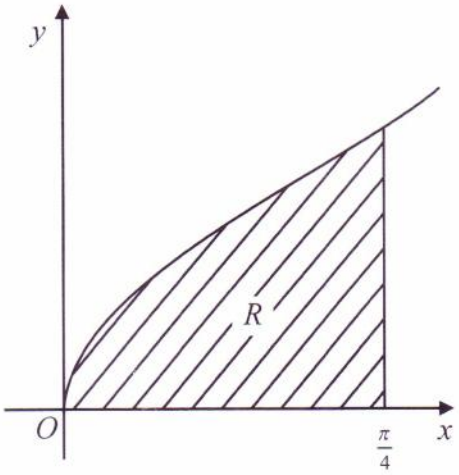

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(a) Given that $y = \sqrt{(\tan x)}$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0	0.44600	0.64359	0.81742	1

(3)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(c) Use integration to find an exact value for the volume of the solid generated.

(4)

b) Area $\simeq \frac{1}{2} \times \frac{\pi}{16} \times \{0 + 1 + 2(0.446 + 0.64359 + 0.81742)\}$

$\simeq 0.4726$

N 2 6 1 1 0 A 0 1 6 2 4

Leave blank

Question 7 continued

$$\text{Volume} = \pi \int y^2 \, dx$$

$$= \pi \int_0^{\pi/4} \tan x \, dx$$

$$= \pi \Big[\ln \sec x \Big]_0^{\pi/4}$$

$$= \pi \ln \sqrt{2}$$

Leave blank

1.

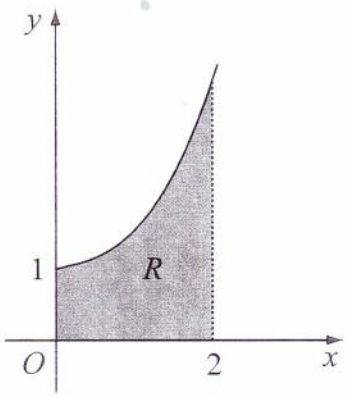

Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

b) Area $= \frac{1}{2}(0.4) \times \left[e^0 + e^2 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right)\right]$

$= 4.922$ (4 s.f.)

H 3 0 4 2 7 A 0 2 2 8

Leave blank

2.

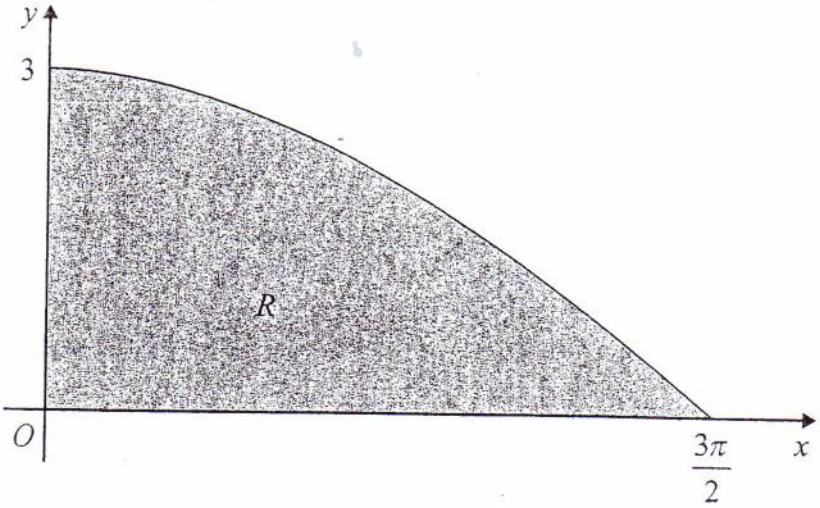

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \leqslant x \leqslant \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132	1.14805	0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(4)

(c) Use integration to find the exact area of R.

(3)

b) $A = \frac{1}{2}\left(\frac{3\pi}{8}\right)\left[3 + 0 + 2(2.77164 + 2.12132 + 1.14805)\right]$

$A = 8.884$

c) $I = \int_0^{3\pi/2} 3\cos\left(\frac{x}{3}\right) = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{3\pi/2}$

$= 9 - 0$

$= 9.$

H 3 4 2 6 5 A 0 4 2 8

1.

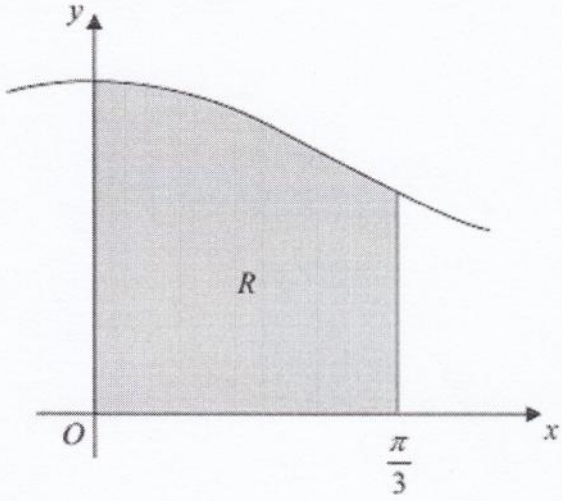

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(*a*) Copy and complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973	1.2247	1.1180	1

(2)

(*b*) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R.

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places.

(6)

bi)

$$I = \frac{\pi}{12}\left(1.3229 + 1 + (2 \times 1.2247)\right)$$

$$= 1.249$$

(ii)

$$I = \frac{1}{2}\left(\frac{\pi}{12}\right)\left(1.3229 + 1 + 2\left(1.2973 + 1.2247 + 1.1180\right)\right)$$

$$= 1.257$$

C4 June 2011

Leave blank

Question 4 continued

(b)

$$\text{Area} = \frac{1}{2} \times \frac{\sqrt{2}}{4}\left(0 + 3.9210 + 2(0.0333 + 0.324 + 1.3596)\right)$$

$$\approx 1.30$$

(c) $$\int_0^{\sqrt{2}} x^3 \ln(x^2+2)\, dx$$

$u = x^2 + 2$

$\frac{du}{dx} = 2x$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

Limits

x	u
0	2
$\sqrt{2}$	4

$$\Rightarrow I = \int_0^{\sqrt{2}} x^2 \ln(x^2+2)\, x\, dx \qquad \text{(split } x^3 \text{ to } x^2 \text{ and } x\text{)}$$

$x^2 = u - 2$

$$I = \int_2^4 (u-2)\ln u \, \frac{1}{2}\, du$$

$$= \frac{1}{2}\int_2^4 (u-2)\ln u \, du$$

C4 June 2011

Leave blank

Question 4 continued

$$= \frac{1}{2}\left\{\left[(8-8)\ln 4 - (2-4)\ln 2\right] - \left[(4-8) - (1-4)\right]\right\}$$

$$= \frac{1}{2}\left\{2\ln 2 - (-1)\right]$$

$$= \ln 2 + \frac{1}{2}$$

Q4

(Total 15 marks)

C4 June 2011

Leave blank

Question 4 continued

(d) by parts

$$I = \frac{1}{2}\int_2^4 (u-2)\ln u \, du$$

as by parts uses u and v change I back to x

$$I = \frac{1}{2}\int_2^4 (x-2)\ln x \, dx$$

$$u = \ln x \rightarrow \frac{du}{dx} = \frac{1}{x}$$

$$v = \frac{x^2}{2} - 2x \leftarrow \frac{dv}{dx} = x - 2$$

$$I = uv - \int v \frac{du}{dx} \, dx$$

$$I = \frac{1}{2}\left\{\left[\left(\frac{x^2}{2} - 2x\right)(\ln x)\right]_2^4 - \int_2^4 \left(\frac{x^2}{2} - 2x\right)\cdot\frac{1}{x} \, dx\right\}$$

$$= \frac{1}{2}\left\{\left[\left(\frac{x^2}{2} - 2x\right)(\ln x)\right]_2^4 - \int_2^4 \frac{x}{2} - 2 \, dx\right\}$$

$$= \frac{1}{2}\left\{\left[\left(\frac{x^2}{2} - 2x\right)(\ln x)\right]_2^4 - \left[\frac{x^2}{4} - 2x\right]_2^4\right\}$$

P 3 8 1 6 0 A 0 1 0 2 4

C4 June 2012

Leave blank

7.

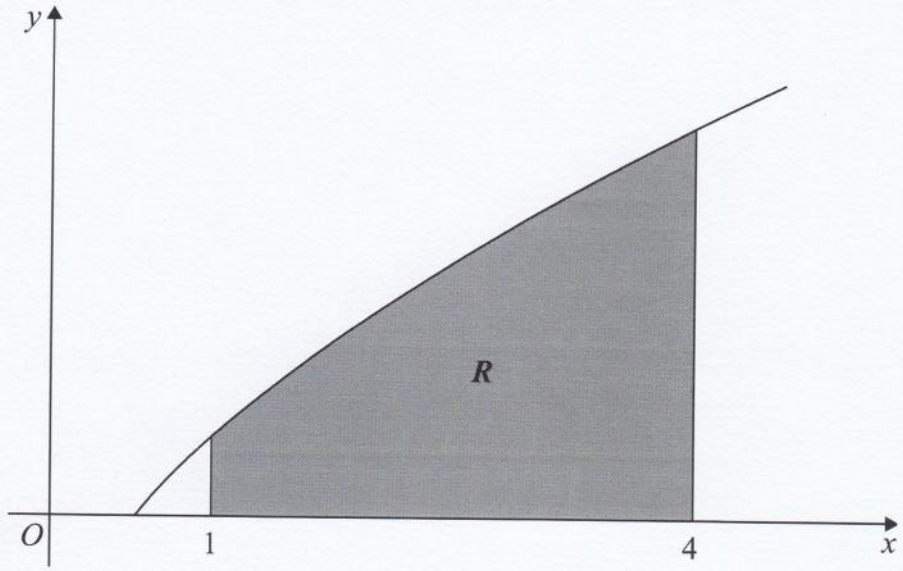

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$.

(4)

(c) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)

a)

x	1	2	3	4
y	$\ln 2$	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	$2\ln 8$

Area $\approx \frac{1}{2}(1)\left[\ln 2 + 2\ln 8 + 2(\sqrt{2}\ln 4 + \sqrt{3}\ln 6)\right]$

Area ≈ 7.49

C4 June 2012

Leave blank

Question 7 continued

(b) $\int x^{1/2} \ln 2x \, dx$ $\qquad u = \ln 2x \rightarrow \frac{du}{dx} = \frac{1}{x}$

$v = \frac{2}{3}x^{3/2} \leftarrow \frac{dv}{dx} = x^{1/2}$

$= uv - \int v \frac{du}{dx} \, dx$

$= \frac{2}{3}x^{3/2}(\ln 2x) - \int \frac{2}{3}x^{3/2} \cdot \frac{1}{x} \, dx$

$= \frac{2}{3}x^{3/2}(\ln 2x) - \int \frac{2}{3}x^{1/2} \, dx$

$= \frac{2}{3}x^{3/2}(\ln 2x) - \frac{4}{9}x^{3/2} + c$

(c)

$A = \int_1^4 x^{1/2} \ln 2x \, dx = \left[\frac{2}{3}x^{3/2} \ln 2x - \frac{4}{9}x^{3/2}\right]_1^4$

$= \left(\frac{16}{3}\ln 8 - \frac{32}{9}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$

$= \frac{16}{3}\ln 8 - \frac{32}{9} - \frac{2}{3}\ln 2 + \frac{4}{9}$

$= 16\ln 2 - \frac{2}{3}\ln 2 - \frac{28}{9}$

$= \frac{46}{3}\ln 2 - \frac{28}{9}$

C4 Jan 2013

Leave blank

4.

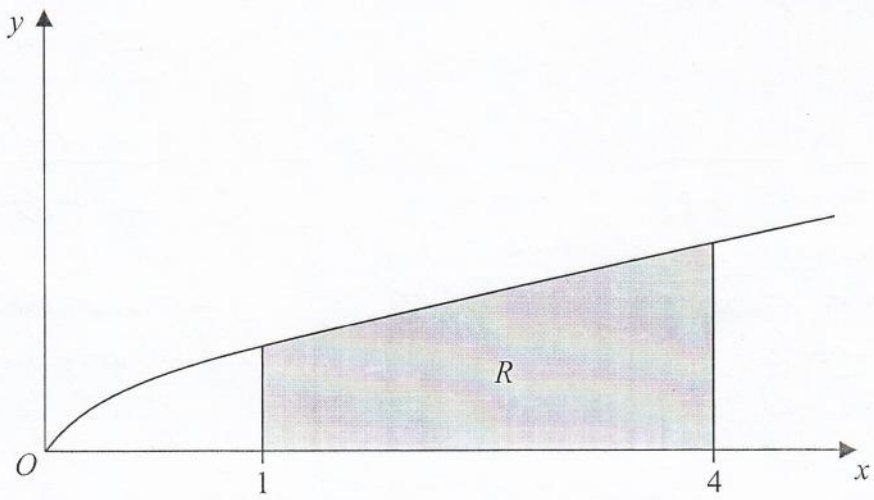

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284	1.0981	1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

b) Area $\approx \frac{1}{2} \times 1 \times [0.5 + 1.3333 + 2(0.8284 + 1.0981)]$

≈ 2.84315

≈ 2.843 (3dp)

C4 Jan 2013

4c) $y = \dfrac{x}{1+\sqrt{x}}$

$$\int_1^4 \frac{x}{1+\sqrt{x}}\,dx$$

using substitution

$$\int_2^3 \frac{(u-1)^2}{u} \times 2 \times (u-1)\,du$$

$$= \int_2^3 \frac{(2u-2)(u^2-2u+1)}{u}\,du$$

$$= \int_2^3 \frac{2u^3-4u^2+2u-2u^2+4u-2}{u}\,du$$

$$= \int_2^3 \frac{2u^3-6u^2+6u-2}{u}\,du$$

$$= \int_2^3 2u^2-6u+6-\frac{2}{u}\,du$$

$$= 2\int_2^3 u^2-3u+3-\frac{1}{u}\,du$$

$$= 2\left[\frac{1}{3}u^3-\frac{3}{2}u^2+3u-\ln u\right]_2^3$$

$$= 2\left[\left(9-\frac{27}{2}+9-\ln 3\right)-\left(\frac{8}{3}-6+6-\ln 2\right)\right]$$

$$= 2\left[\left(\frac{9}{2}-\ln 3\right)-\left(\frac{8}{3}-\ln 2\right)\right]$$

$$= 2\left(\frac{11}{6}-\ln 3+\ln 2\right)$$

$$= 2\left(\frac{11}{6}+\ln\frac{2}{3}\right)$$

$$= \frac{11}{3}+2\ln\frac{2}{3}$$

$u = 1+\sqrt{x}$

$u = 1+x^{\frac{1}{2}}$

$\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$

$\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$u = 1+\sqrt{x}$

$u-1 = \sqrt{x}$

$(u-1)^2 = x$

Limits

$x=4,\ u=1+\sqrt{4}=3$

$x=1,\ u=1+\sqrt{1}=2$

C4 June 2013

Leave blank

3.

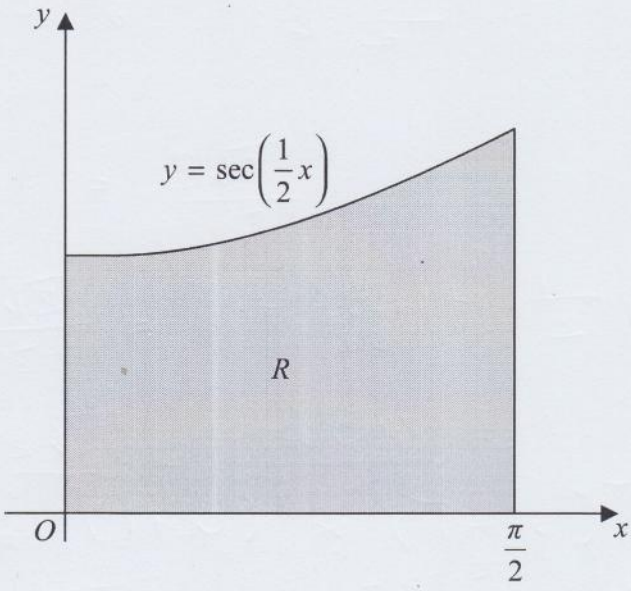

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276	1.154701	1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places.

(1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.

(3)

Region R is rotated through 2π radians about the x-axis.

(c) Use calculus to find the exact volume of the solid formed.

(4)

P 4 3 1 3 7 A 0 8 3 2

C4 June 2013

3b)

$$\text{Area} = \frac{1}{2} \times \frac{\pi}{6}\left[1 + 1.414214 + 2(1.035276 + 1.154701)\right]$$

$$\approx 1.778709$$

$$\approx \underline{\underline{1.7787}} \quad (4\text{dp})$$

c) $$\text{volume} = \pi \int_0^{\frac{\pi}{2}} \sec^2\left(\tfrac{1}{2}x\right)\,dx$$

$$= \pi \left[2\tan\left(\tfrac{1}{2}x\right)\right]_0^{\frac{\pi}{2}}$$

$$= \pi\left(2\tan\left(\tfrac{\pi}{4}\right) - 0\right)$$

$$= \underline{\underline{2\pi}}$$