

6. Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation

$$2\cos^2 x + 1 = 5\sin x,$$

giving each solution in terms of π .

(6)

Identity $\rightarrow \sin^2 x + \cos^2 x = 1$
 $\therefore \cos^2 x = 1 - \sin^2 x$

$$2(1 - \sin^2 x) + 1 = 5\sin x$$

$$2 - 2\sin^2 x + 1 = 5\sin x$$

$$3 - 2\sin^2 x = 5\sin x$$

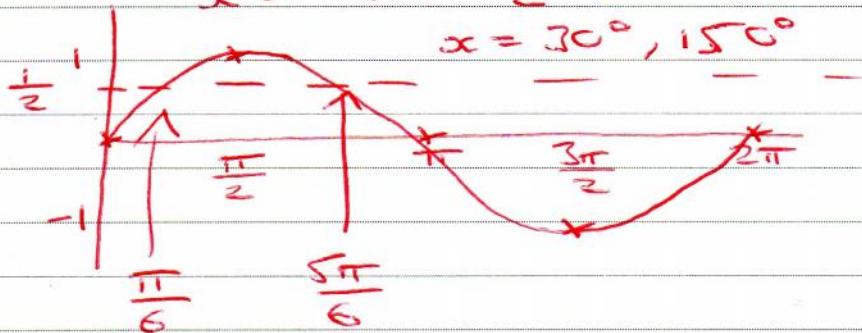
$$2\sin^2 x + 5\sin x - 3 = 0$$

$$(2\sin x - 1)(\sin x + 3) = 0$$

Either $2\sin x - 1 = 0$ or $\sin x + 3 = 0$
(impossible)

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1} \frac{1}{2}$$



In terms of π

$$x = \frac{\pi}{6} \text{ and } x = \frac{5\pi}{6}$$



4. (a) Show that the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1$$

can be written as

$$5 \sin^2 \theta = 3. \quad (2)$$

- (b) Hence solve, for $0^\circ \leq \theta < 360^\circ$, the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1,$$

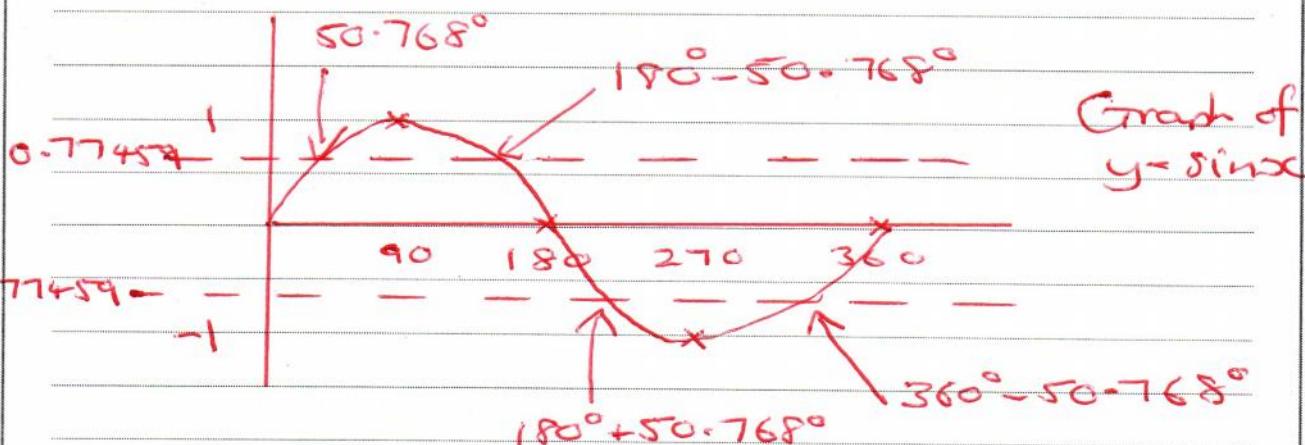
giving your answers to 1 decimal place.

$$\begin{aligned} a) \quad & 3 \sin^2 \theta - 2 \cos^2 \theta = 1 && (7) \\ & 3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1 \\ & 3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1 \\ & 5 \sin^2 \theta = 3 \end{aligned}$$

$$\left| \begin{array}{l} \sin^2 \theta + \cos^2 \theta = 1 \\ \cos^2 \theta = 1 - \sin^2 \theta \end{array} \right.$$

b) From above

$$\begin{aligned} \sin^2 \theta &= \frac{3}{5} \\ \sin \theta &= \pm \sqrt{0.6} \\ \sin \theta &= \pm 0.77459 \end{aligned}$$



Using symmetry of the curve values of θ are $50.8^\circ, 129.2^\circ, 230.8^\circ, 309.2^\circ$
(all to 1dp)

8. (a) Show that the equation

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

can be written as

$$4 \cos^2 x - 9 \cos x + 2 = 0.$$

(2)

- (b) Hence solve, for $0^\circ \leq x < 720^\circ$,

$$4 \sin^2 x + 9 \cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)

Identity $\sin^2 x + \cos^2 x = 1$

$$\therefore \sin^2 x = 1 - \cos^2 x$$

We have $4 \sin^2 x + 9 \cos x - 6 = 0 \quad (1)$

sub $\sin^2 x = 1 - \cos^2 x$ in (1)

$$4(1 - \cos^2 x) + 9 \cos x - 6 = 0$$

$$4 - 4 \cos^2 x + 9 \cos x - 6 = 0$$

$$-2 - 4 \cos^2 x + 9 \cos x = 0$$

$$\therefore 4 \cos^2 x - 9 \cos x + 2 = 0 \text{ as required}$$

b)

$$4 \cos^2 x - 9 \cos x + 2 = 0$$

$$(4 \cos x - 1)(\cos x - 2) = 0$$

Either $4 \cos x - 1 = 0$ or $\cos x - 2 = 0$

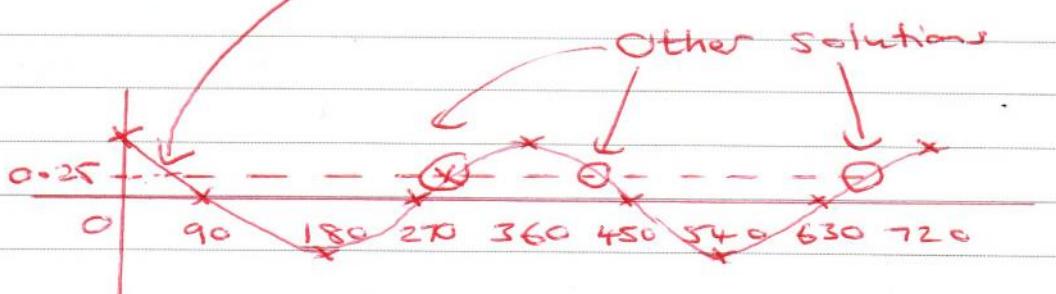
$$\cos x = \frac{1}{4}$$

$$\cos x = 2$$

(impossible)

$$x = \cos^{-1} \frac{1}{4}$$

$$x = 75.522488^\circ = 75.5^\circ \text{ (1dp)}$$



$$90 - 75.522488 = 14.477512^\circ$$

Other solutions are $270 + 14.477512 = 284.5^\circ \text{ (1dp)}$
 $450 - 14.477512 = 435.5^\circ \text{ (1dp)}$
 $630 + 14.477512 = 644.5^\circ \text{ (1dp)}$

2. (a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(2)

- (b) Solve, for $0^\circ \leq x < 360^\circ$,

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(4)

a) Identity $\sin^2 x + \cos^2 x = 1$
 $\therefore \cos^2 x = 1 - \sin^2 x$ ①

$5 \sin x = 1 + 2 \cos^2 x$
Sub $\cos^2 x$ from ① gives

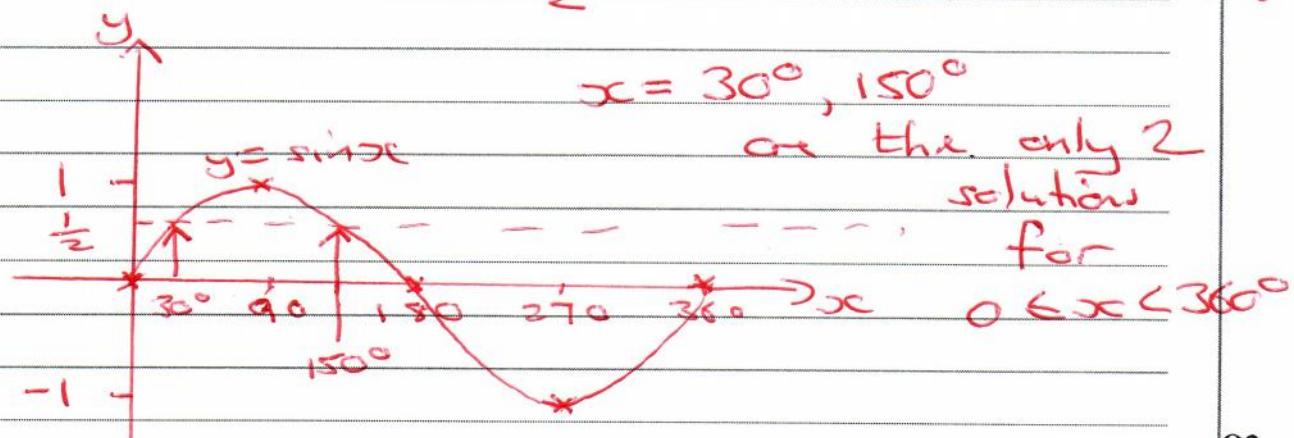
$$5 \sin x = 1 + 2(1 - \sin^2 x)$$

$$5 \sin x = 1 + 2 - 2 \sin^2 x$$

$$2 \sin^2 x + 5 \sin x - 3 = 0 \quad \text{as required}$$

b) $2 \sin^2 x + 5 \sin x - 3 = 0$
 $(2 \sin x - 1)(\sin x + 3) = 0$

Either $2 \sin x - 1 = 0$ or $\sin x + 3 = 0$
 $\sin x = \frac{1}{2}$ (no solution)



Q2

(Total 6 marks)

7. (a) Show that the equation

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0$$

(2)

- (b) Hence solve, for $0^\circ \leq x < 360^\circ$,

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

a)

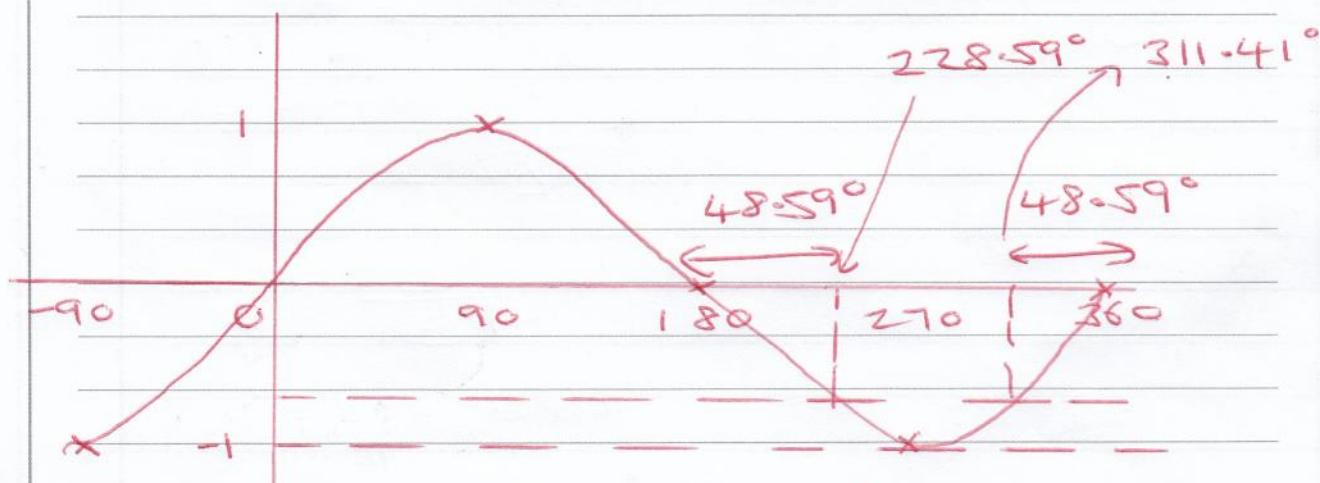
$$\begin{aligned} 3\sin^2 x + 7\sin x &= \cos^2 x - 4 \\ 3\sin^2 x + 7\sin x &= (1 - \sin^2 x) - 4 \\ 3\sin^2 x + 7\sin x - 1 + \sin^2 x + 4 &= 0 \\ 4\sin^2 x + 7\sin x + 3 &= 0 \quad \text{as required.} \end{aligned}$$

b)

Factorise

$$(4\sin x + 3)(\sin x + 1) = 0$$

$$\begin{aligned} \text{Either } \sin x &= -\frac{3}{4} & \text{or } \sin x &= -1 \\ x &= -48.59^\circ & x &= -90^\circ \end{aligned}$$



In range $0^\circ \leq x < 360^\circ$

Solutions are $228.59^\circ, 270^\circ, 311.41^\circ$



9. (i) Find the solutions of the equation $\sin(3x - 15^\circ) = \frac{1}{2}$, for which $0 \leq x \leq 180^\circ$

(6)

(ii)

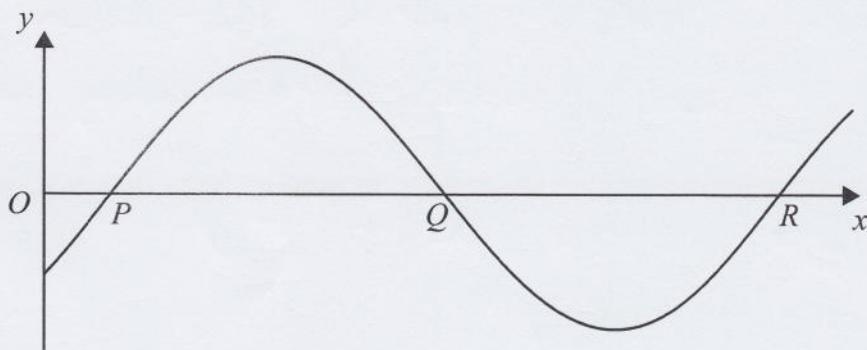


Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0, 0 < b < \pi$$

The curve cuts the x-axis at the points P , Q and R as shown.

Given that the coordinates of P , Q and R are $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{3\pi}{5}, 0\right)$ and $\left(\frac{11\pi}{10}, 0\right)$ respectively, find the values of a and b .

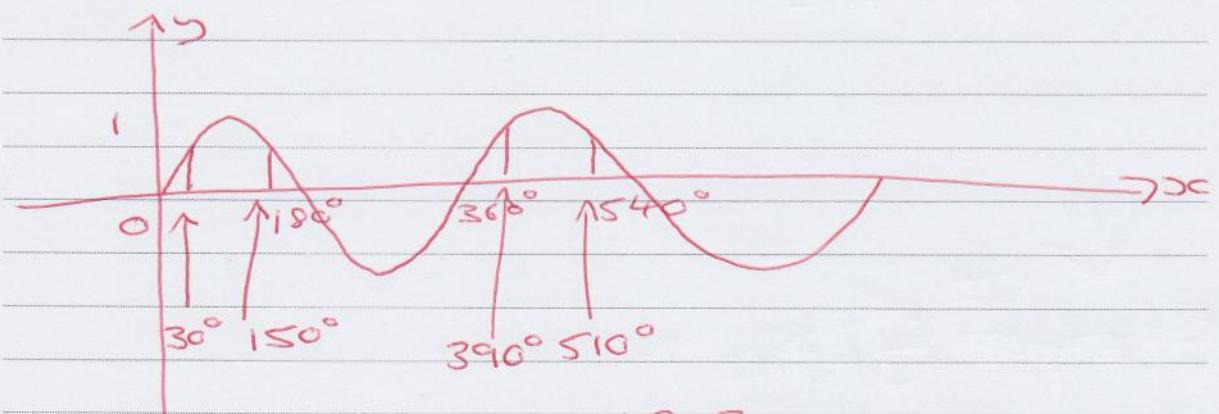
(4)

(i) $\sin(3x - 15) = \frac{1}{2}$

$$0 \leq x \leq 180^\circ$$

$$0 \leq 3x \leq 540^\circ$$

$$\sin^{-1}\left(\frac{1}{2}\right) \\ 30^\circ$$



$$3x - 15 = 30^\circ, x = \frac{15 + 30}{3} = 15^\circ$$

$$3x - 15 = 150^\circ, x = \frac{150 + 15}{3} = 55^\circ$$

$$3x - 15 = 390^\circ, x = \frac{390 + 15}{3} = 135^\circ$$

$$3x - 15 = 510^\circ, x = \frac{510 + 15}{3} = 175^\circ$$



C2 Jan 2012

9(iii)

curve shifted $\frac{\pi}{10}$ units right

Full SINE curve between 0 and π
so twice frequency $\Rightarrow 2x$

$$y = \sin\left(2x - \frac{\pi}{5}\right)$$



When $x = \frac{\pi}{2}$, $2 \times \frac{\pi}{10} - \frac{\pi}{5} = 0$, $\sin 0^\circ = 0$

$x = \frac{3\pi}{5}$, $2 \times \frac{3\pi}{5} - \frac{\pi}{5} = \pi$, $\sin \pi = 0$

$x = \frac{11\pi}{10}$, $2 \times \frac{11\pi}{10} - \frac{\pi}{5} = 2\pi$, $\sin 2\pi = 0$

So $y = (2x - \frac{\pi}{5})$

in form $y = \sin(ax - b)$

where $a = 2$
 $b = \frac{\pi}{5}$

6. (a) Given that $\sin \theta = 5 \cos \theta$, find the value of $\tan \theta$.

(1)

- (b) Hence, or otherwise, find the values of θ in the interval $0^\circ \leq \theta < 360^\circ$ for which

$$\sin \theta = 5 \cos \theta,$$

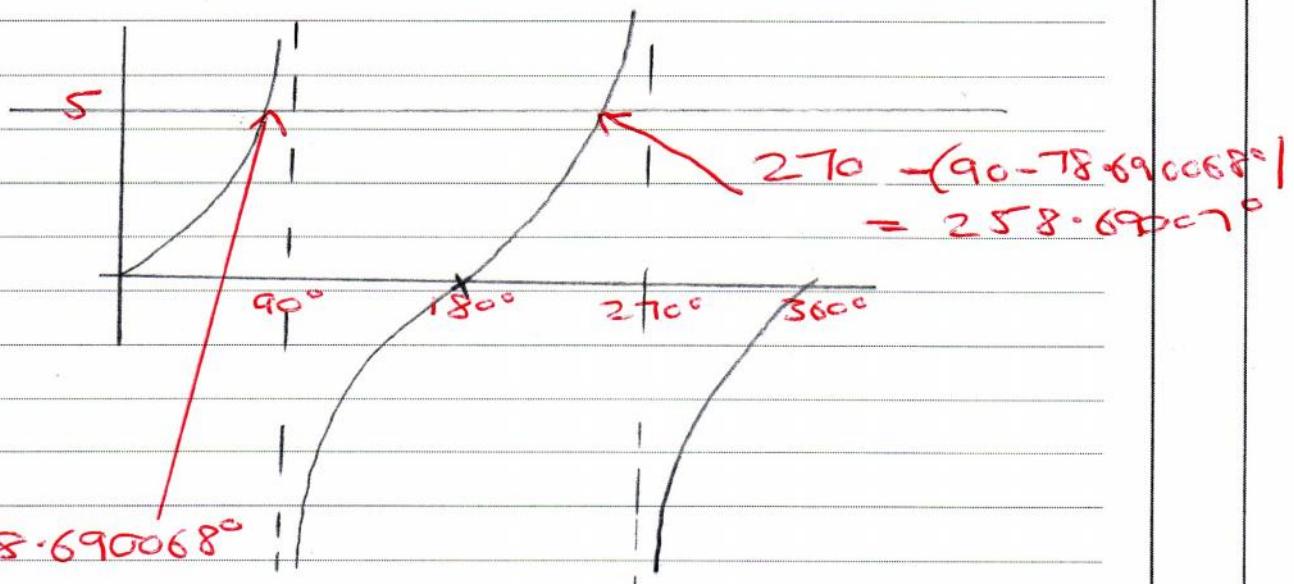
giving your answers to 1 decimal place.

a) $\sin \theta = 5 \cos \theta$
 $\therefore \frac{\sin \theta}{\cos \theta} = 5$
 $\therefore \tan \theta = 5$

(3)

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

b) solve $\sin \theta = 5 \cos \theta$ for $0^\circ \leq \theta < 360^\circ$
 $\tan \theta = 5$
 $\therefore \theta = \tan^{-1} 5$



Values for θ in range $0^\circ \leq \theta < 360^\circ$ are

$$\theta = 78.7^\circ, 258.7^\circ \quad (\text{1dp})$$



9. (a) Sketch, for $0 \leq x \leq 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$.

graph of $\sin x$ shifted left by $\frac{\pi}{6}$ units (2)

- (b) Write down the exact coordinates of the points where the graph meets the coordinate axes.
- (3)

- (c) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

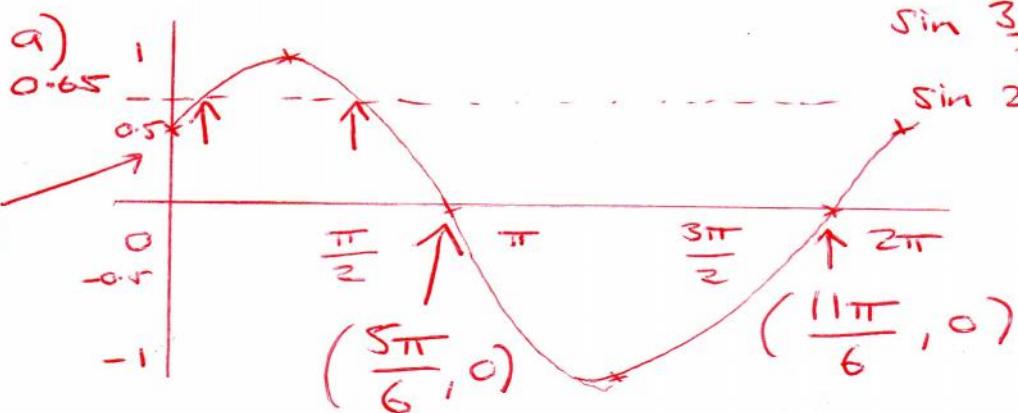
$$\sin 0 = 0$$

$$\sin \frac{\pi}{2} = 1$$

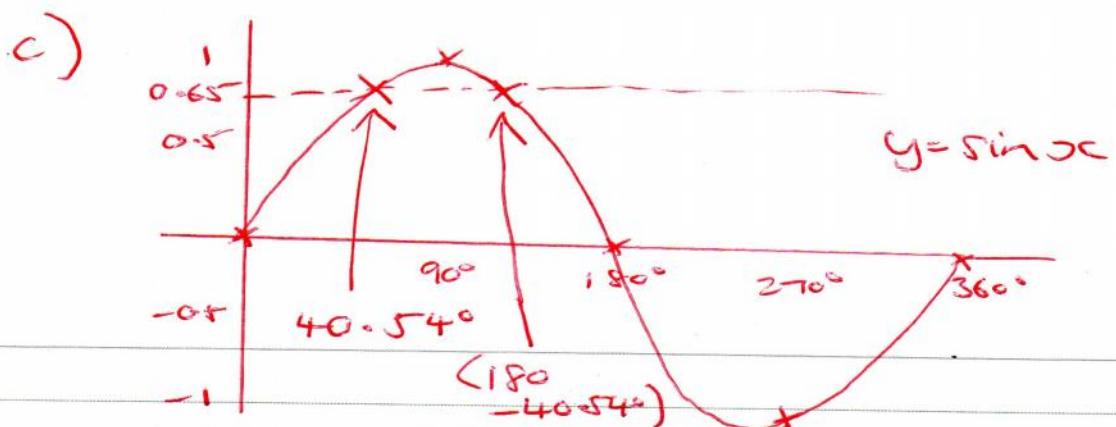
$$\sin \frac{3\pi}{2} = -1 \quad (5)$$

$$\sin 2\pi = 0$$

giving your answers in radians to 2 decimal places.



∴ $(0, \frac{1}{2}), (\frac{5\pi}{6}, 0), (\frac{11\pi}{6}, 0)$



From graph there are two solutions in range $0 \leq x \leq 360^\circ$

$$\sin(x + 30^\circ) = 0.65$$

$$x + 30^\circ = \sin^{-1} 0.65$$

$$9c) \quad 5x + 30^\circ = 40.54160^\circ$$

$$5x = 40.54160 - 30$$

$$x = 10.541602^\circ \quad (1)$$

Other solution by symmetry is

$$180 - 40.54160 = 139.4584^\circ$$

$$\text{so } 5x + 30^\circ = 139.4584^\circ$$

$$5x = 139.4584 - 30$$

$$x = 109.4584^\circ \quad (2)$$

$$(1) \text{ in radians is } \frac{10.541602}{180} \times \pi = 0.18 \text{ radians}$$

$$(2) \text{ in radians is } \frac{109.4584}{180} \times \pi = 1.91 \text{ radians}$$

9. Solve, for $0 \leq x < 360^\circ$,

$$(a) \sin(x - 20^\circ) = \frac{1}{\sqrt{2}} \quad (4)$$

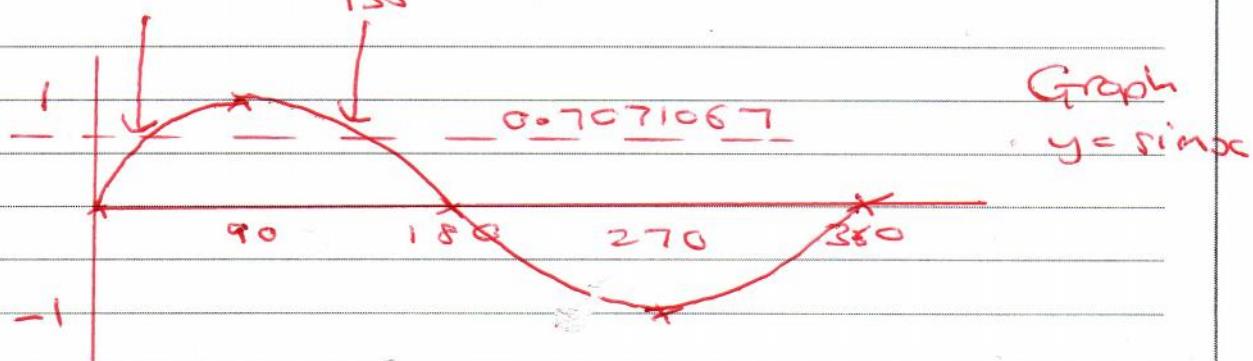
$$(b) \cos 3x = -\frac{1}{2} \quad (6)$$

$$\text{a) } \sin(x - 20^\circ) = \frac{1}{\sqrt{2}}$$

$$x - 20^\circ = \sin^{-1} \frac{1}{\sqrt{2}}$$

$$0 \leq x < 360^\circ$$

$$45^\circ \quad 135^\circ \quad \therefore -20^\circ \leq x - 20^\circ < 340^\circ$$



$$\sin^{-1} \frac{1}{\sqrt{2}} = 0.7071067$$

From graph, solutions in range
 $0 \leq x < 360^\circ$

are 45° and 135° using symmetry

This gives

$$x - 20^\circ = 45^\circ \quad \therefore x = 65^\circ$$

$$x - 20^\circ = 135^\circ \quad \therefore x = 155^\circ$$



$$95) \cos 3x = -\frac{1}{2}$$

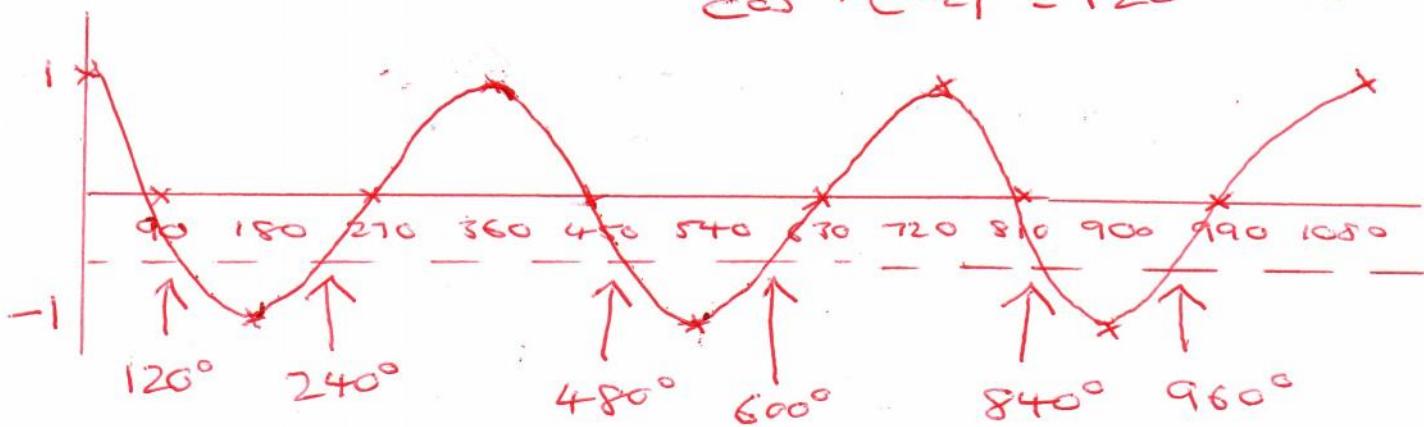
$$3x = \cos^{-1}(-\frac{1}{2})$$

$$0 \leq x < 360^\circ$$

$$0 \leq 3x < 1080^\circ$$

$$\cos^{-1}(-\frac{1}{2}) = 120^\circ$$

Graph
of
 $y = \cos x$



Using symmetry, the values of $3x$ in range $0 \leq 3x < 1080^\circ$

are $120^\circ, 240^\circ, 480^\circ, 600^\circ, 840^\circ, 960^\circ$

$$\therefore 3x = 120^\circ, x = 40^\circ$$

$$3x = 240^\circ, x = 80^\circ$$

$$3x = 480^\circ, x = 160^\circ$$

$$3x = 600^\circ, x = 200^\circ$$

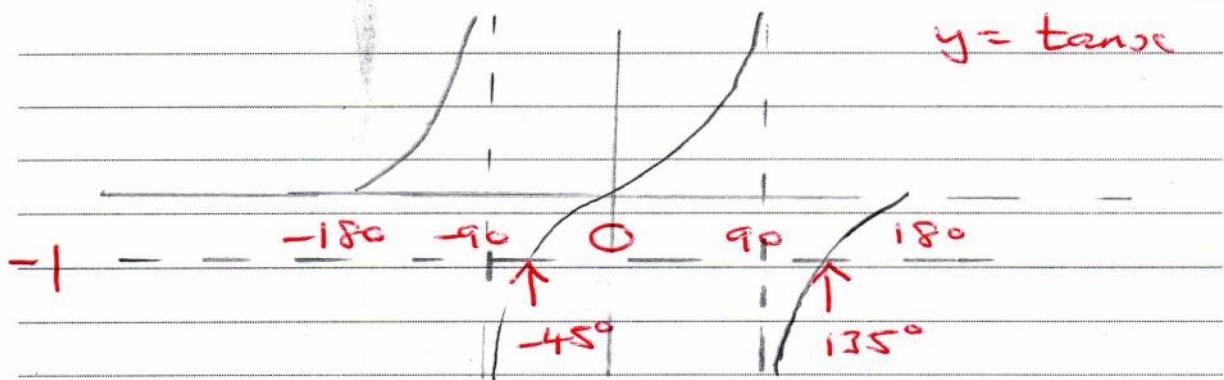
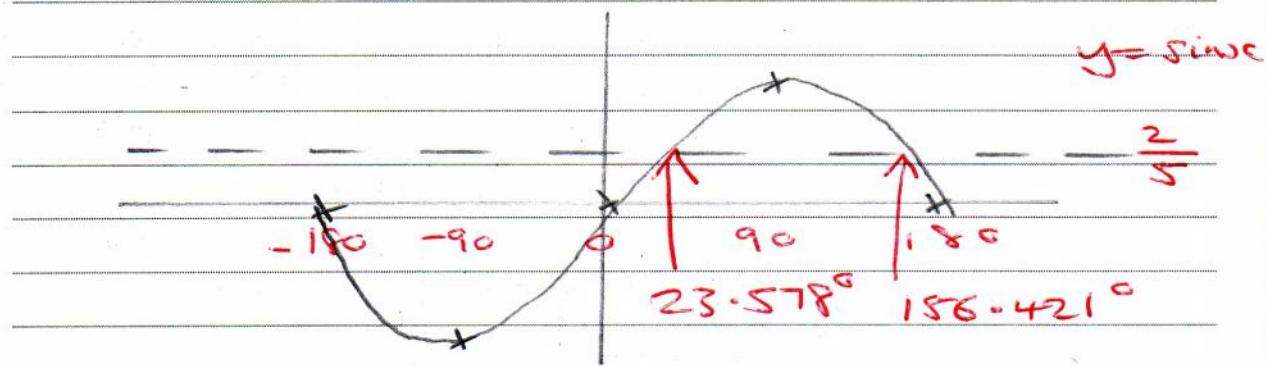
$$3x = 840^\circ, x = 280^\circ$$

$$3x = 960^\circ, x = 320^\circ$$

7. (i) Solve, for $-180^\circ \leq \theta < 180^\circ$,

$$(1 + \tan \theta)(5 \sin \theta - 2) = 0.$$

(4)

 $y = \tan \theta$  $y = \sin \theta$

(i) Either $1 + \tan \theta = 0$ or $5 \sin \theta - 2 = 0$
 $\tan \theta = -1$ $\sin \theta = \frac{2}{5}$
 $\theta = -45^\circ$ or 135° $\theta = 23.6^\circ, 156.4^\circ$

(ii) Solve, for $0 \leq x < 360^\circ$,

$$4\sin x = 3\tan x.$$

(6)

$$4\sin x = \frac{3\sin x}{\cos x}$$

$$4\sin x \cos x = 3\sin x$$

$$4\sin x \cos x - 3\sin x = 0$$

$$\sin x (4\cos x - 3) = 0$$

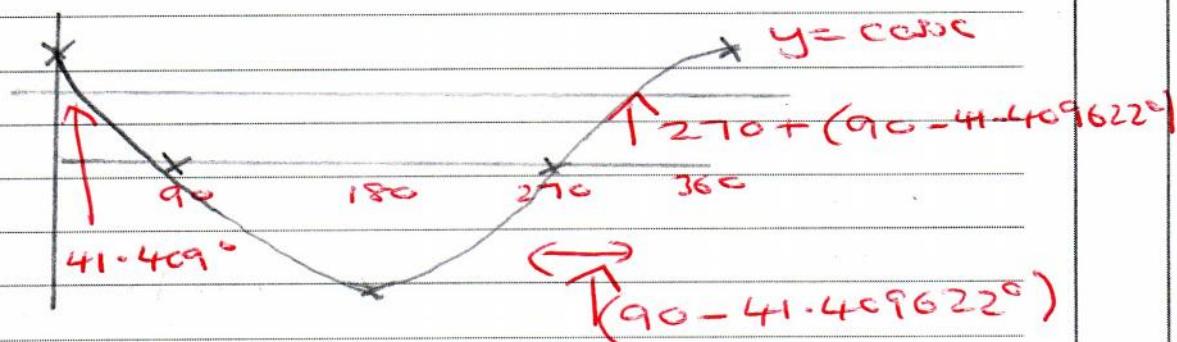
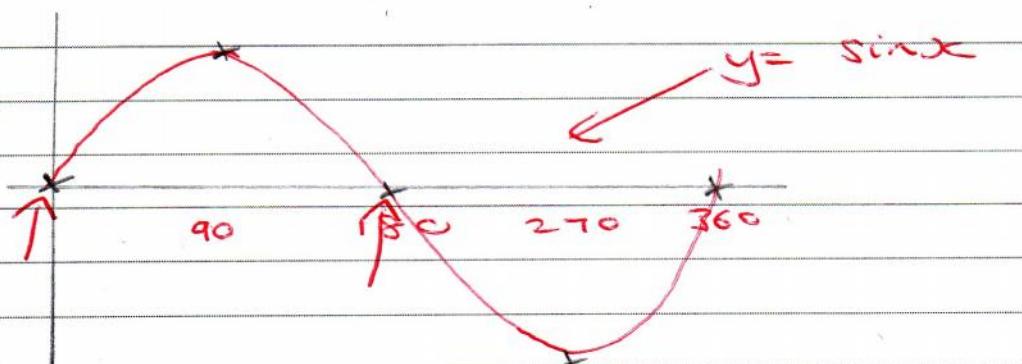
Either

$$\sin x = 0$$

$$x = 0, 180^\circ$$

$$4\cos x - 3 = 0$$

$$\cos x = \frac{3}{4}$$



$$\cos x = \frac{3}{4} \quad \therefore x = \cos^{-1} \frac{3}{4}$$

$$x = 41.409622^\circ, 318.59038^\circ$$

Values of x are $0^\circ, 180^\circ, 41.4^\circ, 318.6^\circ$

Q7

(Total 10 marks)

5. (a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$.

(1)

- (b) Solve, for $0 \leq x < 360^\circ$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place.

(5)

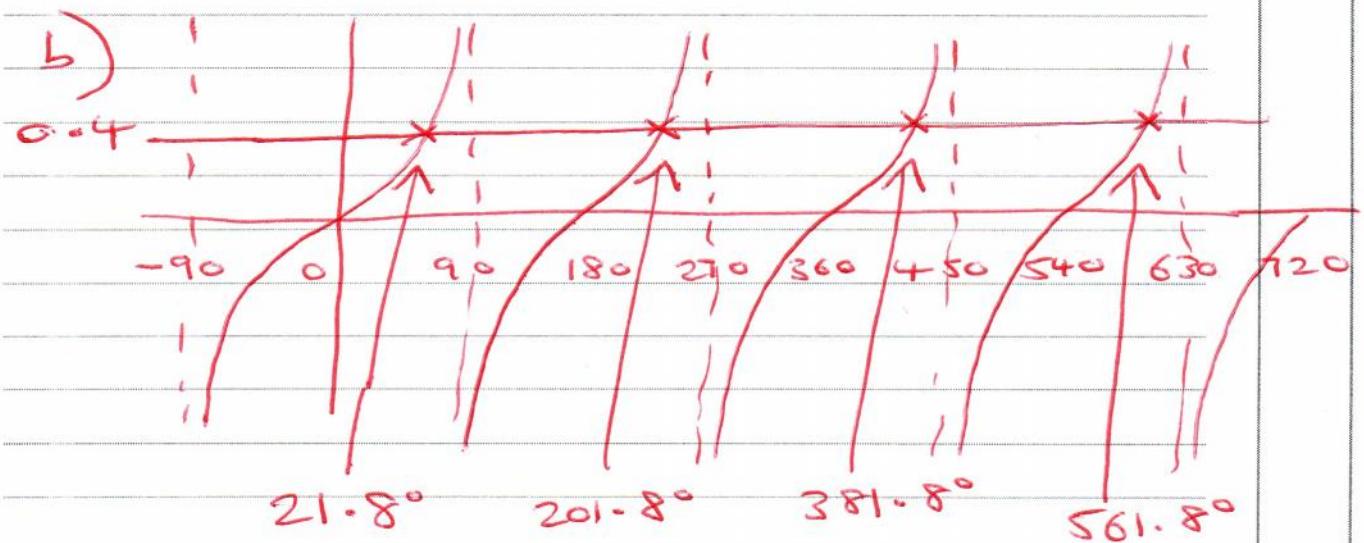
a) $5\sin\theta = 2\cos\theta$

$$\left| \tan\theta = \frac{\sin\theta}{\cos\theta} \right.$$

$$\frac{5\sin\theta}{\cos\theta} = \frac{2\cos\theta}{\cos\theta}$$

$$5\tan\theta = 2$$

$$\tan\theta = \frac{2}{5}$$



$$0 \leq x < 360^\circ$$

$$0 \leq 2x < 720^\circ$$

By symmetry

$$\tan x = \frac{2}{5} \therefore x = \tan^{-1} \frac{2}{5}$$

$$\therefore 2x = 21.8^\circ, \quad x = 10.9^\circ$$

$$2x = 201.8^\circ, \quad x = 100.9^\circ$$

$$2x = 381.8^\circ, \quad x = 190.9^\circ$$

$$2x = 561.8^\circ, \quad x = 280.9^\circ$$

7. (a) Solve for $0 \leq x < 360^\circ$, giving your answers in degrees to 1 decimal place,

~~45~~ $\leq x + 45^\circ < 405^\circ$ $3 \sin(x + 45^\circ) = 2$ (4)

- (b) Find, for $0 \leq x < 2\pi$, all the solutions of

$$2 \sin^2 x + 2 = 7 \cos x$$

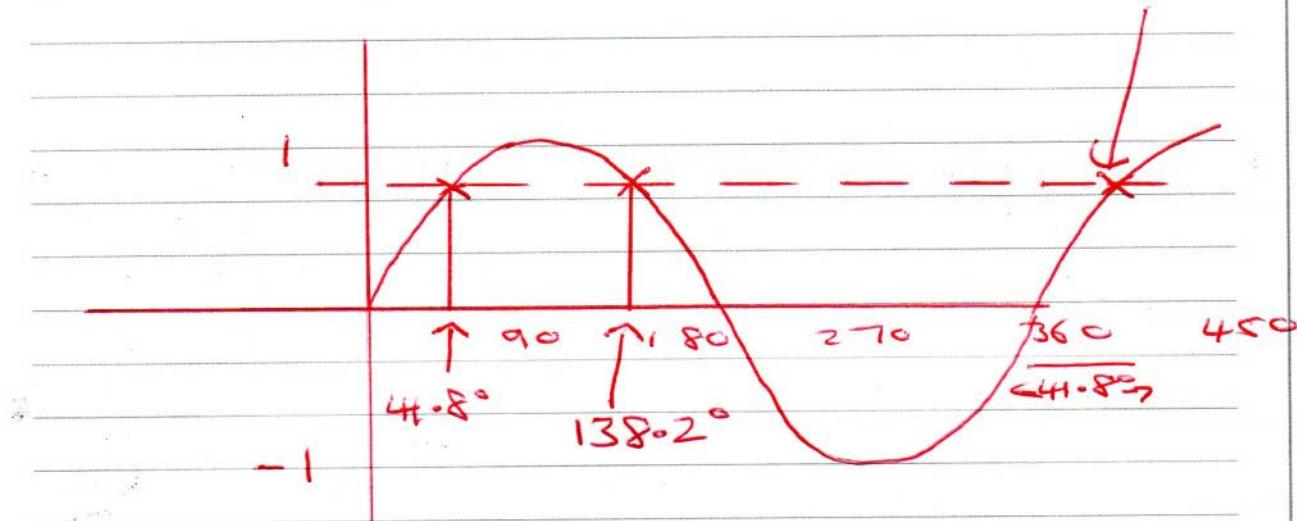
giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

a) $\sin(x + 45^\circ) = \frac{2}{3}$

$x = \sin^{-1} \frac{2}{3} = 41.8^\circ$ (1dp) 401.8°



$$\therefore x + 45 = 41.8^\circ, x = -3.2^\circ \quad (\text{outside range})$$

$$x + 45 = 138.2^\circ, x = 93.2^\circ \text{ (1dp)}$$

$$x + 45 = 401.8^\circ, x = 356.8^\circ \text{ (1dp)}$$



Question 7 continued

b) $2 \sin^2 x + 2 = 7 \cos x$

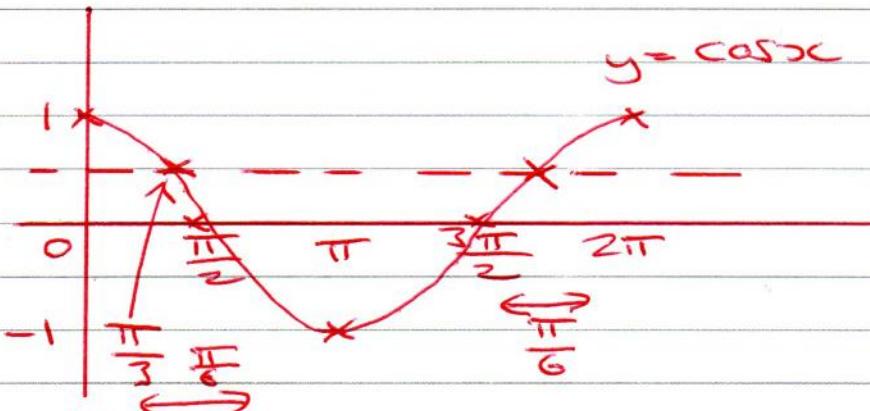
$$\begin{aligned} 2(1 - \cos^2 x) + 2 - 7 \cos x &= 0 & \left| \begin{array}{l} \sin^2 x + \cos^2 x = 1 \\ \sin^2 x = 1 - \cos^2 x \end{array} \right. \\ 2 - 2 \cos^2 x + 2 - 7 \cos x &= 0 \\ 4 - 2 \cos^2 x - 7 \cos x &= 0 \\ 2 \cos^2 x + 7 \cos x - 4 &= 0 \end{aligned}$$

$$(2 \cos x - 1)(\cos x + 4) = 0$$

Either

$$\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -4$$

(impossible)



$$x = \cos^{-1} \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

2nd solution by symmetry

$$\begin{aligned} \text{is } \frac{3\pi}{2} + \frac{\pi}{6} &= \frac{9\pi}{6} + \frac{\pi}{6} = \frac{10\pi}{6} \\ &= \frac{5\pi}{3} \end{aligned}$$

Solution one

$$x = \frac{\pi}{3}, \quad x = \frac{5\pi}{3}$$



6. (a) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0$$

(2)

- (b) Hence solve, for $0^\circ \leq x \leq 180^\circ$,

$$\tan 2x = 5 \sin 2x$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

a) $\tan 2x = 5 \sin 2x$

$$\frac{\sin 2x}{\cos 2x} = 5 \sin 2x$$

$\times \text{ both sides by } \cos 2x$

$$\frac{\cos 2x \sin 2x}{\cos 2x} = 5 \cos 2x \sin 2x$$

$$\sin 2x = 5 \cos 2x \sin 2x$$

$$\sin 2x - 5 \cos 2x \sin 2x = 0$$

$$(1 - 5 \cos 2x) \sin 2x = 0$$

as required.

b) $1 - 5 \cos 2x = 0$ or $\sin 2x = 0$

For $\sin 2x = 0$

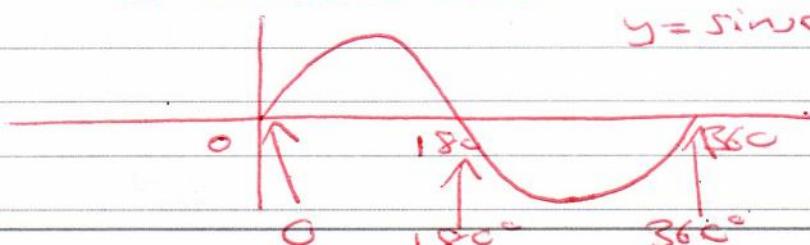
$$2x = 0, x = 0^\circ$$

$$0 < x \leq 180^\circ$$

$$2x = 180^\circ, x = 90^\circ$$

$$0 \leq 2x \leq 360^\circ$$

$$2x = 360^\circ, x = 180^\circ$$



Question 6 continued

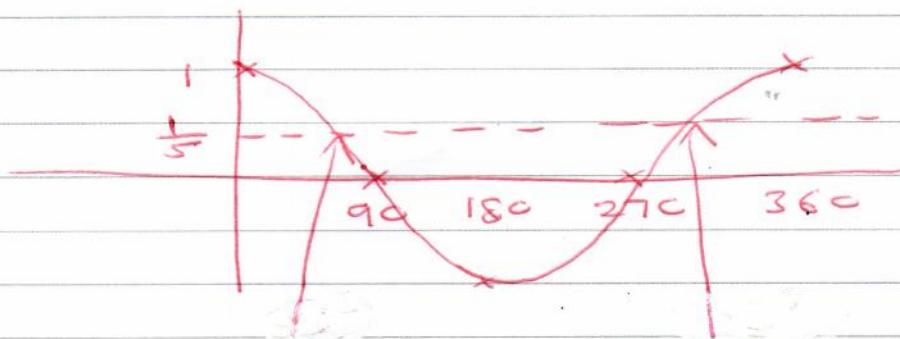
$$1 - 5 \cos 2x = 0$$

$$1 = 5 \cos 2x$$

$$\cos 2x = \frac{1}{5}$$

$$0 \leq x \leq 180^\circ$$

$$0 \leq 2x \leq 360^\circ$$



$$78.463^\circ$$

$$270 + (90 - 78.463^\circ) \\ = 281.53696^\circ$$

$$2x = 78.463$$

$$x = 39.23152$$

$$x = 39.2^\circ \text{ (1dp)}$$

$$2x = 281.53696$$

$$2x = 140.76848$$

$$x = 140.8^\circ \text{ (1dp)}$$

All solutions in range $0 \leq x \leq 180^\circ$
are

$$0^\circ, 39.2^\circ, 90^\circ, 140.8^\circ, 180^\circ$$



4. Solve, for $0 \leq x < 180^\circ$,

$$\cos(3x - 10^\circ) = -0.4$$

giving your answers to 1 decimal place. You should show each step in your working.

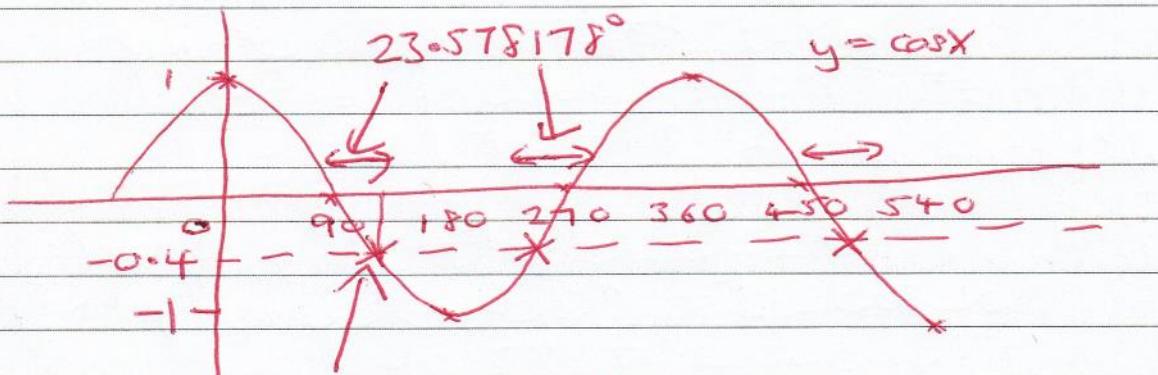
(7)

Let $x = 3x - 10$
 $\cos x = -0.4$

change interval for x

$$x = 0, x = -10$$

$$x = 180, x = 530$$



$$x = 113.57818^\circ$$

Other solutions by symmetry

$$270 - 23.578178 = 246.42182^\circ$$

$$450 + 23.578178 = 473.57818^\circ$$

So $3x - 10 = 113.57818$

$$x = \frac{113.57818 + 10}{3} = 41.192726^\circ$$

$$3x - 10 = 246.42182^\circ$$

$$x = \frac{246.42182 + 10}{3} = 85.47394^\circ$$

$$3x - 10 = 473.57818^\circ$$

$$x = \frac{473.57818 + 10}{3} = 161.19273^\circ$$

Answers to 1dp are $x = 41.2^\circ, 85.5^\circ, 161.2^\circ$



8. (i) Solve, for $-180^\circ \leq x < 180^\circ$,

$$\tan(x - 40^\circ) = 1.5$$

giving your answers to 1 decimal place.

(3)

- (ii) (a) Show that the equation

$$\sin \theta \tan \theta = 3 \cos \theta + 2$$

can be written in the form

$$4\cos^2 \theta + 2\cos \theta - 1 = 0$$

(3)

- (b) Hence solve, for $0 \leq \theta < 360^\circ$,

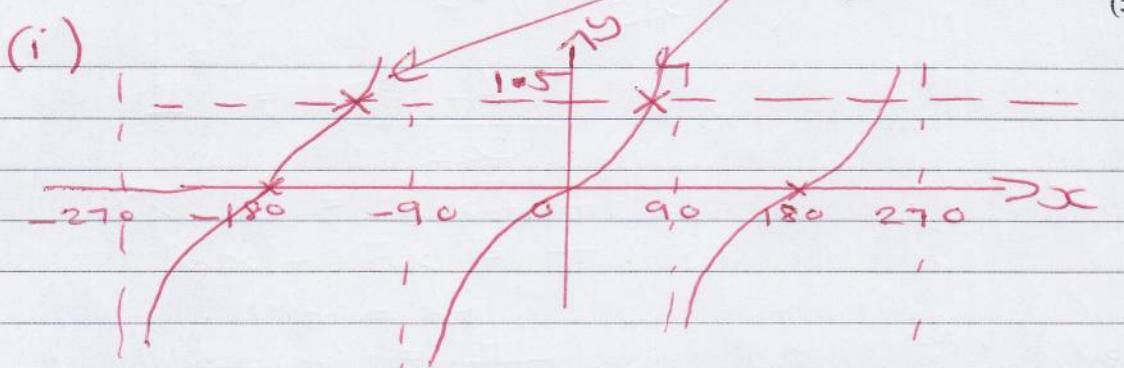
$$\sin \theta \tan \theta = 3 \cos \theta + 2$$

showing each stage of your working.

two solutions
for X

(5)

(i)



$$\text{let } x = \alpha - 40$$

$$\text{so } \tan x = 1.5$$

$$-180^\circ \leq \alpha < 180^\circ$$

$$-220^\circ \leq x < 140^\circ \quad (\text{change limits})$$

$$x = \tan^{-1} 1.5 = 56.309932^\circ$$

$$\text{also } x = -180 + 56.309932^\circ$$

$$= -123.69007^\circ$$

$$\text{So } \alpha - 40 = 56.309932, \alpha = 96.309932^\circ$$

$$\alpha - 40 = -123.69007, \alpha = -83.69007^\circ$$

$$\alpha = \underline{\underline{96.3^\circ}} \text{ or } \underline{\underline{-83.7^\circ}} \quad (\text{1dp})$$



8 (i) a) $\sin \theta \tan \theta = 3 \cos \theta + 2$

$$\sin \theta \times \frac{\sin \theta}{\cos \theta} = 3 \cos \theta + 2$$

$$\frac{\sin^2 \theta}{\cos \theta} = 3 \cos \theta + 2$$

(x through by $\cos \theta$)

$$\sin^2 \theta = 3 \cos^2 \theta + 2 \cos \theta$$

$$(1 - \cos^2 \theta) = 3 \cos^2 \theta + 2 \cos \theta$$

$$0 = 3 \cos^2 \theta + \cos^2 \theta + 2 \cos \theta - 1$$

$$\underline{0 = 4 \cos^2 \theta + 2 \cos \theta - 1} \quad (\text{as required})$$

b) $0 = (4 \cos^2 \theta + 2 \cos \theta - 1)$

$$0 = () ()$$

$$a=4, b=2, c=-1$$

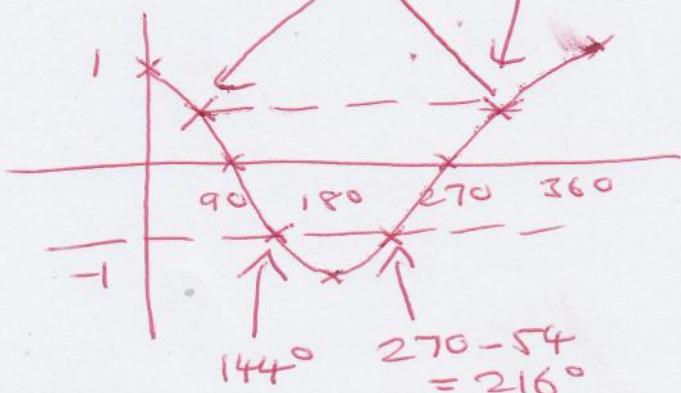
$$\cos \theta = \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times -1}}{8}$$

$$\cos \theta = \frac{-2 + \sqrt{20}}{8} \quad \text{or} \quad \cos \theta = \frac{-2 - \sqrt{20}}{8}$$

$$\theta = \cos^{-1} \left(\frac{-2 + \sqrt{20}}{8} \right) \quad \text{or} \quad \theta = \cos^{-1} \left(\frac{-2 - \sqrt{20}}{8} \right)$$

$$\theta = 72^\circ \quad 270^\circ + 18^\circ = 288^\circ$$

$$\theta = 144^\circ$$



From diagram

Solutions are

$$\underline{\theta = 72^\circ, 288^\circ}$$

and

$$\underline{\theta = 144^\circ, 216^\circ}$$