9.

Figure 2

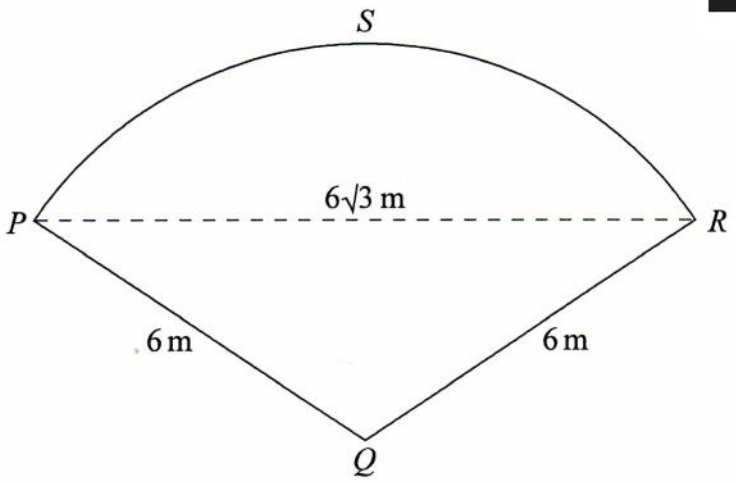


Figure 2 shows a plan of a patio. The patio $PQRS$ is in the shape of a sector of a circle with centre Q and radius 6 m.

Given that the length of the straight line PR is $6\sqrt{3}$ m,

(a) find the exact size of angle PQR in radians. **(3)**

(b) Show that the area of the patio $PQRS$ is 12π m^2. **(2)**

(c) Find the exact area of the triangle PQR. **(2)**

(d) Find, in m^2 to 1 decimal place, the area of the segment PRS. **(2)**

(e) Find, in m to 1 decimal place, the perimeter of the patio $PQRS$. **(2)**

a) Using cosine rule

$$\cos PQR = \frac{6^2+6^2-(6\sqrt{3})^2}{2\times6\times6}$$

$$\cos PQR = \frac{36+36-108}{72} = -\frac{1}{2}$$

$$PQR = \cos^{-1}\left(-\frac{1}{2}\right)$$

$PQR = 120°$ (in degrees)

$= \frac{2}{3}\pi$ radians

9b) Area sector $= \dfrac{\frac{\cancel{2}^{1}\pi}{3}}{\cancel{2\pi}_{1}} \times \cancel{\pi}^{1} \times 6^2$

$= \frac{36}{3}\pi$

$= 12\pi \ m^2$

c) Area of $\Delta = \frac{1}{2} \times 6 \times 6 \times \sin\frac{2\pi}{3}$

$= 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} \ m^2$

d) Area segment PRS $= 12\pi - 9\sqrt{3}$

$= 22.110655$

$= 22.1 \ m^2$ (1dp)

e) arc PR $= \dfrac{\frac{2\pi}{\cancel{3}_{1}}}{\cancel{2\pi}_{1}} \times \cancel{\pi}^{1} \times (\cancel{2}^{1} \times 6)^2$

$= 4\pi \ m$

Perimeter of PQRS

$= 6 + 6 + 4\pi$

$= 24.566371$

$= 24.6 \ m$ (1dp)

Leave blank

6.

Figure 1

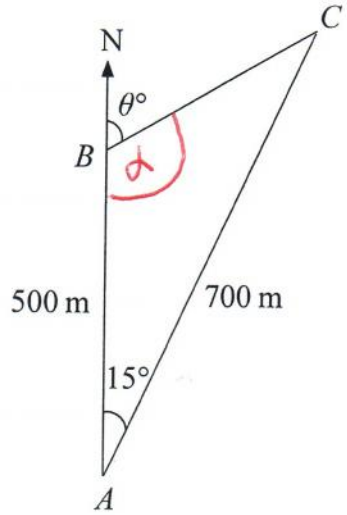


Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A. The bearing of C from A is 015°.

(a) Calculate the distance between yacht B and yacht C, in metres to 3 significant figures.

(3)

The bearing of yacht C from yacht B is $\theta°$, as shown in Figure 1.

(b) Calculate the value of θ.

(4)

a) Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$BC^2 = 500^2 + 700^2 - 2(500)(700)\cos 15°$

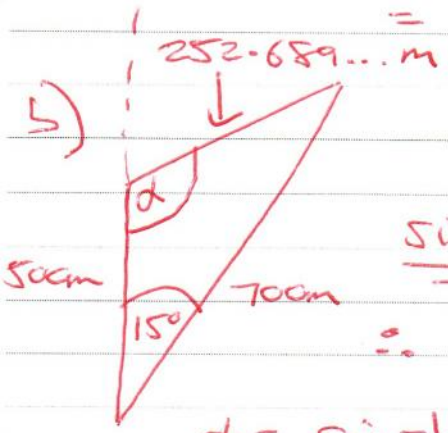
$= 63851.9216...$

$\therefore BC = \sqrt{63851.9216...}$

$= 252.689...$

$= 253\text{m}$ (3sf)

b)



Sine rule

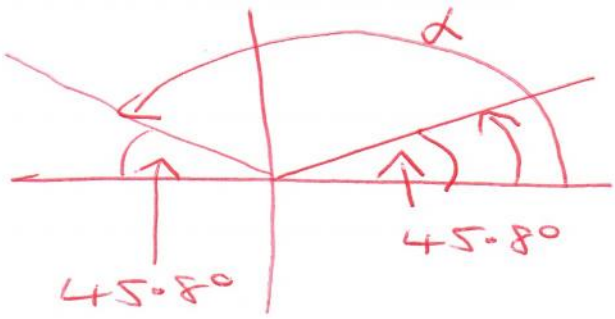
$\frac{\sin\alpha}{700} = \frac{\sin 15°}{252.689...}$

$\therefore \sin\alpha = \frac{700\sin 15°}{252.689...} = 0.71698...$

$\alpha = \sin^{-1} 0.71698... = 45.8057...°$

$, 134.194...°$

6b) continued



From diagram, we need α to be obtuse

so $\alpha = 134.194...°$

$\therefore \theta = 180° - 134.194...°$

$\theta = 45.8057...°$

$\therefore \theta = 046°$ (3 figure bearing)

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7.

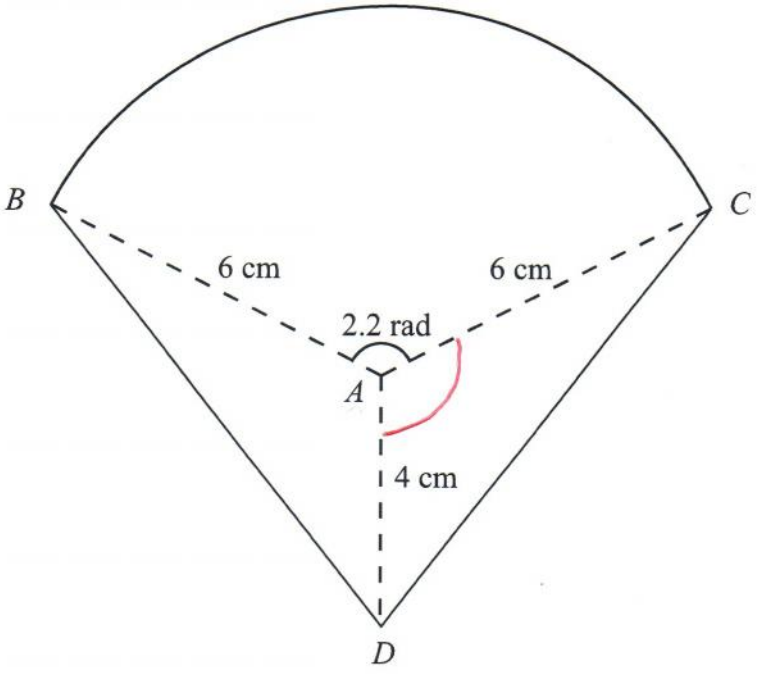

Figure 3

The shape *BCD* shown in Figure 3 is a design for a logo.

The straight lines *DB* and *DC* are equal in length. The curve *BC* is an arc of a circle with centre *A* and radius 6 cm. The size of $\angle BAC$ is 2.2 radians and $AD = 4$ cm.

Find

(a) the area of the sector *BAC*, in cm^2,

(2)

(b) the size of $\angle DAC$, in radians to 3 significant figures,

(2)

(c) the complete area of the logo design, to the nearest cm^2.

(4)

a) Area sector BAC $= \frac{2.2}{2\pi} \times \pi \times 6^2$

$= 39.6 cm^2$

b) As DB and DC are equal

Angle DAC = Angle DAB

$\angle DAC = \frac{(2\pi - 2.2)}{2} = 2.0415927$

$= 2.04$ radians (3 sf)

7c) $\triangle BAD$ and $\triangle CAD$ are congruent

Area $\triangle BAD = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 4 \times 6 \times \sin 2.0415927$

$= 10.694488 \text{ cm}^2$

Area of design $= 39.6 + 2(10.694488)$

$= 60.988976$

$= 61 \text{ cm}^2$ (nearest cm^2)

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4.

π radians = 180°

1 radian = $\frac{180°}{\pi}$

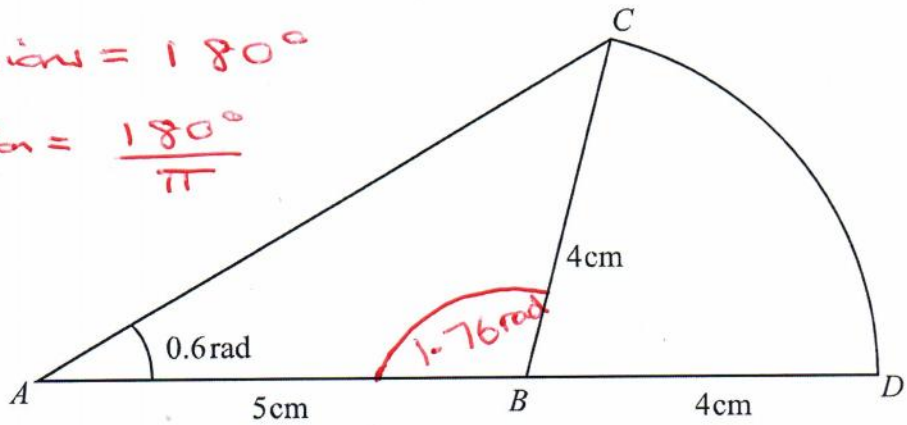


Figure 1

An emblem, as shown in Figure 1, consists of a triangle *ABC* joined to a sector *CBD* of a circle with radius 4 cm and centre *B*. The points *A*, *B* and *D* lie on a straight line with *AB* = 5 cm and *BD* = 4 cm. Angle *BAC* = 0.6 radians and *AC* is the longest side of the triangle *ABC*.

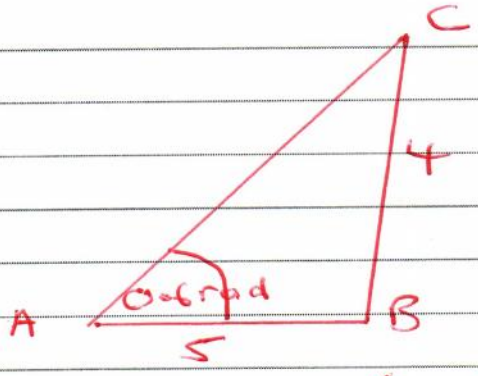
(a) Show that angle *ABC* = 1.76 radians, correct to 3 significant figures.

(4)

(b) Find the area of the emblem.

(3)

a)



Using SINE rule
Find $A\hat{C}B$ first

$\frac{\sin A\hat{C}B}{5} = \frac{\sin 0.6}{4}$

(calculator in radians mode)

$\sin A\hat{C}B = 0.705803$

$A\hat{C}B = 0.7835561$ radians

As angles in triangle add to π radians

$\therefore ABC = \pi - 0.6 - 0.7835561$

$= 1.7580365$

$= 1.76$ radians (3 sf)

4b) Area of ΔABC

$= \frac{1}{2} \times 5 \times 4 \times \sin(1.76)$

$= 9.82154 \text{ cm}^2$

Angle $\angle \hat{B}D = \pi - 1.76 = 1.38159$ radians

Area sector BCD $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 4 \times 4 \times 1.38159$

$= 11.052741$

Total area of emblem $= 9.82154 + 11.052741$

$= 20.874281$

$= 20.9 \text{ cm}^2$ (3 sf)

C2 JAN 2011

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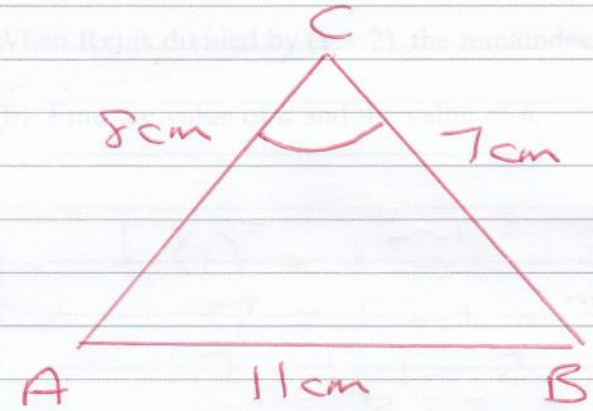
2. In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(a) Find the size of angle C, giving your answer in radians to 3 significant figures. **(3)**

(b) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures. **(3)**



a) $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\cos C = \dfrac{7^2+8^2-11^2}{2\times7\times8}$

$C = 1.6422858$ radian

$= 1.64^c$ to (3sf)

b) Area of triangle $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times7\times8\times\sin 1.6422858^c$

$= 27.92848$

$= 27.9$ cm^2 (3sf)

H 3 5 4 0 3 A 0 4 2 8

C2 Jan 2012

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7.

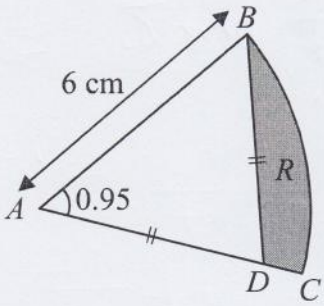


Figure 2

Figure 2 shows ABC, a sector of a circle of radius 6 cm with centre A. Given that the size of angle BAC is 0.95 radians, find

(a) the length of the arc BC,

(2)

(b) the area of the sector ABC.

(2)

The point D lies on the line AC and is such that $AD = BD$. The region R, shown shaded in Figure 2, is bounded by the lines CD, DB and the arc BC.

(c) Show that the length of AD is 5.16 cm to 3 significant figures.

(2)

Find

(d) the perimeter of R,

(2)

(e) the area of R, giving your answer to 2 significant figures.

(4)

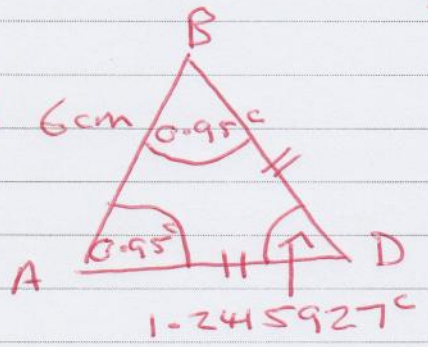
a) $BC = r\theta$

$= 6 \times 0.95 = 5.7\text{cm}$

b) area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 0.95$

$= 17.1\text{cm}^2$

c)



$\frac{AD}{\sin 0.95^c} = \frac{6}{\sin 1.2415927^c}$

$AD = \frac{6 \times \sin 0.95^c}{\sin 1.2415927^c}$

$AD = 5.1574474$

$= 5.16\text{cm}$ (3 sf)

C2 Jan 2012

7d) $DC = 6 - 5.1574474$ cm

Perimeter $= BC + DC + DB$
$= 5.7 + (6 - 5.1574474)$
$+ 5.1574474$
$= 11.7$ cm

7e) area $\triangle ABD$
$= \frac{1}{2} \times 6 \times 5.1574474 \times \sin 0.95$
$= 12.585443$ cm^2

Area R = area sector − area $\triangle ABD$
$= 17.1 - 12.585443$
$= 4.514557$
$= 4.5$ cm^2 (2sf)

Leave blank

8.

Figure 2

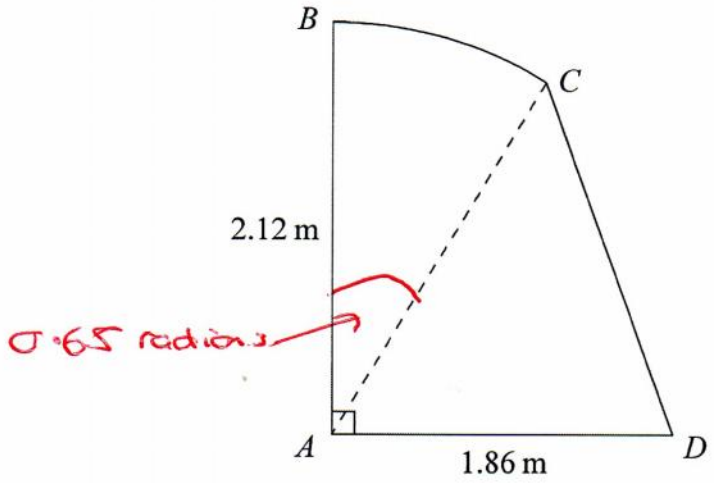


Figure 2 shows the cross section *ABCD* of a small shed.
The straight line *AB* is vertical and has length 2.12 m.
The straight line *AD* is horizontal and has length 1.86 m.
The curve *BC* is an arc of a circle with centre *A*, and *CD* is a straight line.
Given that the size of $\angle BAC$ is 0.65 radians, find

(a) the length of the arc *BC*, in m, to 2 decimal places,

(2)

(b) the area of the sector *BAC*, in m^2, to 2 decimal places,

(2)

(c) the size of $\angle CAD$, in radians, to 2 decimal places,

(2)

(d) the area of the cross section *ABCD* of the shed, in m^2, to 2 decimal places.

(3)

a) length arc BC $= \frac{0.65}{2\pi} \times 2\pi \times 2.12$

$= 1.378$

$= 1.38$ m (2dp)

b) Area of BAC $= \frac{0.65}{2\pi} \times \pi \times 2.12^2$

$= 1.46068$

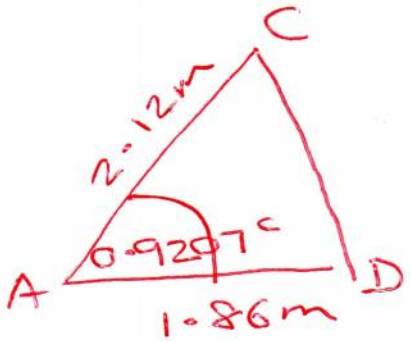
$= 1.46\ m^2$ (2dp)

8c) Angle CAD $= \frac{\pi}{2} - 0.65$

$= 0.9207963$

$= 0.92$ radians (2dp)

d)



Calculator in radians mode

Area $\triangle$ CAD $= \frac{1}{2} \times 1.86 \times 2.12 \times \sin(0.9207963)$

$= 1.5694\ldots$

Area of ABCD $= 1.46668 + 1.5694\ldots$

$= 3.0301$

$= 3.03\ m^2$ (2dp)

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4.

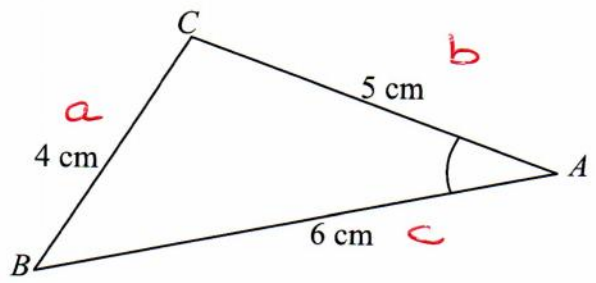

Figure 1

Figure 1 shows the triangle ABC, with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$.

(3)

(b) Hence, or otherwise, find the exact value of $\sin A$.

(2)

a) Cosine Rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6}$$

$$\cos A = \frac{25 + 36 - 16}{60} = \frac{45}{60} = \frac{3}{4}$$

b) $\sin^2 A + \cos^2 A = 1$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin^2 A = 1 - \left(\frac{3}{4} \times \frac{3}{4}\right)$$

$$\sin^2 A = 1 - \frac{9}{16}$$

$$\sin^2 A = \frac{7}{16}$$

$$\sin A = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{\sqrt{16}}$$

$$\sin A = \frac{1}{4}\sqrt{7}$$

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7.

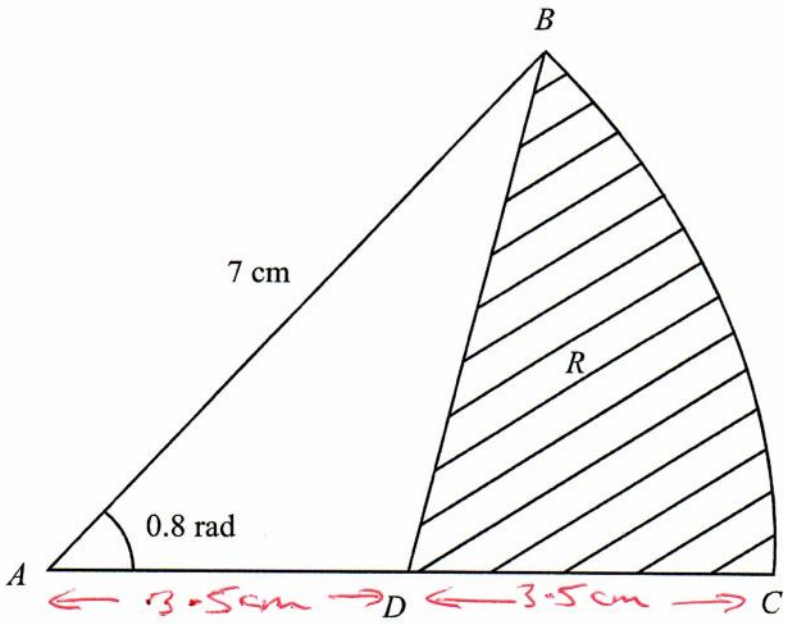


Figure 1

Figure 1 shows ABC, a sector of a circle with centre A and radius 7 cm.

Given that the size of $\angle BAC$ is exactly 0.8 radians, find

(a) the length of the arc BC,

(2)

(b) the area of the sector ABC.

(2)

The point D is the mid-point of AC. The region R, shown shaded in Figure 1, is bounded by CD, DB and the arc BC.

Find

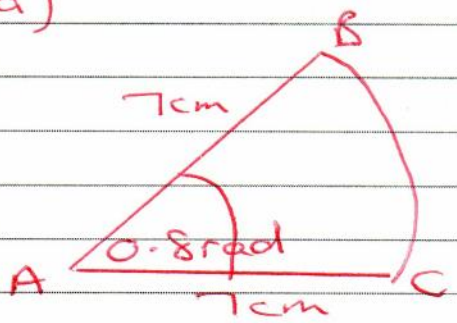
(c) the perimeter of R, giving your answer to 3 significant figures,

(4)

(d) the area of R, giving your answer to 3 significant figures.

(4)

a)



length of arc BC

$= \frac{0.8}{2\pi} \times 2\pi \times 7$

$= 5.6$ cm

$\boxed{\text{Length of arc} = r\theta}$

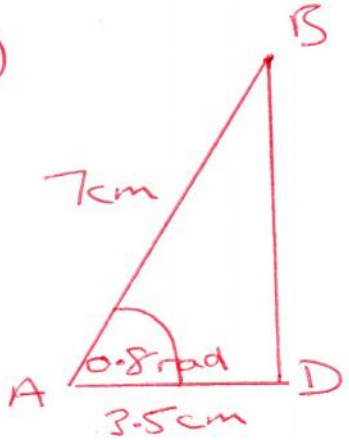
7b) Area of sector $= \frac{0.8}{2\pi} \times \pi \times 7^2$

$= 19.6\text{cm}^2$

$$\boxed{\text{Area of sector} = \frac{1}{2}r^2\theta}$$

$\cos 0.8^c$ → RADIANS on calculator

c)



use Cosine rule to find length BD

$BD^2 = 7^2 + 3.5^2 - 2 \times 7 \times 3.5 \times \cos 0.8^c$

$BD^2 = 27.111371\ldots$

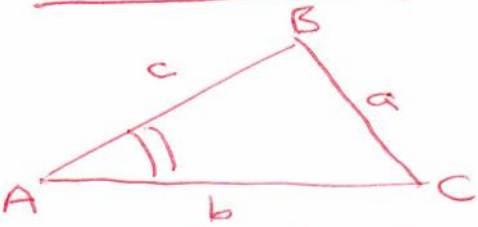
$BD = 5.206858\ldots$

$\therefore$ perimeter of R $= 5.6 + 3.5 + 5.206858$

$= 14.3068\ldots$

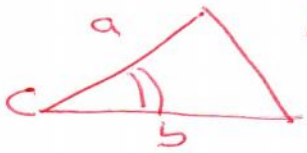
$= 14.3$ cm (3sf)

Cosine rule



$a^2 = b^2 + c^2 - 2bc\cos A$

Area Triangle



Area $\Delta = \frac{1}{2}ab\sin C$

use

d) Area R = Area Sector − Area of Δ ABD

$= 19.6 - \frac{1}{2} \times 7 \times 3.5 \times \sin 0.8^c$

radians mode on calculator

$= 19.6 - 8.78761\ldots$

$= 10.8123\ldots$

$= 10.8\text{ cm}^2$ (3sf)

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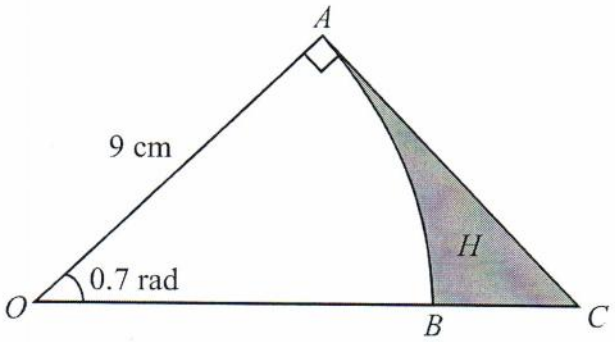


Figure 1

Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc AB.

(2)

(b) Find the area of the sector OAB.

(2)

The line AC shown in Figure 1 is perpendicular to OA, and OBC is a straight line.

(c) Find the length of AC, giving your answer to 2 decimal places.

(2)

The region H is bounded by the arc AB and the lines AC and CB.

(d) Find the area of H, giving your answer to 2 decimal places.

(3)

a) arc length $= r\theta$

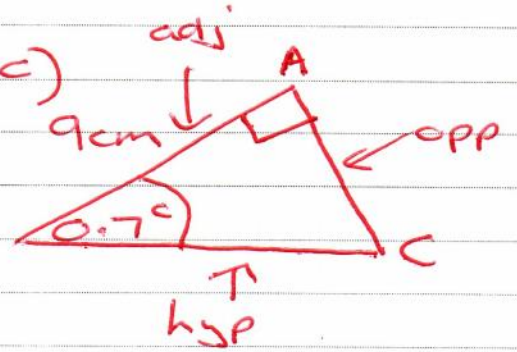
$= 9 \times 0.7 = 6.3\text{cm}$

b) area sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 9^2 \times 0.7$

$= 28.35\text{ cm}^2$

c)



$\tan 0.7^c = \frac{AC}{9}$

calculator in radians

$AC = 9 \times \tan 0.7^c$

$= 7.5805$

$= 7.58\text{ cm}$ (2dp)

C2 June 2010

6d) Area $\Delta OAC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 9 \times 7.5805954$

$= 34.112679 \text{ cm}^2$

Shaded area = Area triangle − Area sector

$= 34.112679 - 28.35$

$= 5.7626794$

$= 5.76 \text{ cm}^2$ (2dp)

May 2011

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5.

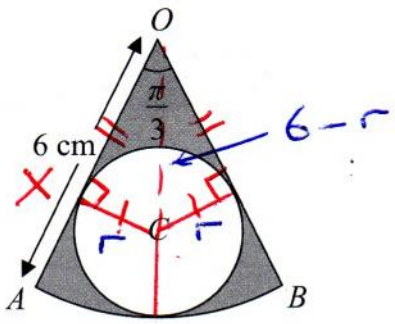


Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector OAB of a circle centre O, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle C, inside the sector, touches the two straight edges, OA and OB, and the arc AB as shown.

Find

(a) the area of the sector OAB,

(2)

(b) the radius of the circle C.

(3)

The region outside the circle C and inside the sector OAB is shown shaded in Figure 1.

(c) Find the area of the shaded region.

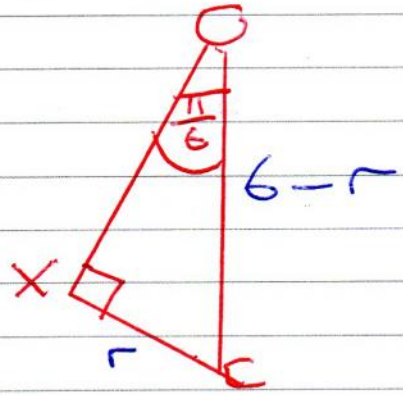
(2)

a) Area $= \frac{1}{2} \times r^2 \times \theta$

$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = 6\pi$

$= 18.8$ square units (3sf)

b)



$\sin \frac{\pi}{6} = \frac{r}{6-r}$

$\frac{1}{2} = \frac{r}{6-r}$

$6 - r = 2r$

$6 = 3r$

$r = 2$

P 3 8 1 5 8 A 0 1 2 3 2

May 2011

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Question 5 continued

c) Area unshaded circle $= \pi \times 2^2$
$= 4\pi$

Area shaded $= 6\pi - 4\pi$
$= 2\pi$
$= 6.28$ sq units (3sf)

P 3 8 1 5 8 A 0 1 3 3 2

Turn over

C2 Jan 2013

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7.

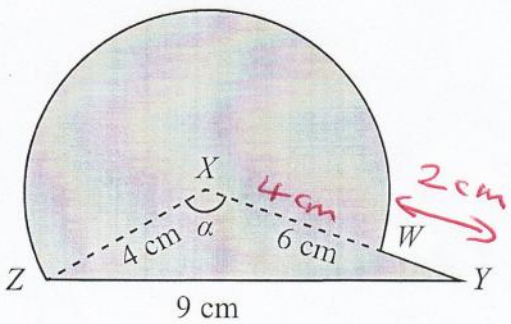


Figure 1

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$. The point W lies on the line XY.

The circular arc ZW, in Figure 1 is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region,

(3)

(d) the perimeter $ZWYZ$ of this shaded region.

(4)

Cosine rule

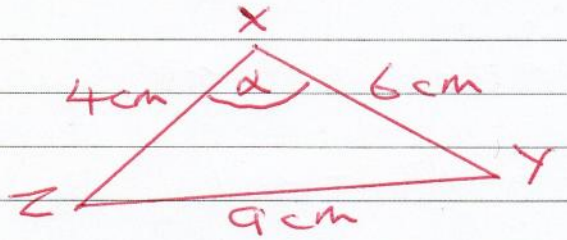
$x^2 = y^2 + z^2 - 2yz\cos\alpha$

or

$\cos x = \dfrac{y^2+z^2-x^2}{2yz}$

(calculator in radians mode)

a)



$\cos\alpha = \dfrac{4^2+6^2-9^2}{2\times4\times6}$

$\cos\alpha = \dfrac{-29}{48}$

$\alpha = \cos^{-1}\left(\dfrac{-29}{48}\right)$

$\alpha = 2.219516$

$\alpha = 2.22^c$ (3 sf) as required

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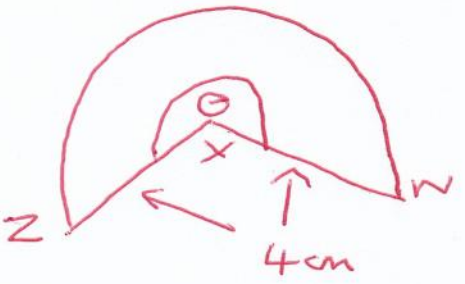
7b)



radius = 4 cm

in degrees 360 − α

$\theta = 2\pi - \alpha$

$\theta = 2\pi - 2.219516$

$= 4.0636693$

Area sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 4^2 \times 4.0636693$

$= 32.509354$

$= \underline{\underline{32.5}}$ cm^2 (3sf)

c) Area $\Delta XYZ = \frac{1}{2} \times 4 \times 6 \times \sin 2.219516$

$= 9.5622958$ units squared

Total shaded $= 9.5622958 + 32.509354$

$= 42.07165$

$= \underline{\underline{42.1}}$ cm^2 (3sf)

d) Arc length $ZW = r\theta$

$= 4 \times 4.0636693$

$= 16.254677$

Perimeter $= ZW + WY + YZ$

$= 16.254677 + 2 + 9$

$= 27.254677$

$= 27.3$ cm (3sf)

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Leave blank

5.

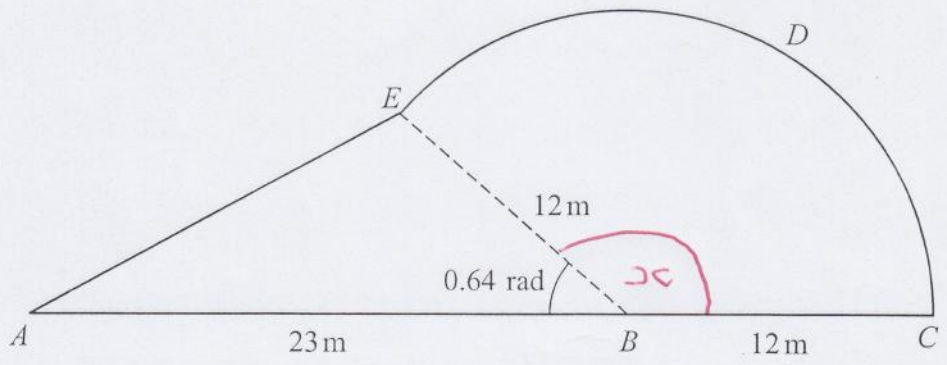

Figure 2

Figure 2 shows a plan view of a garden.
The plan of the garden *ABCDEA* consists of a triangle *ABE* joined to a sector *BCDE* of a circle with radius 12 m and centre *B*.
The points *A*, *B* and *C* lie on a straight line with $AB = 23$ m and $BC = 12$ m.

Given that the size of angle *ABE* is exactly 0.64 radians, find

(a) the area of the garden, giving your answer in m^2, to 1 decimal place,

(4)

(b) the perimeter of the garden, giving your answer in metres, to 1 decimal place.

(5)

a) area triangle AEB $= \frac{1}{2} \times 23 \times 12 \times \sin 0.64$
(calculator in radians)
$= 82.412971 \text{ m}^2$

angle $x = \pi - 0.64 = 2.5015927^c$

area sector BEDC $= \frac{1}{2} \times 12^2 \times 2.5015927$
$= 180.11467 \text{ m}^2$

Area garden $= 82.412971 + 180.11467$
$= 262.52764$
$= \underline{\underline{262.5 \text{ m}^2}}$ (1dp)

b) get AE using COSINE rule

$AE = \sqrt{23^2 + 12^2 - 2 \times 23 \times 12 \times \cos 0.64^c}$
$AE = 15.173765 \text{ m}$

P 4 1 8 5 9 A 0 1 2 3 2

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5b) continued

EDC is arc length using $r\theta$

$EDC = 12 \times 2.5015927$

$= 30.019112$ m

Perimeter $= 15.173765 + 30.019112$
$+ 12 + 23$
$= 80.192877$
$= \underline{\underline{80.2\text{m}}}$ (1dp)