4. Solve the simultaneous equations

$$y = x - 2, \quad (1)$$

$$y^2 + x^2 = 10. \quad (2)$$

(7)

Put $y = x - 2$ in (2)

$(x-2)^2 + x^2 = 10$

$x^2 - 4x + 4 + x^2 - 10 = 0$

$2x^2 - 4x - 6 = 0$

$2(x^2 - 2x - 3) = 0$

$2(x+1)(x-3) = 0$

Either $x = -1$ or $x = 3$

put $x = -1$ in (1)

$y = -1 - 2$

$y = -3$

put $x = 3$ in (1)

$y = 3 - 2 = 1$

The two solutions to the simultaneous equations are

$x = -1, y = -3$ and $x = 3, y = 1$

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6. (a) By eliminating y from the equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0.$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

gives a clue that we need the quadratic formula

a) $y = x - 4$ ①

$2x^2 - xy = 8$ ②

Sub ① into ②

$\therefore 2x^2 - x(x-4) = 8$

$\therefore 2x^2 - x^2 + 4x - 8 = 0$

$\therefore x^2 + 4x - 8 = 0$

b) $a = 1, b = 4, c = -8$

$\therefore x = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 1 \times -8}}{2 \times 1}$

$= \dfrac{-4 \pm \sqrt{48}}{2}$

$= \dfrac{-4 \pm 4\sqrt{3}}{2}$

$= -2 \pm 2\sqrt{3}$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\sqrt{48} = \sqrt{16 \times 3}$

$= \sqrt{16} \times \sqrt{3}$

$= 4\sqrt{3}$

When $x = -2 + 2\sqrt{3}$ sub in ①

$y = -2 + 2\sqrt{3} - 4$

$= -6 + 2\sqrt{3}$

When $x = -2 - 2\sqrt{3}$ sub in ①

$y = -2 - 2\sqrt{3} - 4$

$= -6 - 2\sqrt{3}$

$x = -2 + 2\sqrt{3}$

$y = -6 + 2\sqrt{3}$

or

$x = -2 - 2\sqrt{3}$

$y = -6 - 2\sqrt{3}$

May 2011

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4. Solve the simultaneous equations

$$x + y = 2 \quad (1)$$
$$4y^2 - x^2 = 11 \quad (2)$$

(7)

(1) gives $x = 2 - y$

in (2)

$4y^2 - (2-y)^2 = 11$

$4y^2 - (4 - 4y + y^2) - 11 = 0$

$4y^2 - 4 + 4y - y^2 - 11 = 0$

$3y^2 + 4y - 15 = 0$

$(3y - 5)(y + 3) = 0$

$y = \frac{5}{3}$ | $y = -3$

in (1) gives

$x + \frac{5}{3} = 2$

$x = \frac{1}{3}$

in (1) gives

$x - 3 = 2$

$x = 5$

Solutions are

$x = \frac{1}{3}$, $y = \frac{5}{3}$ or $x = 5$, $y = -3$

C1 Jan 2013

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6.

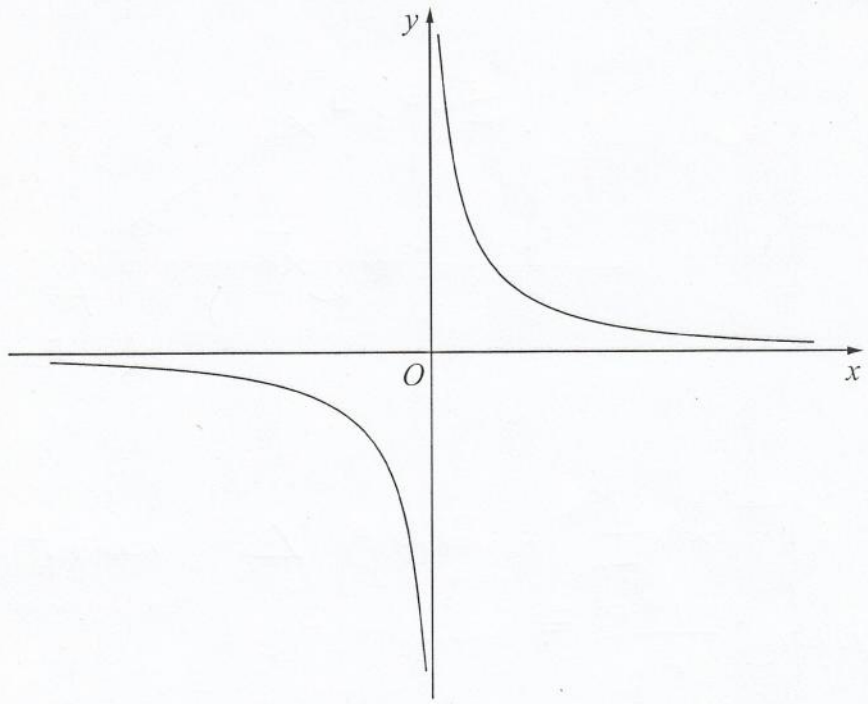


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}$, $x \neq 0$

shifted down 5 units

The curve C has equation $y = \frac{2}{x} - 5$, $x \neq 0$, and the line l has equation $y = 4x + 2$

$y=0,\ 5 = \frac{2}{x}$ so $x = \frac{2}{5}$

$x=0,\ y=2$

$y=0,\ x=-\frac{1}{2}$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

(5)

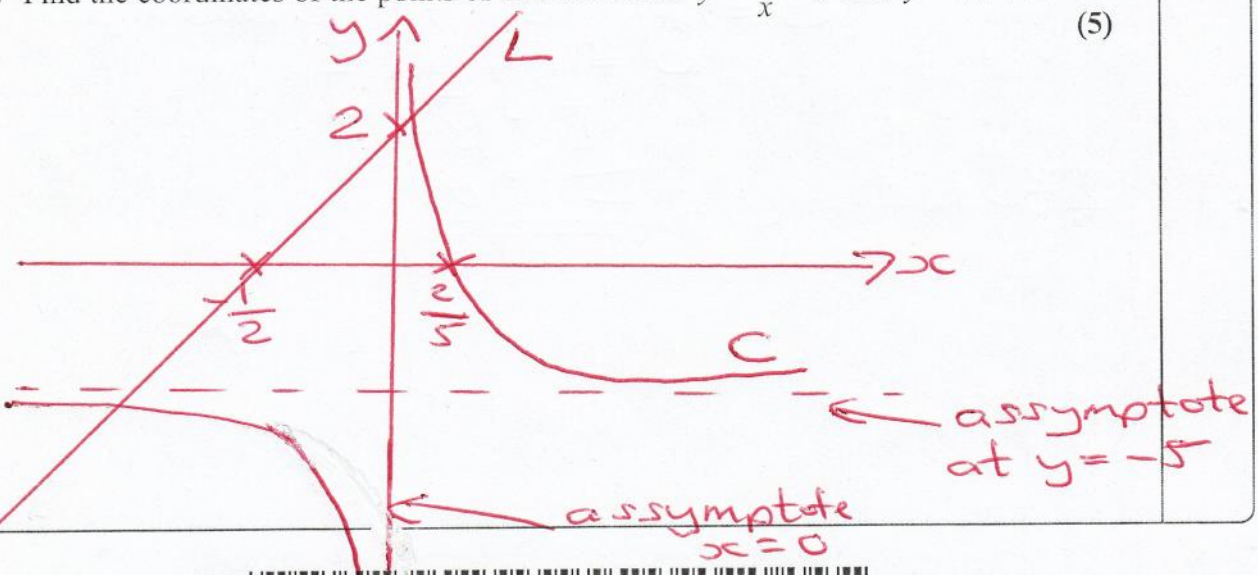
(b) Write down the equations of the asymptotes of the curve C.

(2)

(c) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$

(5)

a)



C1 Jan 2013

6b) $x = 0$ and $y = -5$

c) Solve simultaneous equations

$y = \frac{2}{x} - 5$ ① and $y = 4x + 2$ ②

set ① = ② to solve

$\frac{2}{x} - 5 = 4x + 2$

x through by x

$2 - 5x = 4x^2 + 2x$

$0 = 4x^2 + 2x + 5x - 2$

$0 = 4x^2 + 7x - 2$

$0 = (4x - 1)(x + 2)$

Either $4x - 1 = 0$

$x = \frac{1}{4}$

in ② $y = 4 \times \frac{1}{4} + 2$

$y = 3$

$(\frac{1}{4}, 3)$

or $x + 2 = 0$

$x = -2$

in ② $y = 4x - 2 + 2$

$y = -6$

$(-2, -6)$

C1 MAY 2013

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10. Given the simultaneous equations

$$2x + y = 1 \quad (1)$$
$$x^2 - 4ky + 5k = 0 \quad (2)$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

a) (1) gives $y = 1 - 2x$

sub in (2)

$x^2 - 4k(1-2x) + 5k = 0$

$x^2 - 4k + 8kx + 5k = 0$

$x^2 + 8kx + k = 0$ (as required)

b) equal roots $b^2 - 4ac = 0$

$a = 1,\ b = 8k,\ c = k$

$(8k)^2 - 4 \times 1 \times k = 0$

$64k^2 - 4k = 0$

$4k(16k - 1) = 0$

Either $k = 0$ or $k = \frac{1}{16}$

as k is non-zero constant, $k = \frac{1}{16}$

c) $x^2 + \frac{8}{16}x + \frac{1}{16} = 0$ x through by 16

$16x^2 + 8x + 1 = 0$

$(4x+1)(4x+1) = 0$

$x = -\frac{1}{4}$ or $x = -\frac{1}{4}$

in (1) $y = 1 - 2x - \frac{1}{4} = \frac{3}{2}$

Solution to simultaneous equation

$x = -\frac{1}{4},\ y = \frac{3}{2}$

$(-\frac{1}{4}, \frac{3}{2})$