

4. Solve the simultaneous equations

$$y = x - 2, \quad (1)$$

$$y^2 + x^2 = 10. \quad (2)$$

(7)

Put $y = x - 2$ in (2)

$$(x - 2)^2 + x^2 = 10$$

$$x^2 - 4x + 4 + x^2 - 10 = 0$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x + 1)(x - 3) = 0$$

Either $x = -1$ or $x = 3$ put $x = -1$ in (1)

$$y = -1 - 2$$

$$y = -3$$

put $x = 3$ in (1)

$$y = 3 - 2 = 1$$

The two solutions to the simultaneous equations are

$$x = -1, y = -3 \quad \text{and} \quad x = 3, y = 1$$



6. (a) By eliminating y from the equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0.$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

← gives a clue that we need the quadratic formula (5)

a) $y = x - 4$ (1)
 $2x^2 - xy = 8$ (2)

Sub (1) into (2)

$$\therefore 2x^2 - x(x - 4) = 8$$

$$\therefore 2x^2 - x^2 + 4x - 8 = 0$$

$$\therefore x^2 + 4x - 8 = 0$$

b) $a = 1, b = 4, c = -8$

$$ax^2 + bx + c = 0$$

$$\therefore x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times -8}}{2 \times 1}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{48}}{2}$$

$$= \frac{-4 \pm 4\sqrt{3}}{2}$$

$$= -2 \pm 2\sqrt{3}$$

$$\begin{aligned} \sqrt{48} &= \sqrt{16 \times 3} \\ &= \sqrt{16} \times \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

When $x = -2 + 2\sqrt{3}$ sub in (1)

$$\begin{aligned} y &= -2 + 2\sqrt{3} - 4 \\ &= -6 + 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} x &= -2 + 2\sqrt{3} \\ y &= -6 + 2\sqrt{3} \end{aligned}$$

When $x = -2 - 2\sqrt{3}$ sub in (1)

$$\begin{aligned} y &= -2 - 2\sqrt{3} - 4 \\ &= -6 - 2\sqrt{3} \end{aligned}$$

or

$$\begin{aligned} x &= -2 - 2\sqrt{3} \\ y &= -6 - 2\sqrt{3} \end{aligned}$$



May 2011

Leave blank

4. Solve the simultaneous equations

$$\begin{aligned}x + y &= 2 & \textcircled{1} \\4y^2 - x^2 &= 11 & \textcircled{2}\end{aligned}$$

(7)

$\textcircled{1}$ gives $x = 2 - y$

in $\textcircled{2}$

$$4y^2 - (2 - y)^2 = 11$$

$$4y^2 - (4 - 4y + y^2) - 11 = 0$$

$$4y^2 - 4 + 4y - y^2 - 11 = 0$$

$$3y^2 + 4y - 15 = 0$$

$$(3y - 5)(y + 3) = 0$$

$$y = \frac{5}{3} \quad | \quad y = -3$$

in $\textcircled{1}$ gives

$$\begin{aligned}x + \frac{5}{3} &= 2 \\x &= \frac{1}{3}\end{aligned}$$

in $\textcircled{1}$ gives

$$\begin{aligned}x - 3 &= 2 \\x &= 5\end{aligned}$$

Solutions are

$$\begin{aligned}x &= \frac{1}{3} \\y &= \frac{5}{3}\end{aligned}$$

or

$$\begin{aligned}x &= 5 \\y &= -3\end{aligned}$$



6.

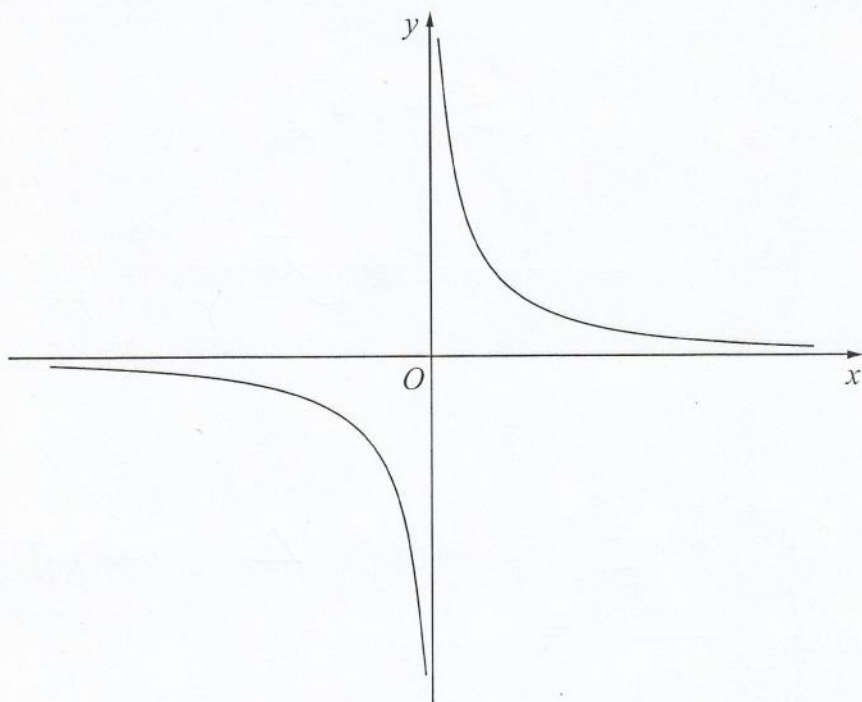


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}$, $x \neq 0$

The curve C has equation $y = \frac{2}{x} - 5$, $x \neq 0$, and the line l has equation $y = 4x + 2$

← shifted down 5 units
 $y=0, 5 = \frac{2}{x} \text{ so } x = \frac{2}{5}$ $x=0, y=2$
 $y=0, x = -\frac{1}{2}$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

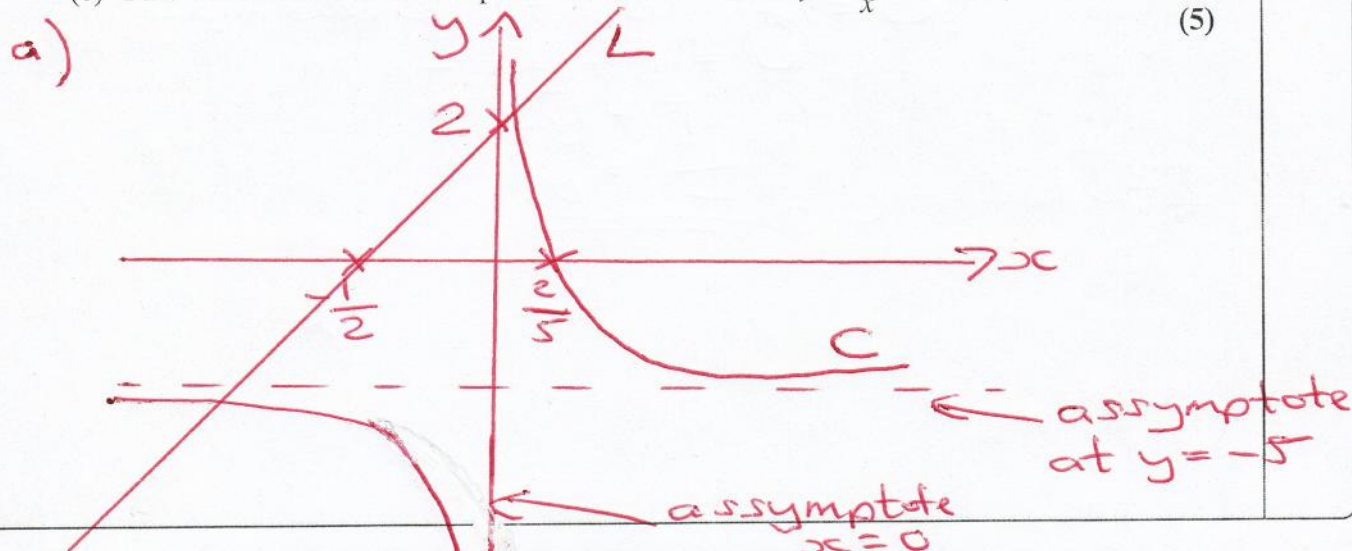
(5)

(b) Write down the equations of the asymptotes of the curve C .

(2)

(c) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$

(5)



01 Jan 2013

6b) $x = 0$ and $y = -5$

c) Solve simultaneous equations

$$y = \frac{2}{x} - 5 \quad (1)$$

$$\text{and } y = 4x + 2 \quad (2)$$

set (1) = (2) to solve

$$\frac{2}{x} - 5 = 4x + 2$$

x through by x

$$2 - 5x = 4x^2 + 2x$$

$$0 = 4x^2 + 2x + 5x - 2$$

$$0 = 4x^2 + 7x - 2$$

$$0 = (4x - 1)(x + 2)$$

Either $4x - 1 = 0$

$$x = \frac{1}{4}$$

in (2) $y = 4 \times \frac{1}{4} + 2$

$$y = 3$$

$$\underline{\underline{\left(\frac{1}{4}, 3\right)}}$$

or $x + 2 = 0$

$$x = -2$$

in (2) $y = 4x - 2 + 2$

$$y = -6$$

$$\underline{\underline{(-2, -6)}}$$

10. Given the simultaneous equations

$$\begin{aligned} 2x + y &= 1 && \textcircled{1} \\ x^2 - 4ky + 5k &= 0 && \textcircled{2} \end{aligned}$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0 \tag{2}$$

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k . (3)

(c) For this value of k , find the solution of the simultaneous equations. (3)

a) $\textcircled{1}$ gives $y = 1 - 2x$
sub in $\textcircled{2}$

$$\begin{aligned} x^2 - 4k(1 - 2x) + 5k &= 0 \\ x^2 - 4k + 8kx + 5k &= 0 \\ x^2 + 8kx + k &= 0 \quad (\text{as required}) \end{aligned}$$

b) equal roots $b^2 - 4ac = 0$
 $a = 1, b = 8k, c = k$

$$\begin{aligned} (8k)^2 - 4 \times 1 \times k &= 0 \\ 64k^2 - 4k &= 0 \\ 4k(16k - 1) &= 0 \end{aligned}$$

Either $k = 0$ or $k = \frac{1}{16}$
as k is non-zero constant, $k = \frac{1}{16}$

c) $x^2 + \frac{8}{16}x + \frac{1}{16} = 0$ x through by 16

$$\begin{aligned} 16x^2 + 8x + 1 &= 0 \\ (4x + 1)(4x + 1) &= 0 \\ x = -\frac{1}{4} \text{ or } x = -\frac{1}{4} \end{aligned}$$

in $\textcircled{1}$ $y = 1 - 2x = 1 - 2(-\frac{1}{4}) = \frac{3}{2}$

Solution to simultaneous equation
 $x = -\frac{1}{4}, y = \frac{3}{2}$

$(-\frac{1}{4}, \frac{3}{2})$

