

5. (a) Find $\int \frac{9x+6}{x} dx, x > 0.$

(2)

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

(6)

5a) $I = \int 9 + \frac{6}{x} dx$
 $= 9x + 6\ln x + C$

(b) $\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$ separate variables

$$\int \frac{1}{y^{\frac{1}{3}}} dy = \int \frac{(9x+6)}{x} dx$$

$$\Rightarrow \frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x + C$$

$$\Rightarrow \frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x + C$$

$$y=8 \quad \frac{3}{2}(8)^{\frac{2}{3}} = 9 + 6\ln 1 + C$$

$$x=1 \quad 6 = 9 + C$$

$$C = -3$$

$$\frac{3}{2} y^{\frac{2}{3}} = 9x + 6 \ln x - 3$$

$$3y^{\frac{2}{3}} = 18x + 12 \ln x - 6$$

$$y^{\frac{2}{3}} = 6x + 4 \ln x - 2$$

$$y^2 = (6x + 4 \ln x - 2)^3$$

3. (a) Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions. (3)

(b) Hence find $\int \frac{5}{(x-1)(3x+2)} dx$, where $x > 1$. (3)

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2) \frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$. (6)

$$\text{Q1} \quad \frac{5}{(x-1)(3x+2)} = \frac{A}{(x-1)} + \frac{B}{(3x+2)}$$

$$5 = A(3x+2) + B(x-1)$$

$$\text{when } x=1 \quad 5 = 5A \quad \underline{A=1}$$

$$\text{when } x = -\frac{2}{3} \quad 5 = -\frac{5}{3}B \quad \underline{B=-3}$$

$$\text{b) } I = \int \frac{1}{(x-1)} - \frac{3}{(3x+2)} dx$$

$$= \ln(x-1) - \ln(3x+2) + C$$

(c) Separating Variables:

$$\int \frac{1}{y} dy = \int \frac{5}{(x-1)(3x+2)} dx \quad \leftarrow \text{part (a)(b)}$$

$$\ln y = \ln(x-1) - \ln(3x+2) + \ln k$$

$$\ln y = \ln \left(\frac{x-1}{3x+2} \right)$$

$$y = k \left(\frac{x-1}{3x+2} \right)$$

$$y=8 \\ \text{at } x=2 \\ 8 = \frac{k}{8}$$

$$k=64$$

$$\Rightarrow y = 64 \left(\frac{x-1}{3x+2} \right)$$

7. (a) Express $\frac{2}{4-y^2}$ in partial fractions.

(3)

- (b) Hence obtain the solution of

$$2 \cot x \frac{dy}{dx} = (4-y^2)$$

for which $y=0$ at $x=\frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)

$$\text{a)} \quad \frac{2}{4-y^2} = \frac{2}{(2-y)(2+y)} = \frac{A}{2-y} + \frac{B}{2+y}$$

$$2 \equiv A(2+y) + B(2-y)$$

$$y=-2 \quad 2=4B \quad \text{and when } y=2 \quad 2=4A$$

$$B=\frac{1}{2}$$

$$A=\frac{1}{2}$$

$$\equiv \frac{1}{2(2-y)} + \frac{1}{2(2+y)}$$

$$\text{b)} \quad 2 \cot x \frac{dy}{dx} = (4-y^2) \quad \text{separate variables.}$$

$$\int \frac{2}{(4-y^2)} dy = \int \frac{1}{\cot x} dx$$

$$\int \frac{1}{2(2-y)} + \frac{1}{2(2+y)} dy = \int \tan x dx$$



Question 7 continued

$$-\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) + c$$

$$y = e^x \quad x = \pi/3$$

$$-\frac{1}{2} \ln 2 + \frac{1}{2} \ln 2 = \ln \left(\frac{1}{\cos(\pi/3)} \right) + c$$

$$0 = \ln 2 + c$$

$$c = -\ln 2$$

$$-\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) - \ln 2$$

$$\frac{1}{2} \ln \left(\frac{2+y}{2-y} \right) = \ln \left(\frac{\sec x}{2} \right)$$

$$\ln \left(\frac{2+y}{2-y} \right) = 2 \ln \left(\frac{\sec x}{2} \right)$$

$$\ln \left(\frac{2+y}{2-y} \right) = \ln \left(\frac{\sec^2 x}{4} \right)$$

$$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$$

$$\sec^2 x = \frac{8+4y}{2-y}$$



4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y \cos^2 x}$$

(5)

$$\int y \, dy = \int \frac{3}{\cos^2 x} \, dx$$

$$\frac{1}{2} y^2 = \int 3 \sec^2 x \, dx$$

$$\frac{1}{2} y^2 = 3 \tan x + C$$

$$\text{At } (\frac{\pi}{4}, 2) \quad 2 = 3 \tan(\frac{\pi}{4}) + C$$

$$2 = 3 + C$$

$$C = -1$$

$$\therefore \frac{1}{2} y^2 = 3 \tan x - 1$$

$$y^2 = 6 \tan x - 2$$

$$y = \sqrt{6 \tan x - 2}$$

