Jan 2007

4. (a) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions.

(3)

(b) Given that $x \geqslant 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{dy}{dx} = (2x-1)y.$$

(5)

(c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = f(x)$.

(4)

2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\dfrac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

3. (a) Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(b) Hence find the exact value of $\displaystyle\int_{2}^{6} \dfrac{5x+3}{(2x-3)(x+2)}\,dx$, giving your answer as a single logarithm.

(5)

4.

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence show that the exact value of $\displaystyle\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\ dx$ is $2 + \ln k$, giving the value of the constant k.

(6)

7. (a) Express $\dfrac{2}{4-y^2}$ in partial fractions.

(3)

(b) Hence obtain the solution of

$$2\cot x \,\frac{dy}{dx} = (4-y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)
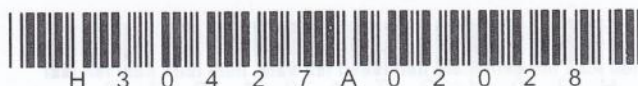
3.
$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)} .$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int f(x) \, dx$.

(3)

(ii) Find $\int_0^2 f(x) \, dx$ in the form $\ln k$, where k is a constant.

(3)

6.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \dfrac{1}{2} + \dfrac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}} \theta\, f(\theta)\; d\theta$.

(7)

1.
$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A, B and C.

(4)

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

1.
$$f(x) = \frac{1}{x(3x-1)^2} \equiv \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int f(x)\, dx$.

(ii) Find $\int_1^2 f(x)\, dx$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

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3. Express $\dfrac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$ in partial fractions.

(4)

5. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)}\, dx = \int \frac{2}{u(2u - 1)}\, du$$

(3)

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)}\, dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)