

Question Number	Scheme	
4. (a)	$\frac{2x-1}{(x-1)(2x-3)} = \frac{A}{(x-1)} + \frac{B}{(2x-3)}$ <p>$2x-1 \equiv A(2x-3) + B(x-1)$</p> <p>Let $x = \frac{3}{2}$, $2 = B\left(\frac{1}{2}\right) \Rightarrow B = 4$</p> <p>Let $x = 1$, $1 = A(-1) \Rightarrow A = -1$</p> <p>giving $\frac{-1}{(x-1)} + \frac{4}{(2x-3)}$</p>	Forming this identity. NB: A & B are not assigned in this question M1
		either one of $A = -1$ or $B = 4$. both correct for their A, B. A1 A1
		[3]
(b) & (c)	$\begin{aligned} \frac{dy}{y} &= \int \frac{(2x-1)}{(2x-3)(x-1)} dx \\ &= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)} dx \end{aligned}$ <p>$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$</p> <p>$y = 10, x = 2$ gives $c = \ln 10$</p> <p>$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$</p> <p>$\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$</p> <p>$\ln y = \ln\left(\frac{(2x-3)^2}{(x-1)}\right) + \ln 10 \text{ or}$</p> <p>$\ln y = \ln\left(\frac{10(2x-3)^2}{(x-1)}\right)$</p> <p>$y = \frac{10(2x-3)^2}{(x-1)}$</p>	Separates variables as shown Can be implied Replaces RHS with their partial fraction to be integrated. <i>At least</i> two terms in ln's <i>At least</i> two ln terms correct All three terms correct and '+ c' M1 A1 ✓ A1 [5]
		$c = \ln 10$ B1
		M1
		A1 aef
		[4]
		12 marks

Question Number	Scheme	Marks
<p>Aliter 4. (b) & (c)</p> <p>Way 2</p>	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)} dx$ $\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$ <p><i>See below for the award of B1</i></p> $\ln y = -\ln(x-1) + \ln(2x-3)^2 + c$ $\ln y = \ln \left(\frac{(2x-3)^2}{x-1} \right) + c$ $\ln y = \ln \left(\frac{A(2x-3)^2}{x-1} \right) \quad \text{where } c = \ln A$ $\text{or } e^{\ln y} = e^{\ln \left(\frac{A(2x-3)^2}{x-1} \right) + c} = e^{\ln \left(\frac{(2x-3)^2}{x-1} \right)} e^c$ $y = \frac{A(2x-3)^2}{(x-1)}$ $y = 10, x = 2 \text{ gives } A = 10$ $y = \frac{10(2x-3)^2}{(x-1)}$	<p>Separates variables as shown Can be implied</p> <p>Replaces RHS with their partial fraction to be integrated.</p> <p><i>At least</i> two terms in ln's <i>At least</i> two ln terms correct All three terms correct and '+ c'</p> <p><i>decide to award B1 here!!</i></p> <p>Using the power law for logarithms</p> <p>Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.</p> <p>A = 10 for B1</p> <p>y = $\frac{10(2x-3)^2}{(x-1)}$ or aef & isw</p>

Note: The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

Question Number	Scheme	Marks
Aliter (b) & (c) Way 3	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{(x-1)} + \frac{2}{(x-\frac{3}{2})} dx$ $\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + c$ $y = 10, x = 2 \text{ gives } c = \underline{\ln 10 - 2\ln(\frac{1}{2})} = \underline{\ln 40}$ $\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + \ln 40$ $\ln y = -\ln(x-1) + \ln(x-\frac{3}{2})^2 + \ln 10$ $\ln y = \ln\left(\frac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40 \text{ or}$ $\ln y = \ln\left(\frac{40(x-\frac{3}{2})^2}{(x-1)}\right)$ $y = \underline{\frac{40(x-\frac{3}{2})^2}{(x-1)}}$	Separates variables as shown Can be implied B1

Replaces RHS with their partial fraction to be integrated.

At least two terms in ln's
At least two ln terms correct
All three terms correct and '+ c'

M1

M1 √

A1

A1

[5]

$$c = \ln 10 - 2\ln(\frac{1}{2}) \text{ or } c = \ln 40$$

B1 oe

Using the power law for logarithms

M1

Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.

M1

$$y = \underline{\frac{40(x-\frac{3}{2})^2}{(x-1)}} \text{ or aef. isw}$$

A1 aef

[4]

Note: Please mark parts (b) and (c) together for any of the three ways.

2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \quad t \geq 0.$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

$$\frac{dI}{dt} = -16(0.5)^t \ln(0.5)$$

$$t=3 \quad \frac{dI}{dt} = -16(0.5)^3 \ln(0.5) \\ = -2 \ln 0.5$$

$$\frac{dI}{dt} = \ln 4$$

Question Number	Scheme	Marks
3.	<p>(a) $\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$</p> $5x+3 = A(x+2) + B(2x-3)$ <p>Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B.</p> $A = 3, B = 1$ <p>If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.</p> <p>(b) $\int \frac{5x+3}{(2x-3)(x+2)} dx = \frac{3}{2} \ln(2x-3) + \ln(x+2)$</p> $\left[\dots \right]_2^6 = \frac{3}{2} \ln 9 + \ln 2$ $= \ln 54$	M1 A1, A1 (3)
		M1 A1ft M1 A1 cao A1 (5) [8]

4.

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(a) Find the values of the constants A , B and C .

(4)

(b) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)} dx$ is $2 + \ln k$, giving the value of the constant k .

(6)

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv \frac{A}{2x+1} + \frac{B}{2x-1}$$

$$2(4x^2+1) = A(2x+1)(2x-1) + B(2x-1) + C(2x+1)$$

Equate x^2 :- $8x^2 \equiv A \cdot 4x^2$

$$\underline{A = 2}$$

$$x = 1/2 \Rightarrow 4 \equiv 2C$$

$$\underline{2 = C}$$

$$x = -1/2 \Rightarrow 4 \equiv -2B$$

$$\underline{-2 = B}$$

$$\Rightarrow \frac{2}{2x+1} - \frac{2}{2x-1}$$

b) $\int_1^2 \frac{2}{2x+1} - \frac{2}{2x-1} dx$

$$= \left[2x - \ln|2x+1| + \ln|2x-1| \right]_1^2$$

$$= (4 - \ln 5 + \ln 3) - (2 - \ln 3)$$

$$= 2 + 2\ln 3 - \ln 5$$

$$= 2 + \ln(9/5)$$



7. (a) Express $\frac{2}{4-y^2}$ in partial fractions.

(3)

- (b) Hence obtain the solution of

$$2 \cot x \frac{dy}{dx} = (4-y^2)$$

for which $y=0$ at $x=\frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)

$$\text{a)} \quad \frac{2}{4-y^2} = \frac{2}{(2-y)(2+y)} = \frac{A}{2-y} + \frac{B}{2+y}$$

$$2 \equiv A(2+y) + B(2-y)$$

$$y=-2 \quad 2=4B \quad \text{and when } y=2 \quad 2=4A$$

$$B=\frac{1}{2}$$

$$A=\frac{1}{2}$$

$$\equiv \frac{1}{2(2-y)} + \frac{1}{2(2+y)}$$

$$\text{b)} \quad 2 \cot x \frac{dy}{dx} = (4-y^2) \quad \text{separate variables.}$$

$$\int \frac{2}{(4-y^2)} dy = \int \frac{1}{\cot x} dx$$

$$\int \frac{1}{2(2-y)} + \frac{1}{2(2+y)} dy = \int \tan x dx$$



Question 7 continued

$$-\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) + c$$

$$y = e^x \quad x = \pi/3$$

$$-\frac{1}{2} \ln 2 + \frac{1}{2} \ln 2 = \ln \left(\frac{1}{\cos(\pi/3)} \right) + c$$

$$0 = \ln 2 + c$$

$$c = -\ln 2$$

$$-\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) - \ln 2$$

$$\frac{1}{2} \ln \left(\frac{2+y}{2-y} \right) = \ln \left(\frac{\sec x}{2} \right)$$

$$\ln \left(\frac{2+y}{2-y} \right) = 2 \ln \left(\frac{\sec x}{2} \right)$$

$$\ln \left(\frac{2+y}{2-y} \right) = \ln \left(\frac{\sec^2 x}{4} \right)$$

$$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$$

$$\sec^2 x = \frac{8+4y}{2-y}$$



$$3. \quad f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants A , B and C .

(4)

(b) (i) Hence find $\int f(x) dx$.

(3)

(ii) Find $\int_0^2 f(x) dx$ in the form $\ln k$, where k is a constant.

(3)

$$a) \quad 4 - 2x^2 = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$$

$$\text{When } x = -1 \quad 6 = B(-1)(2) \quad \text{When } x = -3 \quad 10 = C(-5)(-2)$$

$$B = -3 \qquad \qquad \qquad C = 1$$

$$\text{When } x = -\frac{1}{2} \quad S = A\left(\frac{1}{2}\right)\left(2\frac{1}{2}\right)$$

$$A = 4$$

$$f(x) = \frac{4}{2x+1} + \frac{3}{x+1} + \frac{1}{x+3}$$

b.)

$$I = \int \frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{2x+3} dx$$

$$= \left[\frac{4}{2} \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) \right]_0^2 + C$$

(ii)

$$\left[2 \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) \right]_0^2$$

$$= (2\ln 5 - 3\ln 3 + \ln 5) - (2\ln(1) - 3\ln(1) + \ln 3)$$

$$= 3 \ln 5 - 3 \ln 3 - \ln 3$$

$$= 3 \ln 5 - 4 \ln 3$$

$$= \ln \left(\frac{5^3}{3^4} \right)$$

$$= \ln\left(\frac{125}{81}\right)$$



6.

$$f(\theta) = 4 \cos^2 \theta - 3 \sin^2 \theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2} \cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta) d\theta$.

(7)

6(a)

$$\cos^2 \theta = \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) \quad \sin^2 \theta = \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right)$$

$$f(\theta) = 4 \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) - 3 \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right)$$

$$= \frac{4}{2} + \frac{4}{2} \cos 2\theta - \frac{3}{2} + \frac{3}{2} \cos 2\theta$$

$$= \underline{\frac{1}{2} + \frac{7}{2} \cos 2\theta}$$

(b)

$$I = \int_0^{\pi/2} \theta \left(\frac{1}{2} + \frac{7}{2} \cos 2\theta \right) d\theta$$

$$= \int_0^{\pi/2} \underline{\theta} \frac{1}{2} + \frac{7}{2} \theta \cos 2\theta d\theta$$

θ is just ∞

$$= \int_0^{\pi/2} \underline{\frac{\theta}{2}} + \int_0^{\pi/2} \underline{\frac{7}{2} \theta \cos 2\theta} d\theta$$

$$= \left[\frac{\theta^2}{4} \right]_0^{\pi/2} + \int_0^{\pi/2} \frac{7}{2} \theta \cos 2\theta \, d\theta \quad \leftarrow \text{by parts}$$

$$u = \theta \quad \frac{du}{d\theta} = 1$$

$$v = \frac{1}{2} \sin 2\theta \quad \frac{dv}{d\theta} = \cos 2\theta$$

$$= \left[\frac{\theta^2}{4} \right]_0^{\pi/2} + \frac{7}{2} \left[\left(\frac{1}{2} \theta \sin 2\theta \right)_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2} \sin 2\theta \, d\theta \right]$$

$$= \left[\frac{\theta^2}{4} \right]_0^{\pi/2} + \frac{7}{2} \left[\left(\frac{1}{2} \theta \sin 2\theta \right)_0^{\pi/2} + \left[\frac{\cos 2\theta}{4} \right]_0^{\pi/2} \right]$$

$$= \frac{\pi^2}{16} + \frac{7}{2} \left[(0 - 0) + \left(-\frac{1}{4} - \frac{1}{4} \right) \right]$$

$$= \frac{\pi^2}{16} + \frac{7}{2} \left(-\frac{2}{4} \right)$$

$$= \frac{\pi^2}{16} - \frac{7}{4}$$

1. $\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$

Find the values of the constants A , B and C .

(4)

$$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$$

when $x = 1$

$$9 = 3B$$

$$\underline{B = 3}$$

when $x = -\frac{1}{2}$

$$\frac{9}{4} = \left(-\frac{3}{2}\right)^2 C$$

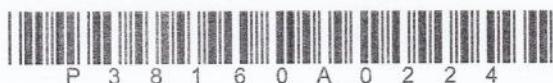
$$\underline{C = 1}$$

when $x = 0$

$$0 = -A + B + C$$

$$0 = -A + 3 + 1$$

$$\underline{A = 4}$$



1. $f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$

(a) Find the values of the constants A , B and C .

(4)

(b) (i) Hence find $\int f(x) dx$.

(ii) Find $\int_1^2 f(x) dx$, leaving your answer in the form $a + \ln b$,
where a and b are constants.

(6)

a) $\frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$

$$\frac{1}{x(3x-1)^2} = \frac{A(3x-1)^2 + B(3x-1) + Cx}{x(3x-1)^2}$$

so $1 \equiv A(3x-1)^2 + B(3x-1)x + Cx$

when $x = 0$ $1 \equiv A(-1)^2$
 $\underline{A = 1}$

when $x = \frac{1}{3}$ $1 \equiv \frac{1}{3}C$
 $\Rightarrow \underline{C = 3}$

when $x = 1$ $1 \equiv 4A + 2B + C$

$$1 \equiv 4(1) + 2B + 3$$

$$2B = -6$$

$$\underline{B = -3}$$



Question 1 continued

$$(b) \int \frac{1}{x} + \frac{-3}{(3x-1)} + \frac{3}{(3x-1)^2} dx$$

$$= \int \frac{1}{x} - \frac{3}{(3x-1)} + 3(3x-1)^{-2} dx$$

$$= \ln x - \ln(3x-1) - (3x-1)^{-1} + c$$

$$\text{iii) } \left[\ln x - \ln(3x-1) - (3x-1)^{-1} \right]_1^2$$

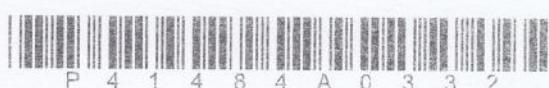
$$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$$

$$= \ln 2 - \ln 5 - \frac{1}{5} + \ln 2 + \frac{1}{2}$$

$$= 2\ln 2 - \ln 5 + \frac{3}{10}$$

$$= \ln 4 - \ln 5 + \frac{3}{10}$$

$$= \ln \left(\frac{4}{5} \right) + \frac{3}{10}$$



3. Express $\frac{9x^2 + 20x - 10}{(x+2)(3x-1)}$ in partial fractions. (4)

Power of numerator and denominator equal so improper fraction. First divide to change into number and proper fraction.

$$\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} = \frac{9x^2 + 20x - 10}{3x^2 + 5x - 2}$$

$$9x^2 + 20x - 10 = 3(3x^2 + 5x - 2) + 5x - 4$$

So

$$\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} = 3 + \frac{5x - 4}{(x+2)(3x-1)}$$

$$\frac{5x - 4}{(x+2)(3x-1)} = \frac{A}{x+2} + \frac{B}{3x-1}$$

$$5x - 4 = A(3x-1) + B(x+2)$$

$$\text{let } x = \frac{1}{3}, \frac{5}{3} - 4 = \frac{7}{3}B$$

$$-\frac{1}{3} = \frac{7}{3}B$$

$$B = -1$$

$$\text{let } x = -2, -10 - 4 = -7A$$

$$-14 = -7A$$

$$A = 2$$

$$\therefore \frac{9x^2 + 20x - 10}{(x+2)(3x-1)} = 3 + \frac{2}{x+2} - \frac{1}{3x-1}$$



5. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)} dx = \int \frac{2}{u(2u - 1)} du \quad (3)$$

- (b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)} dx = 2 \ln \left(\frac{a}{b} \right)$$

where a and b are integers to be determined.

(7)

a) $x = u^2$
 $\sqrt{x} = u$

$$\frac{dx}{du} = 2u$$

$$dx = 2u du$$

$$\int \frac{1}{x(2\sqrt{x} - 1)} dx = \int \frac{1}{u^2(2u - 1)} \times 2u du$$

$$= \int \frac{2}{u(2u - 1)} du \quad \text{as required}$$

b) limits $x = 9$, $u = \sqrt{9} = 3$
 $x = 1$, $u = \sqrt{1} = 1$

$$\int_1^3 \frac{2}{u(2u-1)} du \quad \text{split into partial fractions}$$

$$\frac{2}{u(2u-1)} = \frac{A}{u} + \frac{B}{2u-1}$$

$$2 = A(2u-1) + Bu$$

$$\text{when } u=0, 2 = -A \quad \therefore A = -2$$

$$\text{when } u=\frac{1}{2}, 2 = \frac{1}{2}B \quad \therefore B = 4$$

$$\int_1^3 -\frac{2}{u} + \frac{4}{2u-1} du$$



C4 June 2013

5b (cont)

$$\begin{aligned} &= \int_1^3 -\frac{2}{u} du + \int \frac{4}{2u-1} du \\ &= \left[-2\ln u + \frac{4}{2} \ln(2u-1) \right]_1^3 \\ &= (-2\ln 3 + 2\ln 5) - (2\ln 1 + 2\ln 1) \\ &= 2\ln 5 - 2\ln 3 = 0 \\ &< 2(\ln 5 - \ln 3) \\ &= 2 \ln \left(\frac{5}{3} \right) \\ &\underline{\underline{\underline{\quad}}} \end{aligned}$$