

4. Solve the equation

$$5^x = 17,$$

giving your answer to 3 significant figures.

(3)

$$\begin{aligned}5^x &= 17 \\ \log 5^x &= \log 17 \\ x \log 5 &= \log 17 \\ x &= \frac{\log 17}{\log 5}\end{aligned}$$

$$\begin{aligned}x &= 1.7603744 \\ x &= 1.76 \quad (3 \text{ sf})\end{aligned}$$

Q4

(Total 3 marks)



5. Given that  $a$  and  $b$  are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

$$\begin{aligned} a &= 3b \\ \log_3 a + \log_3 b &= 2 \end{aligned}$$

①  
②

$$\left. \begin{aligned} \log_c A + \log_c B \\ = \log_c AB \end{aligned} \right\} (6)$$

sub ① into ②

$$\therefore \log_3 3b + \log_3 b = 2$$

$$\therefore \log_3 3b^2 = 2$$

$$\therefore 3b^2 = 3^2$$

$$\therefore 3b^2 = 9$$

$$\therefore b^2 = 3$$

$$\therefore b = \sqrt{3}$$

$$\begin{aligned} \text{Sub } b = \sqrt{3} \text{ into ①} \\ a = 3\sqrt{3}, \quad b = \sqrt{3} \end{aligned}$$



4. Given that  $0 < x < 4$  and

$\log_5(4-x) - 2\log_5 x = 1$ ,  
find the value of  $x$ .

(6)

$$\log_5(4-x) - 2\log_5 x = 1$$

$$\log_5(4-x) - \log_5 x^2 = 1$$

$$\log_5 \left( \frac{4-x}{x^2} \right) = 1$$

Take antilog

$$\frac{4-x}{x^2} = 5^1 \quad \text{base}$$

$$4-x = 5x^2$$

$$0 = 5x^2 + x - 4$$

$$0 = (5x-4)(x+1)$$

$$\text{Either } 5x-4=0 \quad \text{or } x+1=0$$

$$x = \frac{4}{5}$$

$$x = -1$$

as  $0 < x < 4$

$$x = \frac{4}{5}$$



5. (a) Find the positive value of  $x$  such that

$$\log_x 64 = 2 \quad (2)$$

- (b) Solve for  $x$

$$\log_2(11 - 6x) = 2 \log_2(x - 1) + 3 \quad (6)$$

a)  $\log_x 64 = 2$       |  $8^2 = 64$   
 $x = 8$

b)  $\log_2(11 - 6x) = 2 \log_2(x - 1) + 3$   
 $\log_2(11 - 6x) - 2 \log_2(x - 1) = 3$   
 $\log_2(11 - 6x) - \log_2(x - 1)^2 = 3$   
 $\frac{\log_2(11 - 6x)}{(x - 1)^2} = 3$   
 $\frac{(11 - 6x)}{(x - 1)^2} = 2^3$

$$\begin{aligned} 11 - 6x &= 8(x - 1)^2 \\ 11 - 6x &= 8(x^2 - 2x + 1) \\ 11 - 6x &= 8x^2 - 16x + 8 \\ 0 &= 8x^2 - 10x - 3 \\ 0 &= (4x + 1)(2x - 3) \end{aligned}$$

Either  $4x + 1 = 0$  or  $2x - 3 = 0$   
 $x = -\frac{1}{4}$  or  $x = \frac{3}{2}$



8. (a) Sketch the graph of  $y = 7^x$ ,  $x \in \mathbb{R}$ , showing the coordinates of any points at which the graph crosses the axes.

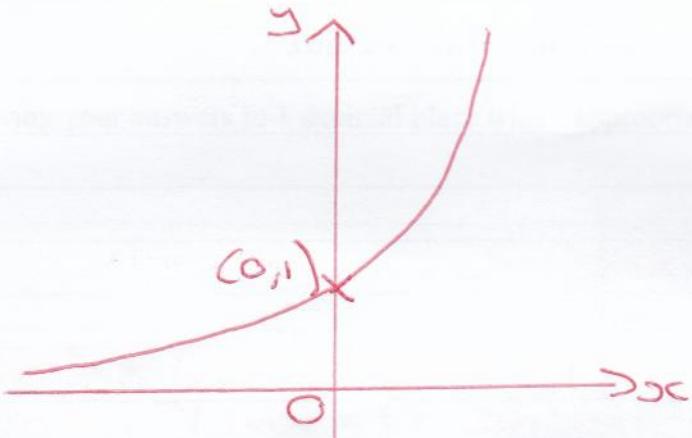
(2)

- (b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

(6)



b) Let  $y = 7^x$

$$y^2 - 4(y) + 3 = 0$$

$$(y - 3)(y - 1) = 0$$

Either  $y = 3$  or  $y = 1$

$$\therefore 3 = 7^x \quad \left| \begin{array}{l} \text{or } 1 = 7^x \\ x = 0 \end{array} \right.$$

$$\log 3 = \log 7^x$$

$$\log 3 = x \log 7$$

$$x = \frac{\log 3}{\log 7}$$

$$x = 0.5645$$

$$x = 0.56 \text{ (2dp)}$$

Two solutions are  $x = 0$  or  $x = 0.56$   
(2dp)



4. Given that  $y = 3x^2$ ,

(a) show that  $\log_3 y = 1 + 2 \log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2 \log_3 x = \log_3(28x - 9)$$

(3)

a)  $y = 3x^2$   
 $\log_3 y = \log_3 3x^2$   
 $\log_3 y = \log_3 3 + \log_3 x^2$   
 $\log_3 y = 1 + \log_3 x^2$   
 $\log_3 y = 1 + 2 \log_3 x$  (as required)

b)  $1 + 2 \log_3 x = \log_3(28x - 9)$   
 $\log_3 y = \log_3(28x - 9)$

$$\log_3 3x^2 = \log_3 (28x - 9)$$

$$\begin{aligned} 3x^2 &= 28x - 9 \\ 3x^2 - 28x + 9 &= 0 \\ (3x - 1)(x - 9) &= 0 \\ x = \frac{1}{3} \text{ or } x &= 9 \end{aligned}$$



3. (i) Write down the value of  $\log_6 36$ .

(1)

- (ii) Express  $2 \log_a 3 + \log_a 11$  as a single logarithm to base  $a$ .

(3)

(i) As  $6^2 = 36$  then  
 $\log_6 36 = 2$

(ii)  $2 \log_a 3 + \log_a 11$

$$\begin{aligned} &= \log_a 3^2 + \log_a 11 \\ &= \log_a (9 \times 11) \\ &= \log_a 99 \end{aligned}$$

Q3

(Total 4 marks)



6. (a) Find, to 3 significant figures, the value of  $x$  for which  $8^x = 0.8$ .

(2)

- (b) Solve the equation

$$2 \log_3 x - \log_3 7x = 1.$$

(4)

a)  $8^x = 0.8$

$$\log 8^x = \log 0.8$$

$$x \log 8 = \frac{\log 0.8}{\log 8}$$

$$x = -0.1073093$$

$$x = -0.107 \text{ (3 sf)}$$

b)

$$2 \log_3 x - \log_3 7x = 1$$

$$\log_3 x^2 - \log_3 7x = 1$$

$$\log_3 \frac{x^2}{7x} = 1$$

$$\log_3 \frac{x^2}{7x} = \log_3 3 \quad \left| \log_3 3 = 1 \right.$$

$$\therefore \frac{x^2}{7x} = 3$$

$$x = 21$$



4. (a) Find, to 3 significant figures, the value of  $x$  for which  $5^x = 7$ .

(2)

- (b) Solve the equation  $5^{2x} - 12(5^x) + 35 = 0$ .

(4)

a)  $5^x = 7$   
 $\therefore \log 5^x = \log 7$   
 $\therefore x \log 5 = \log 7$   
 $\therefore x = \frac{\log 7}{\log 5}$   
 $\therefore x = 1.209062$   
 $\therefore x = 1.21 \quad (3 \text{ sf})$

b)  $5^{2x} - 12(5^x) + 35 = 0$

$$\therefore (5^x)^2 - 12(5^x) + 35 = 0$$

Let  $y = 5^x$

$$\therefore y^2 - 12y + 35 = 0$$
$$\therefore (y - 7)(y - 5) = 0$$
$$\therefore y - 7 = 0 \quad \text{or} \quad y - 5 = 0$$
$$\therefore y = 7 \quad \text{or} \quad y = 5$$
$$\therefore 5^x = 7 \quad \text{or} \quad 5^x = 5$$
$$\therefore x = 1.21 \quad (3 \text{ sf}) \quad \text{or} \quad x = 1$$

from part (a)



8. (a) Find the value of  $y$  such that

$$\log_2 y = -3$$

(2)

- (b) Find the values of  $x$  such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)

a)  $\log_2 y = -3$

$$\text{So } y = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

b)  $\log_2 32 = 5$  as  $2^5 = 32$   
 $\log_2 16 = 4$  as  $2^4 = 16$

$$\text{so } \frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

$$\frac{5+4}{\log_2 x} = \log_2 x$$

$$9 = (\log_2 x)^2$$

$$\therefore \log_2 x = 3 \quad \text{or} \quad \log_2 x = -3$$

$$x = 2^3 \quad \quad \quad x = 2^{-3}$$

$$x = 8 \quad \quad \quad x = \frac{1}{8}$$



7. (a) Given that

$$2\log_3(x-5) - \log_3(2x-13) = 1,$$

show that  $x^2 - 16x + 64 = 0$ .

(5)

- (b) Hence, or otherwise, solve  $2\log_3(x-5) - \log_3(2x-13) = 1$ .

(2)

a)  $2\log_3(x-5) - \log_3(2x-13) = 1$

$$\log_3 \frac{(x-5)^2}{2x-13} = 1$$

$$\frac{(x-5)^2}{2x-13} = 3^1$$

$$(x-5)^2 = 3(2x-13)$$

$$x^2 - 10x + 25 = 6x - 39$$

$$x^2 - 10x - 6x + 25 + 39 = 0$$

$$x^2 - 16x + 64 = 0 \quad \text{as required}$$

b)

Factorise  $x^2 - 16x + 64 = 0$

$$(x-8)(x-8) = 0$$

$$x = 8 \quad \text{or} \quad x = 8$$



3. Find, giving your answer to 3 significant figures where appropriate, the value of  $x$  for which

(a)  $5^x = 10$ , (2)

(b)  $\log_3(x-2) = -1$ . (2)

$$\text{a)} \quad 5^x = 10 \\ \log 5^x = \log 10$$

$$x \log 5 = \log 10$$

$$x = \frac{\log 10}{\log 5}$$

$$x = 1.43 \quad (\text{3sf})$$

b)  $\log_3(x-2) = -1$

$$x-2 = 3^{-1} \\ x-2 = \frac{1}{3}$$

$$x = 2\frac{1}{3}$$



2. Find the values of  $x$  such that

$$2 \log_3 x - \log_3(x-2) = 2$$

(5)

$$2 \log_3 x - \log_3(x-2) = 2$$

$$\log_3 x^2 - \log_3(x-2) = 2$$

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

$$\frac{x^2}{x-2} = 3^2$$

$$x^2 = 9(x-2)$$

$$x^2 = 9x - 18$$

$$x^2 - 9x + 18 = 0$$

$$(x-6)(x-3) = 0$$

$$x = 6 \quad \text{or} \quad x = 3$$



6. Given that

$$2 \log_2(x+15) - \log_2 x = 6$$

(a) Show that

$$x^2 - 34x + 225 = 0$$

(5)

(b) Hence, or otherwise, solve the equation

$$2 \log_2(x+15) - \log_2 x = 6$$

(2)

a)  $2 \log_2(x+15) - \log_2 x = 6$

$$\log_2(x+15)^2 - \log_2 x = 6$$

$$\log_2 \frac{(x+15)^2}{x} = 6$$

$$\frac{(x+15)^2}{x} = 2^6$$

$$\frac{(x+15)^2}{x} = 64$$

$$(x+15)^2 = 64x$$

$$x^2 + 30x + 225 = 64x$$

$$x^2 + 30x - 64x + 225 = 0$$

$$x^2 - 34x + 225 = 0 \quad \text{as required}$$

b) solve  $x^2 - 34x + 225 = 0$

$$(x-25)(x-9) = 0$$

$$\underline{\underline{x=25}} \quad \text{or} \quad \underline{\underline{x=9}}$$



3.

$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where  $a$  is a constant.

Given that  $(x - 3)$  is a factor of  $f(x)$ ,

(a) show that  $a = -9$

(2)

(b) factorise  $f(x)$  completely.

(4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

(c) find the values of  $y$  that satisfy  $g(y) = 0$ , giving your answers to 2 decimal places where appropriate.

(3)

a) If  $(x - 3)$  is a factor,  $f(3) = 0$

$$0 = 2 \times 3^3 - 5 \times 3^2 + 3a + 18$$

$$0 = 54 - 45 + 3a + 18$$

$$-54 + 45 - 18 = 3a$$

$$-27 = 3a$$

$$\underline{\underline{a = -9}}$$

as required

b)

$$\begin{array}{r} 2x^2 + x - 6 \\ \hline x - 3 | 2x^3 - 5x^2 - 9x + 18 \\ \quad - \underline{2x^3 - 6x^2} \\ \quad \quad \quad \downarrow \\ \quad \quad \quad x^2 - 9x \\ \quad \quad \quad - \underline{x^2 - 3x} \\ \quad \quad \quad \downarrow \\ \quad \quad \quad -6x + 18 \\ \quad \quad \quad -6x + 18 \\ \hline \quad \quad \quad 0 \end{array}$$

$$2x^2 + x - 6 = (2x - 3)(x + 2)$$

$$f(x) = (x - 3)(x + 2)(2x - 3)$$

fully factorised



$$3c) g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

$$\text{let } x = 3^y$$

and then it matches

$$f(x) = 2x^3 - 5x^2 - 9x + 18$$

$$f(x) = (x-3)(x+2)(2x-3)$$

$$0 = (x-3)(x+2)(2x-3)$$

$$\text{solutions are } x=3, x=-2, x=\frac{3}{2}$$

$$\text{but } 3^y = x$$

$$3^y = 3 \quad , \quad \log 3^y = \log 3 \\ y \log 3 = \log 3 \\ y = \frac{\log 3}{\log 3} = 1$$

$$3^y = -2$$

$$\log 3^y = \log(-2) \\ y \log 3 = \log(-2)$$

$$y = \frac{\log(-2)}{\log 3} \quad (\text{error})$$

-no  
solution,  
can't have  
 $\log(-2)$ )

$$3^y = 1.5$$

$$y \log 3 = \log 1.5$$

$$y = \frac{\log 1.5}{\log 3} = 0.3690702$$

Solutions are  $y=1$  and  $y=0.37$  (2dp)

7. (i) Find the exact value of  $x$  for which

$$\log_2(2x) = \log_2(5x+4) - 3 \quad (4)$$

- (ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express  $y$  in terms of  $a$ .

Give your answer in its simplest form.

(i)  $\log_2(2x) = \log_2(5x+4) - 3$

$$3 = \log_2(5x+4) - \log_2(2x)$$

$$3 = \log_2\left(\frac{5x+4}{2x}\right)$$

$$2^3 = \frac{5x+4}{2x}$$

$$8 \times 2x = 5x+4$$

$$16x - 5x = 4$$

$$11x = 4$$

$$x = \frac{4}{11}$$

(ii)  $\log_a y + 3\log_a 2 = 5$

$$\log_a y + \log_a 2^3 = 5$$

$$\log_a 8y = 5$$

$$8y = a^5$$

$$y = \frac{a^5}{8}$$

