4. The point A $(-6, 4)$ and the point B $(8, -3)$ lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.
(4)

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.
(3)

a) $A\ (-6, 4)$ $\quad B\ (8, -3)$

Gradient $= \dfrac{4-(-3)}{-6-8} = \dfrac{7}{-14} = -\dfrac{1}{2}$

Equation of line is. Use $A(-6,4)$

$y - y_1 = m\,(x - x_1)$

$y - 4 = -\frac{1}{2}(x - (-6))$

$y - 4 = -\frac{1}{2}x - 3$

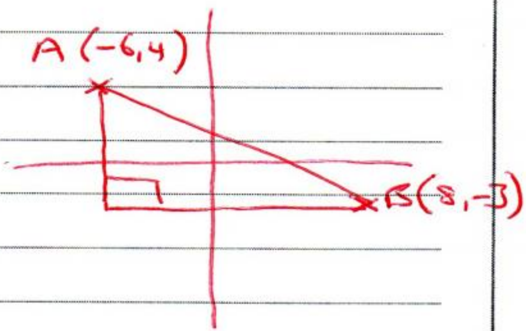
$\times$ through by 2

$2y - 8 = -x - 6$

$2y + x - 8 + 6 = 0$

$x + 2y - 2 = 0$

b) Using Pythagoras



$AB = \sqrt{(-6-8)^2 + (4-(-3))^2}$

$= \sqrt{(-14)^2 + 7^2}$

$= \sqrt{196 + 49}$

$= \sqrt{245}$

$= \sqrt{49} \times \sqrt{5}$

$AB = 7\sqrt{5}$

14
$14 \times$
56
$140 +$
196

49
$5\ \overline{)\,245}$

Q4

(Total 7 marks)

Leave blank

10. The curve C has equation

$$y = (x+3)(x-1)^2.$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

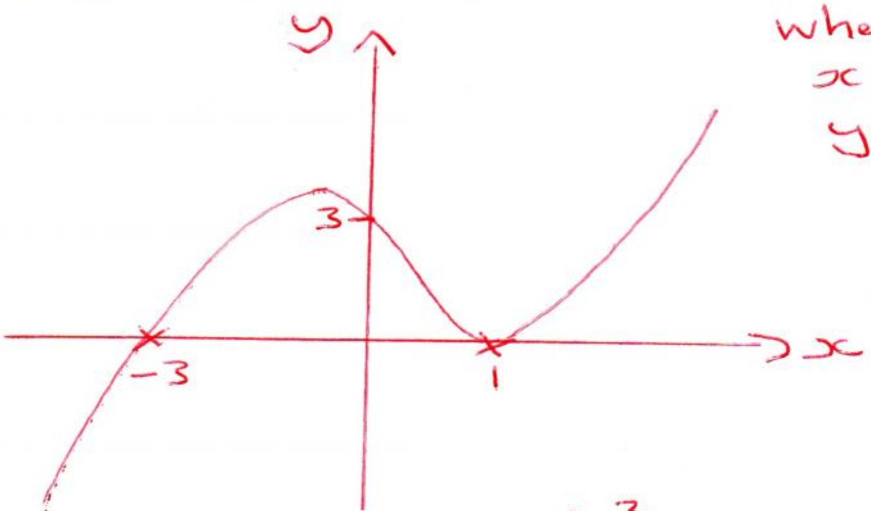
(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

a)



when $x = 0$, $y = 3$

b) $y = (x+3)(x-1)^2$
$= (x+3)(x^2 - 2x + 1)$
$= x^3 - 2x^2 + x + 3x^2 - 6x + 3$
$= x^3 + x^2 - 5x + 3$
(where $k = 3$)

c) Gradient, need to find $\frac{dy}{dx} = 3$

$\frac{dy}{dx} = 3x^2 + 2x - 5$

$\therefore 3 = 3x^2 + 2x - 5$
$0 = 3x^2 + 2x - 8$
$0 = (3x-4)(x+2)$

Either $3x - 4 = 0$ or $x + 2 = 0$
$x = \frac{4}{3}$ $\qquad$ $x = -2$

Leave blank

10. The line l_1 passes through the point A (2, 5) and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$.

(3)

The point B has coordinates (–2, 7).

(b) Show that B lies on l_1.

(1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$

(4)

a) $y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 2)$

$y - 5 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 1 + 5$

$y = -\frac{1}{2}x + 6$ (1)

b) put $x = -2$ in (1)

$y = -\frac{1}{2} \times (-2) + 6$

$y = 1 + 6$

$y = 7$

So point $B(-2, 7)$ lies on L_1

c) $A(2, 5)$ $B(-2, 7)$

$AB = \sqrt{(2 - -2)^2 + (5 - 7)^2}$

$= \sqrt{4^2 + (-2)^2}$

$= \sqrt{16 + 4}$

$= \sqrt{20}$

$= \sqrt{4}\sqrt{5}$

$= 2\sqrt{5}$ square units

10d) C lies on l_1 with x-coordinate p

To find y-coordinate, put it in ①

$y = -\frac{1}{2}p + 6$

$\therefore$ $C\left(p, \left(-\frac{1}{2}p+6\right)\right)$ $\quad$ $A(2,5)$

$AC = \sqrt{(2-p)^2 + \left(5 - \left(-\frac{1}{2}p+6\right)\right)^2}$

$AC^2 = (2-p)^2 + \left(5 + \frac{1}{2}p - 6\right)^2$

$5^2 = (2-p)^2 + \left(+\frac{1}{2}p - 1\right)^2$

$25 = 4 - 4p + p^2 + \frac{1}{4}p^2 - p + 1$

$0 = p^2 + \frac{1}{4}p^2 - 5p + 5 - 25$

$0 = \frac{5}{4}p^2 - 5p - 20$

$\times$ through by $\frac{4}{5}$

$0 = p^2 - 4p - 16$ $\quad$ as required

Leave blank

9. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point A $(1, 4)$ lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(2)

a) $2(4) - 3(1) - k = 0$

$8 - 3 = k$

$k = 5$

b) $2y = 3x - 5$

$y = \frac{3}{2}x - \frac{5}{2}$

$y = mx + c$ so gradient $= \frac{3}{2}$

$\frac{dy}{dx} = \frac{3}{2}$

c) Gradient of L_2 is $-\frac{2}{3}$ as perpendicular

- goes through $A(1,4)$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{2}{3}(x - 1)$

$y - 4 = -\frac{2}{3}x + \frac{2}{3}$

$3y - 12 = -2x + 2$

$2x + 3y - 14 = 0$ is equation of L_2

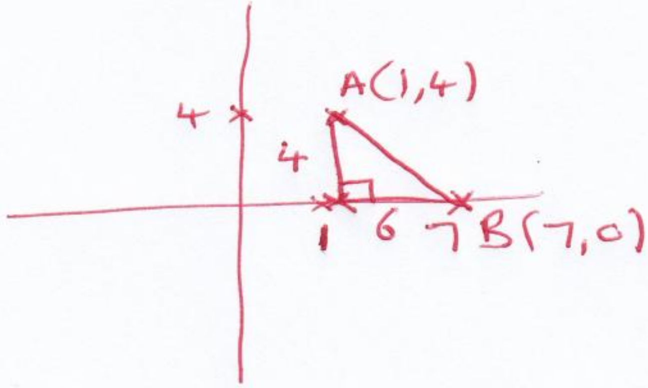
C1 JAN 2011

9d) at x-axis $y = 0$
in equation for l_2

$2x - 14 = 0$

$x = 7$ $\qquad$ $\underline{\underline{B(7,0)}}$

e) A is $(1,4)$ $\qquad$ $B(7,0)$



By Pythagoras Theorem

$AB = \sqrt{4^2 + 6^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$ units

(or $\sqrt{52} = \sqrt{4}\sqrt{13}$
$= 2\sqrt{13}$ units)

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6.

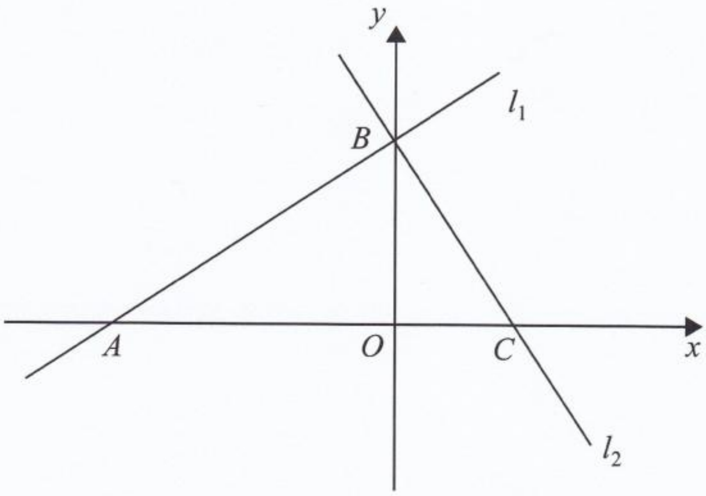

Figure 1

The line l_1 has equation $2x-3y+12=0$

(a) Find the gradient of l_1.

(1)

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2.

(3)

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC.

(4)

a) $2x - 3y + 12 = 0$

$2x + 12 = 3y$

$\frac{2}{3}x + 4 = y$

gradient of l_1 is $\frac{2}{3}$

b) l_1 meets y-axis at $B(0,4)$

gradient of $l_2 = -\frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{3}{2}(x - 0)$

$y - 4 = -\frac{3}{2}x$

$y = -\frac{3}{2}x + 4$

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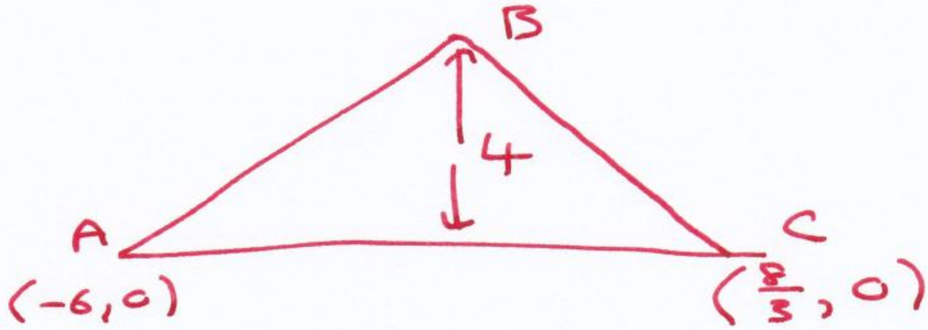
6c) l_2 meets x-axis $(y=0)$

$$0 = -\frac{3}{2}x + 4$$

$$\frac{3}{2}x = 4$$

$$x = \frac{8}{3}$$



l_1 meets x axis at $y=0$

$$2x - 0 + 12 = 0$$

$$x = -6 \quad A \text{ is } (-6,0)$$

$$\text{Area } \triangle ABC = \frac{1}{2} \times \left(6 + \frac{8}{3}\right) \times 4$$

$$= 2 \times \left(\frac{18}{3} + \frac{8}{3}\right)$$

$$= 2 \times \frac{26}{3} = \frac{52}{3} \text{ square units}$$

Leave blank

11. The line l_1 has equation $y = 3x + 2$ and the line l_2 has equation $3x + 2y - 8 = 0$.

(a) Find the gradient of the line l_2.

(2)

The point of intersection of l_1 and l_2 is P.

(b) Find the coordinates of P.

(3)

The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively.

(c) Find the area of triangle ABP.

(4)

a) $3x + 2y - 8 = 0$ | $y = mx + 1$

$\therefore 2y = -3x + 8$

$\therefore y = -\frac{3}{2}x + 4$

$\therefore$ gradient of $l_2 = -\frac{3}{2}$

b) For intersection, simultaneous equations

$y = 3x + 2$ ①

$3x + 2y - 8 = 0$ ②

Sub ① into ②

$\therefore 3x + 2(3x+2) - 8 = 0$

$\therefore 3x + 6x + 4 - 8 = 0$

$\therefore 9x - 4 = 0$

$\therefore 9x = 4$

$\therefore x = \frac{4}{9}$

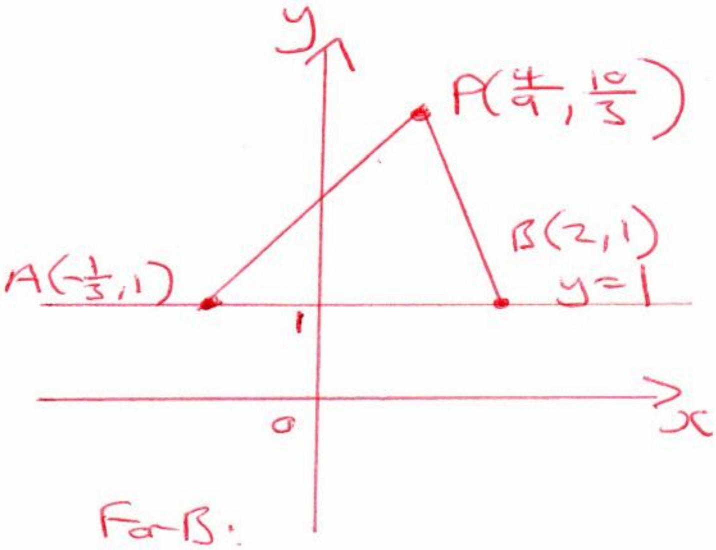
$\therefore$ Sub $x = \frac{4}{9}$ into ①

$\therefore y = \cancel{3}^{1}\left(\frac{4}{\cancel{9}_3}\right) + 2$

$y = \frac{10}{3}$

$\therefore P\left(\frac{4}{9}, \frac{10}{3}\right)$

11c)



For A:

Sub $y=1$ into L_1

$\therefore 1 = 3x + 2$

$\therefore -1 = 3x$

$\therefore x = -\frac{1}{3}$

$\therefore A\left(-\frac{1}{3}, 1\right)$

For B:

sub $y=1$ into L_2

$3x + 2\times(1) - 8 = 0$

$3x + 2 - 8 = 0$

$3x = 6$

$x = 2$

$\therefore B(2,1)$

Area $\triangle ABP = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times \left(2 - -\frac{1}{3}\right) \times \left(\frac{10}{3} - 1\right)$

$= \frac{1}{2} \times \frac{7}{3} \times \frac{7}{3}$

$= \frac{49}{18}$ units^2

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10.

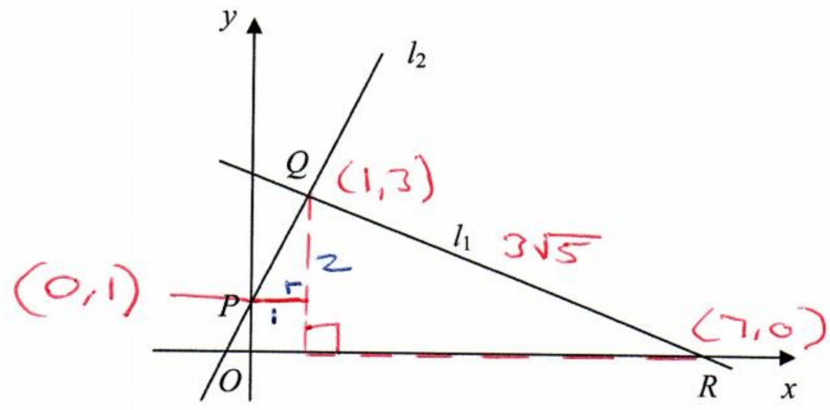

Figure 2

The points Q (1, 3) and R (7, 0) lie on the line l_1, as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(a) Find the value of a.

(3)

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2.

Find

(b) an equation for l_2,

(5)

(c) the coordinates of P,

(1)

(d) the area of ΔPQR.

Pythagoras Theorem

(4)

a) $QR^2 = 6^2 + 3^2$

$QR^2 = 36 + 9$

$QR^2 = 45$

$QR = \sqrt{45}$

$QR = \sqrt{9} \times \sqrt{5}$

$QR = 3\sqrt{5}$

$\equiv a\sqrt{5}$

where $a = 3$

10b) First work out gradient of QR (Line L_1)

$$= \frac{3-0}{1-7} = \frac{3}{-6} = -\frac{1}{2}$$

Gradient of L_2 is the negative reciprocal of this as it is perpendicular.

$\therefore$ Gradient of $L_2 = 2$

$\therefore$ Equation of L_2 is

$y - y_1 = m(x - x_1)$ as we know point $(1,3)$ is on line

$y - 3 = 2(x-1)$

$y - 3 = 2x - 2$

$y = 2x - 2 + 3$

$\therefore$ $y = 2x + 1$

c) coordinates of P

when $x = 0$, $y = 2 \times 0 + 1$

$y = 1$

coordinates of $P = (0,1)$

d) Use Pythagoras to get length PQ

$PQ^2 = 1^2 + 2^2$

$PQ^2 = 5$

$PQ = \sqrt{5}$

Area $\triangle PQR = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3\sqrt{5} \times \sqrt{5} = \frac{1}{2} \times 3 \times 5$

$= \frac{15}{2}$ sq units

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8.

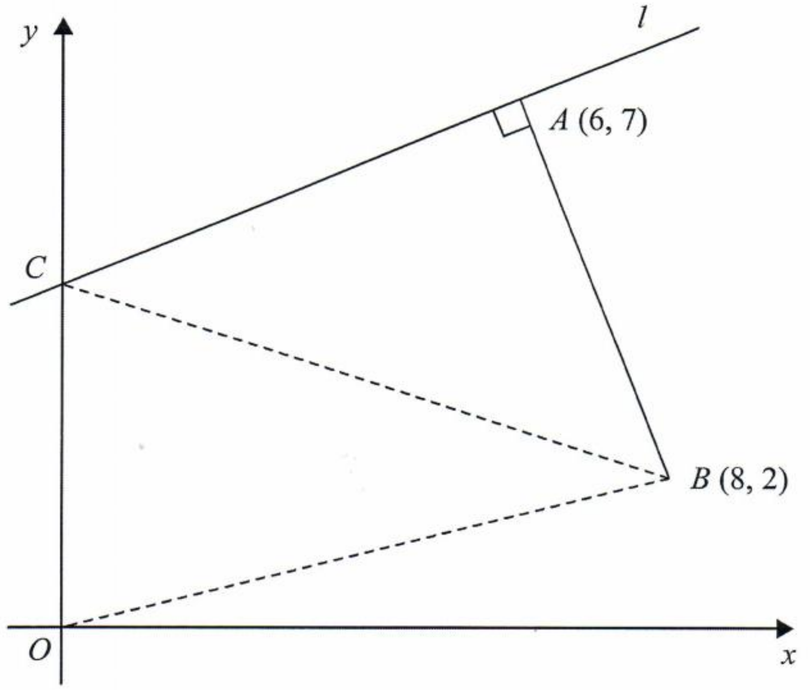


Figure 1

The points A and B have coordinates (6, 7) and (8, 2) respectively.

The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 1.

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.
(4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C,
(2)

(c) the area of ΔOCB, where O is the origin.
(2)

a) gradient of $AB = \frac{7-2}{6-8} = \frac{5}{-2} = -\frac{5}{2}$

As l is perpendicular to AB
gradient of l is $\frac{2}{5}$.

use A(6,7)

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{2}{5}(x - 6)$

(× through by 5)

$5y - 35 = 2(x - 6)$

$5y - 35 = 2x - 12$

$0 = 2x - 5y + 23$ (in form $ax+by+c=0$)

$2x - 5y + 23 = 0$

8b) Equation of l is $2x - 5y + 23 = 0$

at C, $x = 0$

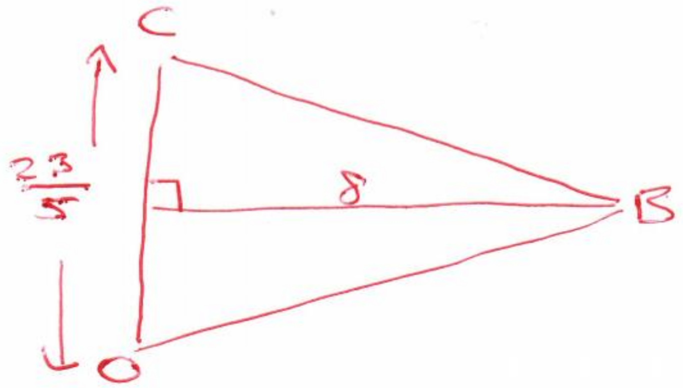
$$-5y + 23 = 0$$

$$5y = 23$$

$$y = \frac{23}{5}$$

Coordinates of C $\left(0, \frac{23}{5}\right)$

c)



Area Δ OCB = $\frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{\cancel{2}_1} \times \frac{23}{5} \times \cancel{8}^4$$

$$= \frac{92}{5} \text{ square units}$$

$$\begin{array}{r} 23 \\ \times\ 4 \\ \hline 92 \end{array}$$

Leave blank

11. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0$$

The point P has coordinates (2, 7).

(a) Show that P lies on C.

(1)

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}\left(2+\sqrt{6}\right)$.

(5)

a) put $x = 2$ in equation

$y = 2^3 - 2(2^2) - 2 + 9$

$= 8 - 8 - 2 + 9$

$= 7$

Which proves $P(2,7)$ lies on C

b) Find gradient at $P(2,7)$ by differentiating

$\frac{dy}{dx} = 3x^2 - 4x - 1$

at $x = 2$

$\frac{dy}{dx} = 3(2^2) - 4(2) - 1$

$= 12 - 8 - 1$

$= 3$

Use $y - y_1 = m(x - x_1)$ to get equation of tangent

$y - 7 = 3(x - 2)$

$y - 7 = 3x - 6$

$y = 3x + 1$ is equation of tangent

11c) Gradient perpendicular to tangent is $-\frac{1}{3}$

Equation of C is $y = x^3 - 2x^2 - x + 9$

$$\frac{dy}{dx} = 3x^2 - 4x - 1$$

$$-\frac{1}{3} = 3x^2 - 4x - 1$$

$$0 = 3x^2 - 4x - \frac{2}{3}$$

$\times$ through by 3

$$0 = 9x^2 - 12x - 2$$

Using quadratic formula with $a=9$, $b=-12$, $c=-2$

$$x = \frac{-(-12) \pm \sqrt{144 - 4(9)(-2)}}{2 \times 9}$$

$$x = \frac{12 \pm \sqrt{216}}{18}$$

Either

$x = \frac{12 + \sqrt{216}}{18}$ or $x = \frac{12 - \sqrt{216}}{18}$

$x = \frac{12 + \sqrt{36}\sqrt{6}}{18}$ $\quad$ $x = \frac{12 - \sqrt{36}\sqrt{6}}{18}$

$x = \frac{12 + 6\sqrt{6}}{18}$ $\quad$ $x = \frac{12 - 6\sqrt{6}}{18}$

$x = \frac{1}{3}(2 + \sqrt{6})$ $\quad$ $x = \frac{1}{3}(2 - \sqrt{6})$

which shows coordinate of Q is $\frac{1}{3}(2 + \sqrt{6})$ as required

Leave blank

11. The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

(a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. **(4)**

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

(b) Calculate the coordinates of S. **(5)**

(c) Show that the length of RS is $3\sqrt{5}$. **(2)**

(d) Hence, or otherwise, find the exact area of triangle PQR. **(4)**

a) gradient $= \frac{8-2}{11--1} = \frac{6}{12} = \frac{1}{2}$

$y - y_1 = m(x - x_1)$
$y - 2 = \frac{1}{2}(x - -1)$

$y - 2 = \frac{1}{2}x + \frac{1}{2}$
$y = \frac{1}{2}x + \frac{5}{2}$ ① L_1

b) gradient of l_2 is -2 as it is perpendicular to L_1
equation of l_2 using $R(10, 0)$
$y - y_1 = m(x - x_1)$
$y - 0 = -2(x - 10)$
$y = -2x + 20$ ② l_2

To find coordinates of intersection solve ① and ② simultaneously

① − ② gives $0 = \frac{1}{2}x - (-2x) + \frac{5}{2} - 20$
$0 = \frac{5}{2}x - \frac{35}{2}$

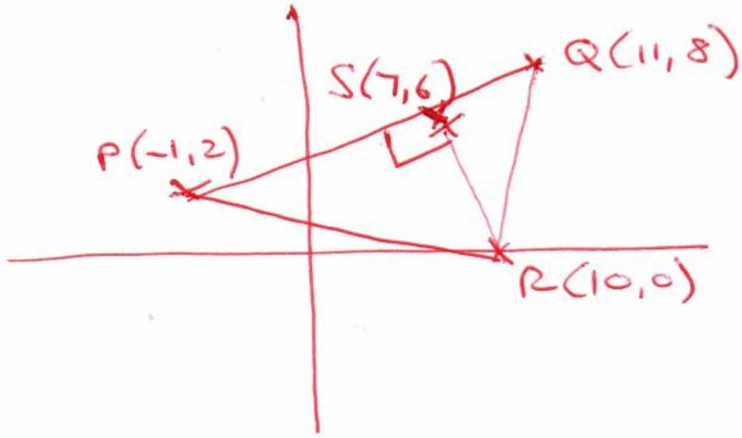
$5x = 35$

$x = 7$
Put $x = 7$ in ① $y = \frac{1}{2} \times 7 + \frac{5}{2} = 6$

11 b continued

Coordinates of S are (7, 6)

c)



$$\text{Length } RS = \sqrt{(6-0)^2+(7-10)^2}$$
$$= \sqrt{36+9}$$
$$= \sqrt{45}$$
$$= \sqrt{9}\sqrt{5}$$
$$= 3\sqrt{5} \quad \text{as required}$$

d) Area $\Delta PQR = \frac{1}{2} \times \text{base} \times \text{height}$

$\uparrow 3\sqrt{5}$

$$\text{Length of base} = PQ = \sqrt{(8-2)^2+(11--1)^2}$$
$$= \sqrt{36+144}$$
$$= \sqrt{180}$$
$$= \sqrt{36}\sqrt{5}$$
$$= 6\sqrt{5}$$

$$\text{Area } \Delta PQR = \frac{1}{2} \times 6\sqrt{5} \times 3\sqrt{5}$$
$$= 9 \times 5$$
$$= 45 \text{ square units}$$

May 2010

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8. (a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax+by+c=0$, where a, b and c are integers.
(3)

(b) Find the length of AB, leaving your answer in surd form.
(2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t.
(1)

(d) Find the area of triangle ABC.
(2)

gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

a) gradient $= \dfrac{4-0}{7-2} = \dfrac{4}{5}$

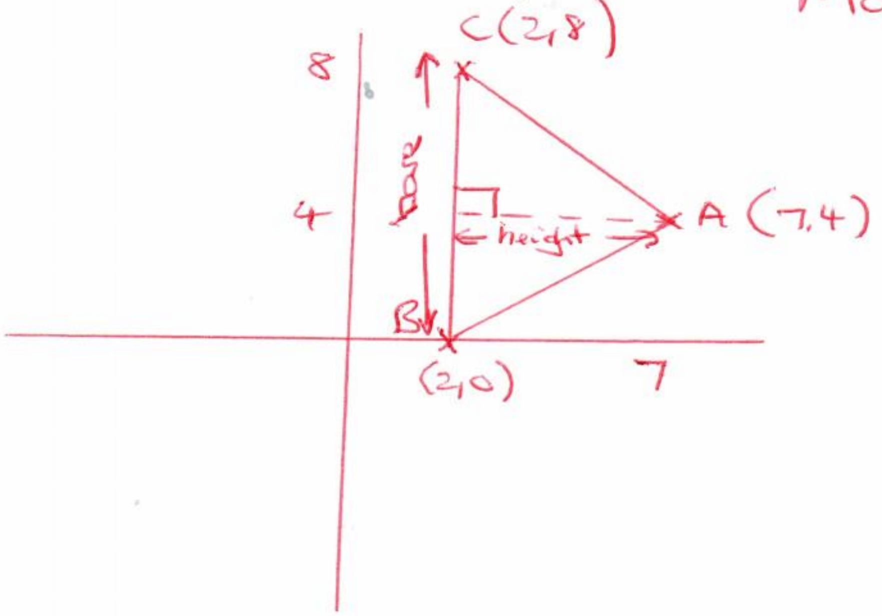
Equation of line is
$y - y_1 = m(x - x_1)$
using point $B(2,0)$
$y - 0 = \frac{4}{5}(x-2)$

$y = \frac{4}{5}(x-2)$
$5y = 4(x-2)$
$5y = 4x - 8$
$0 = 4x - 5y - 8$
in form $ax + by + c = 0$
where $a = 4$, $b = -5$, $c = -8$

b) $AB = \sqrt{(4-0)^2 + (7-2)^2}$
$= \sqrt{4^2 + 5^2}$
$= \sqrt{16+25}$
$= \sqrt{41}$

c) $AC = \sqrt{41}$
$AC = \sqrt{41} = \sqrt{(4-t)^2 + (7-2)^2}$
$41 = (4-t)^2 + 25$
$41 = 16 - 8t + t^2 + 25$
$0 = t^2 - 8t$
$0 = t(t-8)$
as $t > 0$ from question,
$t = 8$

8d) May 2010



Area $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 8 \times 5$

$= 20$ square units

May 2011

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10. The curve C has equation

touches at $x = -3$

$$y = (x+1)(x+3)^2$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

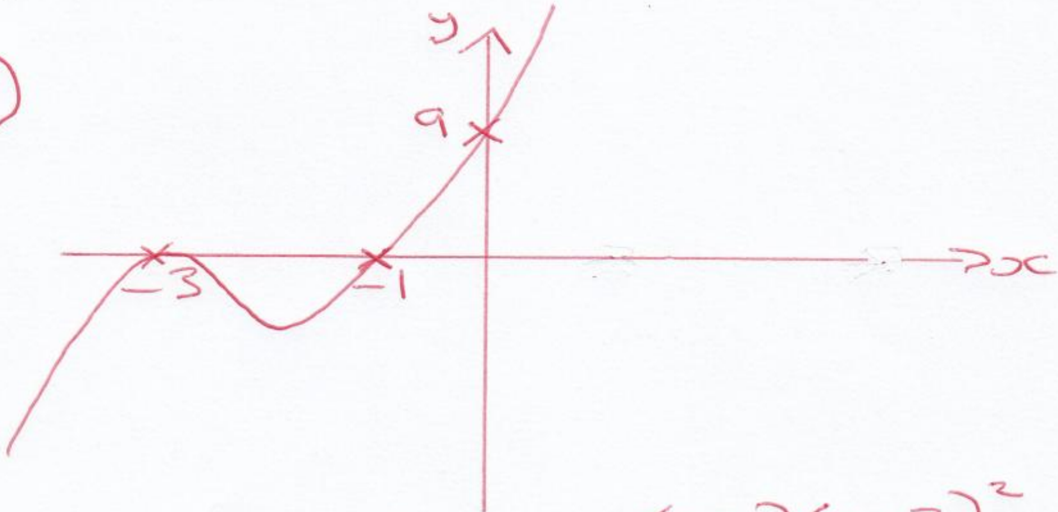
(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B.

(3)

a)



when $x = 0$, $y = (0+1)(0+3)^2$

$y = 9$

b) $y = (x+1)(x^2+6x+9)$

$y = x^3 + 6x^2 + 9x$

$+ x^2 + 6x + 9$

$y = x^3 + 7x^2 + 15x + 9$

$\frac{dy}{dx} = 3x^2 + 14x + 15$ as required

May 2011

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Question 10 continued

c) A when $x = -5$

$y = (-5+1)(-5+3)^2$
$= (-4)(-2)^2$
$= -16$

A is point $(-5, -16)$

$\frac{dy}{dx}$ at $x = -5$ to get gradient of tangent

$\frac{dy}{dx} = 3(-5)^2 + 14(-5) + 15$
$= 75 - 70 + 15$
$= 20$

Equation of tangent at A

$y - y_1 = m(x - x_1)$

$y - (-16) = 20(x - -5)$

$y + 16 = 20x + 100$
$y = 20x + 84$

d) If tangents are parallel, $\frac{dy}{dx} = 20$ at B

$\therefore 3x^2 + 14x + 15 = 20$
$3x^2 + 14x - 5 = 0$
$(3x - 1)(x + 5) = 0$
either $x = \frac{1}{3}$ or $x = -5$

equation for $\frac{dy}{dx}$

when $x = \frac{1}{3}$, $y = (1 + \frac{1}{3})(\frac{1}{3} + 3)^2$
$= \frac{4}{3} \times (\frac{10}{3})^2 = \frac{400}{27}$

x-coord at B is $\frac{1}{3}$

Leave blank

9. The line L_1 has equation $4y + 3 = 2x$

The point A $(p, 4)$ lies on L_1

(a) Find the value of the constant p. **(1)**

The line L_2 passes through the point C $(2, 4)$ and is perpendicular to L_1

(b) Find an equation for L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

The line L_1 and the line L_2 intersect at the point D.

(c) Find the coordinates of the point D. **(3)**

(d) Show that the length of CD is $\frac{3}{2}\sqrt{5}$ **(3)**

A point B lies on L_1 and the length of $AB = \sqrt{(80)}$

The point E lies on L_2 such that the length of the line $CDE = 3$ times the length of CD.

(e) Find the area of the quadrilateral $ACBE$. **(3)**

a) $4y + 3 = 2x$

$4y = 2x - 3$

$y = \frac{1}{2}x - \frac{3}{4}$ ①

When $y = 4$

$4 \times 4 + 3 = 2x$

$16 + 3 = 2x$

$19 = 2x$

$x = 9.5$

$p = 9.5$

b) From ① gradient of L_1 is $\frac{1}{2}$

gradient of $L_2 = -2$

$C(2,4)$ $\quad y - y_1 = m(x - x_1)$

$y - 4 = -2(x - 2)$

$y - 4 = -2x + 4$

P 4 0 6 8 4 A 0 1 8 2 4

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Question 9 continued

$2x + y - 8 = 0$

is equation of L_2

c) L_1 $4y + 3 = 2x$ (1)

L_2 $2x + y - 8 = 0$ (2)

(2) gives $y = 8 - 2x$

in (1) $4(8-2x) + 3 = 2x$

$32 - 8x + 3 = 2x$

$35 = 10x$

$x = 3 \cdot 5$

$y = 8 - 2 \times 3 \cdot 5$

$y = 1$

Coordinates of D are $(3 \cdot 5, 1)$

d) $C(2,4)$

$D(3 \cdot 5, 1)$

$CD = \sqrt{(2-3 \cdot 5)^2 + (4-1)^2}$

$CD = \sqrt{\left(-\frac{3}{2}\right)^2 + 3^2}$

$CD = \sqrt{\frac{9}{4} + 9}$

$CD = \sqrt{\frac{9+36}{4}} = \sqrt{\frac{45}{4}}$

$= \frac{\sqrt{45}}{\sqrt{4}} = \frac{\sqrt{45}}{2} = \frac{\sqrt{9} \times \sqrt{5}}{2}$

$= \frac{3\sqrt{5}}{2}$ as required

P 4 0 6 8 4 A 0 1 9 2 4

Turn over

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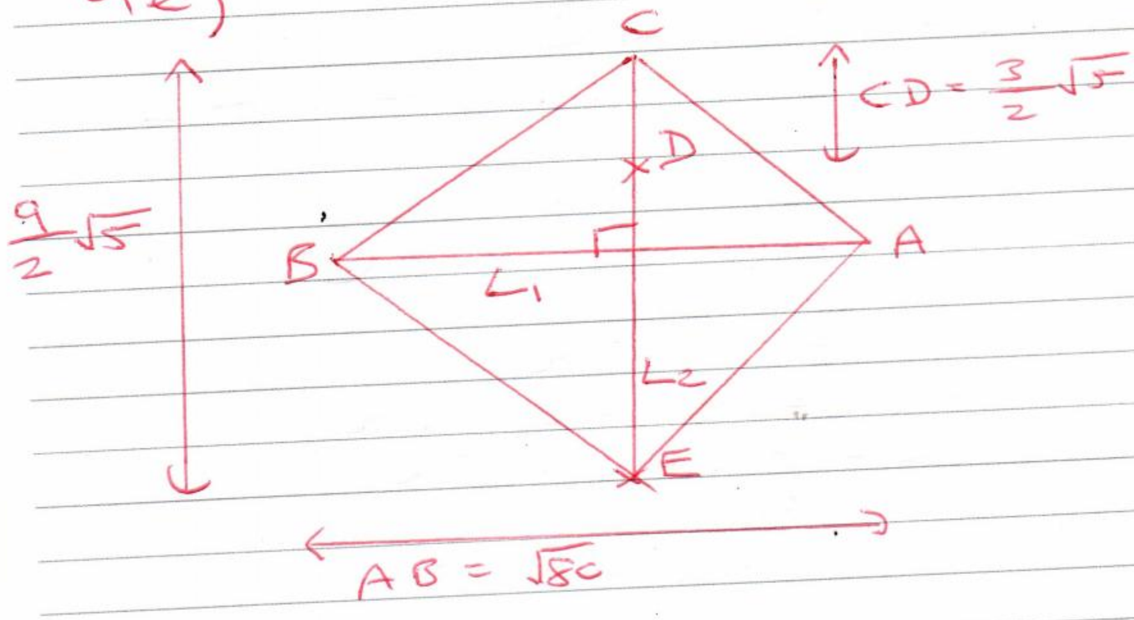
Question 9 continued

9e)



$$CE = 3 \times \frac{3}{2}\sqrt{5} = \frac{9}{2}\sqrt{5}$$

$$\text{Area } ACBE = \frac{1}{2} \times \left(\sqrt{80} \times \frac{9}{2} \times \sqrt{5} \right)$$

$$= \frac{9}{4} \times \sqrt{80} \times \sqrt{5}$$

$$= \frac{9}{4} \times \sqrt{16} \times \sqrt{5} \times \sqrt{5}$$

$$= \frac{9}{\cancel{4}_{1}} \times \cancel{4}^{1} \times 5$$

$$= 45 \text{ square units}$$

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5. The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.
(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B.
(2)

Given that O is the origin,

(c) find the area of the triangle OAB.
(2)

a) L_1 $y = -2x + 3$, gradient $= -2$

l_2 gradient is perpendicular $= \frac{1}{2}$

$y - y_1 = m(x - x_1)$ using $(5, 6)$

$y - 6 = \frac{1}{2}(x - 5)$ $\times$ by 2

$2(y - 6) = x - 5$

$2y - 12 = x - 5$

$0 = x - 2y - 5 + 12$

$x - 2y + 7 = 0$ is equation of l_2

b) x-axis at A, $y = 0$

$x + 7 = 0$

$\underline{\underline{x = -7}}$ x-coord of A

y-axis at B, $x = 0$

$-2y + 7 = 0$

$2y = 7$

$\underline{\underline{y = \frac{7}{2}}}$ y-coord of B

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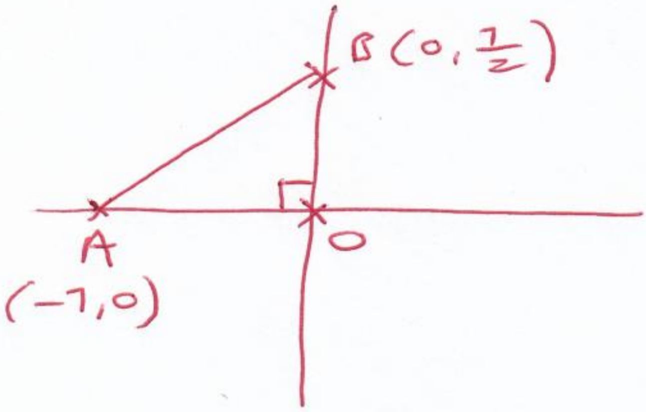
5c)



Area $\triangle AOB = \frac{1}{2} \times 7 \times \frac{7}{2}$

$= \frac{49}{4}$ square units

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6. The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$,

where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2.

(3)

a) gradient $= \dfrac{12-3}{11--1} = \dfrac{9}{12} = \dfrac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{3}{4}(x - -1)$

$4(y-3) = 3(x+1)$

$4y - 12 = 3x + 3$

$0 = 3x - 4y + 3 + 12$

$\underline{0 = 3x - 4y + 15}$ $\quad L_1$

b) $0 = 4x + 3y - 30$ $\quad L_2$

Solve L_1 and L_2 simultaneously

$3x - 4y = -15$ $\quad L_1 \; (\times 3)$

$4x + 3y = 30$ $\quad L_2 \; (\times 4)$

$9x - 12y = -45$ ①

$16x + 12y = 120$ ②

①+② $25x = 75$

$x = 3$

in ② $48 + 12y = 120$

$12y = 120 - 48$

$12y = 72$

$y = 6$

Coordinates of intersection of L_1 and L_2 are $\underline{(3, 6)}$

P 4 1 8 0 2 A 0 1 0 2 8

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11.

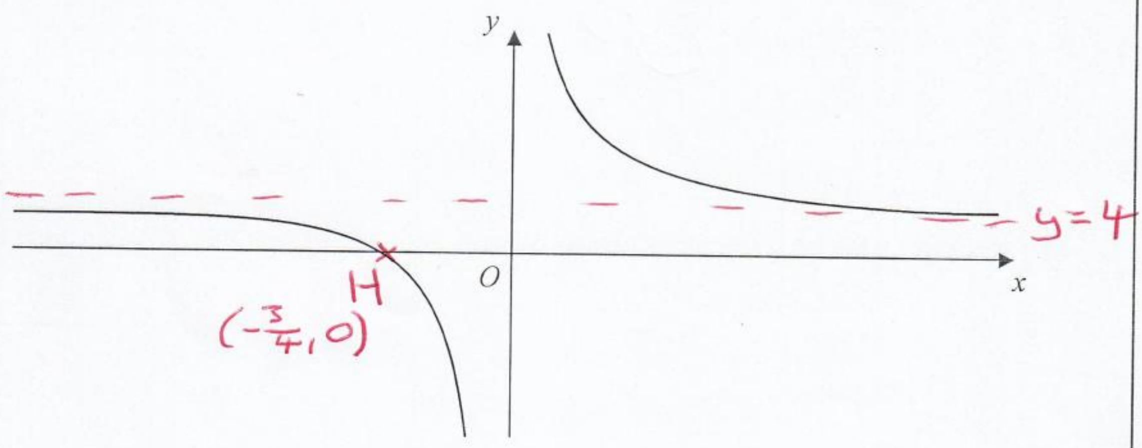


Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.

(1)

(b) Give the equations of the asymptotes to H.

(2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$.

(5)

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd.

(3)

a) $0 = \frac{3}{x} + 4$

$-4 = \frac{3}{x}$ so $x = -\frac{3}{4}$

H is $(-\frac{3}{4}, 0)$

b) $x = 0$, $y = 4$

c) $y = 3x^{-1} + 4$

$\frac{dy}{dx} = -3x^{-2} = -\frac{3}{x^2}$

at $x = -3$, $\frac{dy}{dx} = \frac{-3}{(-3)^2} = -\frac{1}{3}$

Gradient at P is $-\frac{1}{3}$, so gradient of normal at P is 3

P 4 1 8 0 2 A 0 2 4 2 8

11 c) continued

$y - y_1 = m(x - x_1)$ using $m = 3$, $(-3, 3)$

$y - 3 = 3(x - -3)$

$y - 3 = 3x + 9$

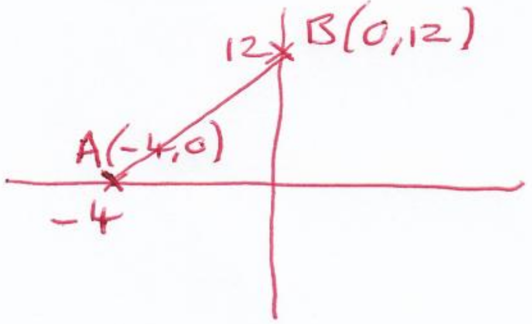
$\underline{y = 3x + 12}$ is equation of normal

d) at A, $y = 0$, $0 = 3x + 12$

$x = -4$

at B, $x = 0$ $y = 3 \times 0 + 12$

$y = 12$



$AB = \sqrt{(12-0)^2 + (0--4)^2}$

$= \sqrt{12^2 + 4^2}$

$= \sqrt{144 + 16}$

$= \sqrt{160}$

$= \sqrt{16} \times \sqrt{10}$

$\underline{AB = 4\sqrt{10}}$ units