3. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}}\,dx.$$

(8)

C4 Jan 2012

Leave blank

2. (a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

C4 June 2005

Leave blank

4. Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx.$$

(7)

N 2 0 2 3 2 B 0 6 2 4

C4 June 2007

Leave blank

2. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx.$$

(6)

June 2010

2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \ \mathrm{d}x = e(e - 1).$$

(6)

C4 June 2011

Leave blank

8. (a) Find $\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}y$

(2)

(b) Given that $y=1.5$ at $x=-2$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form $y=\mathrm{f}(x)$.

(6)

C4 June 2012

Leave blank

1. $$f(x) = \frac{1}{x(3x-1)^2} \equiv \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int f(x)\, dx$.

(ii) Find $\int_1^2 f(x)\, dx$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

C4 June 2012

Leave blank

7.

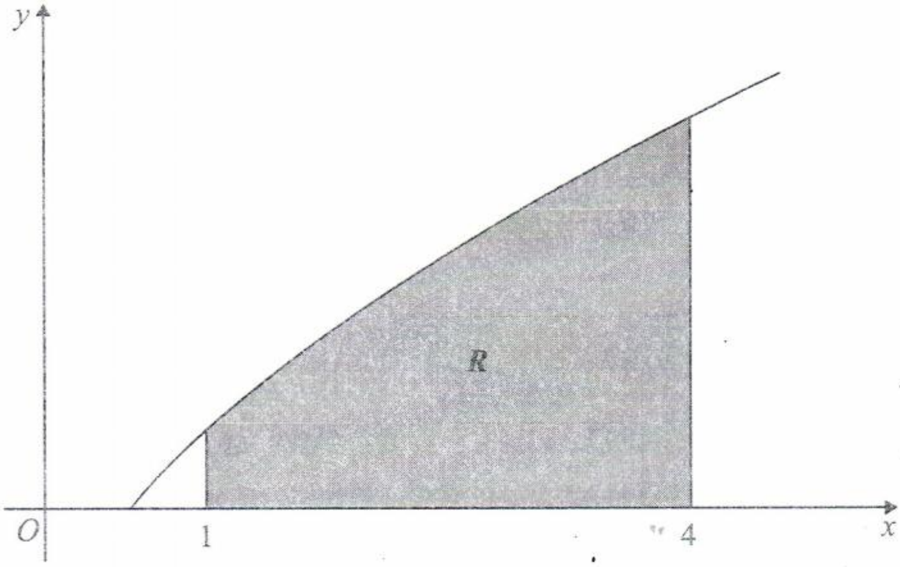

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$.

(4)

(c) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)

Leave blank

2. (a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, \mathrm{d}x$$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, \mathrm{d}x$$

(2)

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4.

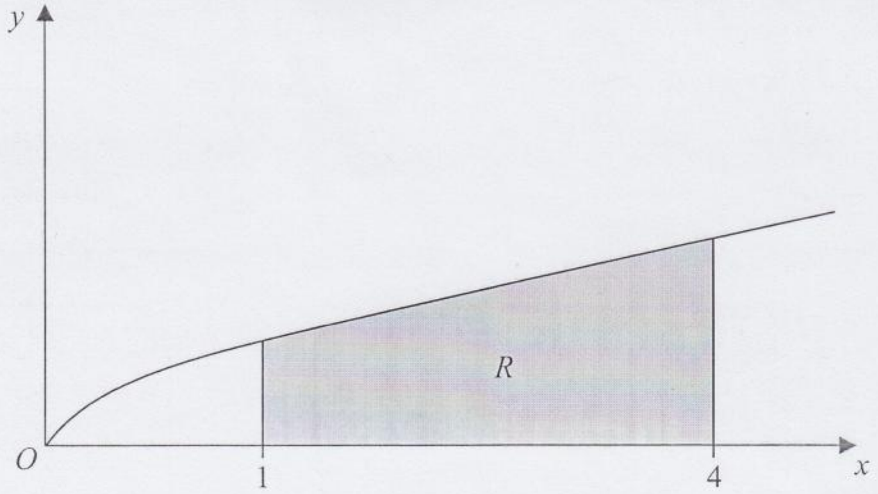

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

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5.

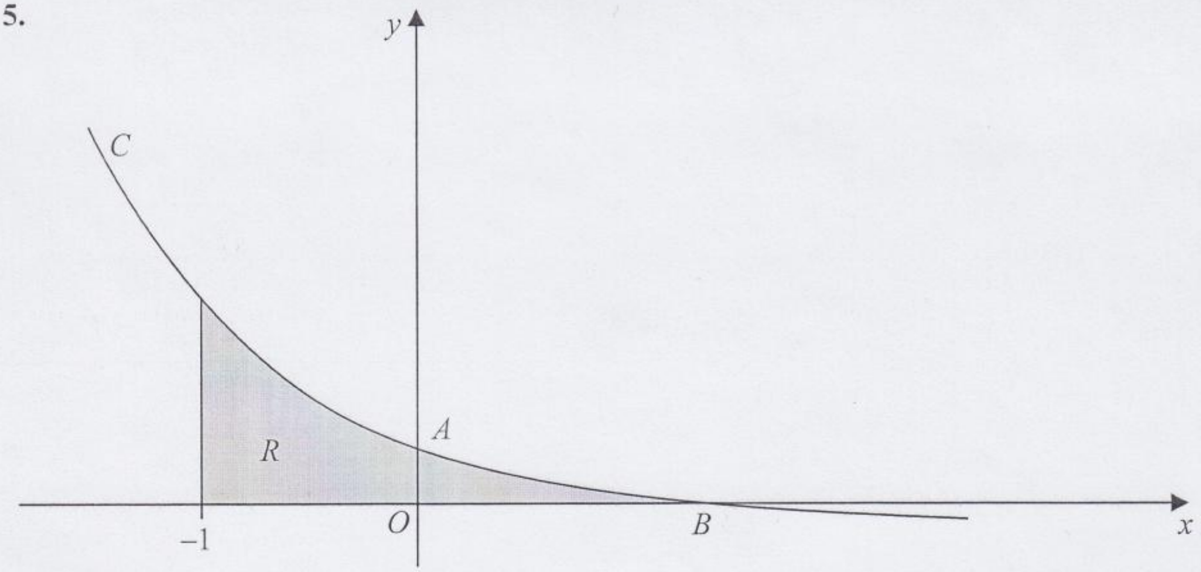

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).
(2)

(b) Find the x coordinate of the point B.
(2)

(c) Find an equation of the normal to C at the point A.
(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.
(6)

C4 June 2013

Leave blank

1. (a) Find $\int x^2 e^x \mathrm{d}x$.

(5)

(b) Hence find the exact value of $\int_0^1 x^2 e^x \mathrm{d}x$.

(2)

C4 June 2013

Leave blank

5. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)} \, dx = \int \frac{2}{u(2u - 1)} \, du$$

(3)

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)} \, dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)