C4 Jan 2012

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2. (a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

a) $u = x \rightarrow \frac{du}{dx} = 1$

$v = -\frac{\cos 3x}{3} \leftarrow \frac{dv}{dx} = \sin 3x$

$$I = -\frac{x\cos 3x}{3} - \int -\frac{\cos 3x}{3} \, dx$$

$$= -\frac{x\cos 3x}{3} + \int \frac{\cos 3x}{3} \, dx$$

$$= -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + c$$

b) $u = x^2 \rightarrow \frac{du}{dx} = 2x$

$v = \frac{\sin 3x}{3} \leftarrow \frac{dv}{dx} = \cos 3x$

$$I = x^2 \frac{\sin 3x}{3} - \int \frac{2}{3} x \sin 3x \, dx$$

already know $\int x \sin 3x \, dx$

$$= \frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[-\frac{x\cos 3x}{3} + \frac{\sin 3x}{9}\right]$$

P 4 0 0 8 5 A 0 4 2 8

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Question 2 continued

$$= \frac{x^2 \sin 3x}{3} + \frac{2}{9} x \cos 3x - \frac{2}{27} \sin 3x + c$$

Q2

(Total 6 marks)

P 4 0 0 8 5 A 0 5 2 8

Turn over

C4 June 2005

Question Number	Scheme		Marks
4.	$\int \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx = \int \frac{1}{(1-\sin^2\theta)^{\frac{3}{2}}}\cos\theta\,d\theta$ Use of $x=\sin\theta$ and $\frac{dx}{d\theta}=\cos\theta$		M1
	$= \int \frac{1}{\cos^2\theta}\,d\theta$		M1 A1
	$= \int \sec^2\theta\,d\theta = \tan\theta$		M1 A1
	Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral		M1
	$[\tan\theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}}\ \left(=\frac{\sqrt{3}}{3}\right)$	cao	A1
			[7]
	Alternative for final M1 A1		
	Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral		M1
	$\left[\frac{x}{\sqrt{(1-x^2)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}}\ \left(=\frac{\sqrt{3}}{3}\right)$	cao	A1

June 07

edexcel

Question Number	Scheme		Marks
2.	$\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx$, with substitution $u = 2^x$		
	$\dfrac{du}{dx} = 2^x.\ln 2 \Rightarrow \dfrac{dx}{du} = \dfrac{1}{2^x.\ln 2}$	$\frac{du}{dx} = 2^x.\ln 2$ or $\frac{du}{dx} = u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{du}{dx} = \ln 2$	B1
	$\displaystyle\int \frac{2^x}{(2^x+1)^2}\,dx = \left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,du$	$k\displaystyle\int \frac{1}{(u+1)^2}\,du$ where k is constant	M1 *
	$= \left(\dfrac{1}{\ln 2}\right)\left(\dfrac{-1}{(u+1)}\right) + c$	$(u+1)^{-2} \rightarrow a(u+1)^{-1}$	M1
		$(u+1)^{-2} \rightarrow -1.(u+1)^{-1}$	A1
	change limits: when $x = 0$ & $x = 1$ then $u = 1$ & $u = 2$		
	$\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$		
	$= \dfrac{1}{\ln 2}\left[\left(-\dfrac{1}{3}\right) - \left(-\dfrac{1}{2}\right)\right]$	Correct use of limits $u = 1$ and $u = 2$	depM1 *
	$= \dfrac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ Exact value only!	A1 aef [6]
	Alternatively candidate can revert back to x …		
	$\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$		
	$= \dfrac{1}{\ln 2}\left[\left(-\dfrac{1}{3}\right) - \left(-\dfrac{1}{2}\right)\right]$	Correct use of limits $x = 0$ and $x = 1$	depM1 *
	$= \dfrac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ Exact value only!	A1 aef
			6 marks

If you see this **integration** applied anywhere in a candidate's working then you can award M1, A1

There are other acceptable answers for A1, eg: $\frac{1}{2\ln 8}$ or $\frac{1}{\ln 64}$

NB: Use your calculator to check eg. 0.240449…

2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \ \ dx = e(e-1).$$

(6)

$\frac{du}{dx} = -\sin x$

$dx = \frac{1}{-\sin x} du$

Limits

x	$u = \cos x + 1$
0	2
$\frac{\pi}{2}$	1

$\int_2^1 e^u \cancel{\sin x} \cdot \frac{1}{-\cancel{\sin x}} du$

$= \int_2^1 -e^u \, du$

minus changes limits

$= \int_1^2 e^u \, du$

$= \left[e^u\right]_1^2$

$= e^2 - e$

$= e(e-1)$

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8. (a) Find $\int (4y+3)^{-\frac{1}{2}}\,dy$ **(2)**

(b) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form $y = f(x)$. **(6)**

a) $\dfrac{(4y+3)^{1/2}}{(4)(1/2)}$

$= \frac{1}{2}(4y+3)^{1/2}$

(b) $\int \frac{1}{\sqrt{4y+3}}\,dy = \int \frac{1}{x^2}\,dx$

$\Rightarrow \int (4y+3)^{-1/2} = \int x^{-2}\,dx$

from (a) $\Rightarrow$

$\Rightarrow \frac{1}{2}(4y+3)^{1/2} = -x^{-1} \quad (+c)$

At $y = 1.5$ $\quad \frac{1}{2}\left(4(1.5)+3\right)^{1/2} = -(-2)^{-1} + c$

$x = -2$

$c = 1$

P 3 8 1 6 0 A 0 2 2 2 4

C4 June 2011

Question 8 continued

$$\frac{1}{2}(4y+3)^{1/2} = -\frac{1}{x} + 1$$

$$(4y+3)^{1/2} = -\frac{2}{x} + 2$$

$$4y+3 = \left(\frac{-2}{x}+2\right)^2$$

$$4y = \left(\frac{-2}{x}+2\right)^2 - 3$$

$$y = \frac{1}{4}\left(\frac{-2}{x}+2\right)^2 - \frac{3}{4}$$

Leave blank

P 3 8 1 6 0 A 0 2 3 2 4