8. The curve with equation $y = \mathrm{f}(x)$ passes through the point (1, 6). Given that

YORKSHIRE MATHS TUTOR

$$\mathrm{f}'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \quad x > 0,$$

find f(x) and simplify your answer. **(7)**

(integrate f'(x) to get f(x))

$$f'(x) = 3 + 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$$

$$f(x) = 3x + \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$f(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + c$$

We are given it passes through
p (1, 6) so

$$6 = 3(1) + 2(1^{\frac{5}{2}}) + 4(1^{\frac{1}{2}}) + c$$

$$6 = 3 + 2 + 4 + c$$

$$c = -3$$

$$\therefore \; f(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 3$$

Leave blank

4. A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find f(x), giving each term in its simplest form.

(5)

$f(x) = \int (3x^2 - 3x^{\frac{1}{2}} - 7)\,dx$

$f(x) = \frac{3x^3}{3} - \frac{2 \times 3}{3} x^{\frac{3}{2}} - 7x + c$

$f(x) = x^3 - 2x^{\frac{3}{2}} - 7x + c$ ①

at (4, 22) then put $x = 4, y = 22$ in ①

$22 = 4^3 - 2 \times 4^{\frac{3}{2}} - 7(4) + c$

$22 = 64 - 16 - 28 + c$

$c = 22 - 64 + 16 + 28$

$c = 2$

$\therefore$

$f(x) = x^3 - 2x^{\frac{3}{2}} - 7x + 2$

C1 JAN 2011

Leave blank

2. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\,dx$$

giving each term in its simplest form.

(5)

$$= \frac{12x^6}{6} - \frac{3x^3}{3} + \frac{3 \cancel{\times} 4x^{\frac{4}{3}}}{4} + c$$

$$= 2x^6 - x^3 + 3x^{\frac{4}{3}} + c$$

Q2

(Total 5 marks)

C1 Jan 2012

Leave blank

7. A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of $f(1)$.

(5)

$f'(x) = 3x^2 - 3x + 5$

$f(x) = \frac{3x^3}{3} - \frac{3x^2}{2} + 5x + c$

$= x^3 - \frac{3}{2}x^2 + 5x + c$

goes through (2,10)

$10 = 2^3 - \frac{3}{2} \times 4 + 10 + c$

$10 = 8 - 6 + 10 + c$

$10 - 8 + 6 - 10 = c$

$c = -2$

$f(x) = x^3 - \frac{3}{2}x^2 + 5x - 2$

$f(1) = 1^3 - \frac{3}{2} + 5 - 2$

$= 4 - \frac{3}{2} = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}$

Leave blank

9. The curve C with equation $y = \mathrm{f}(x)$ passes through the point (5, 65).

Given that $\mathrm{f}'(x) = 6x^2 - 10x - 12$,

(a) use integration to find $\mathrm{f}(x)$.

(4)

(b) Hence show that $\mathrm{f}(x) = x(2x+3)(x-4)$.

(2)

(c) In the space provided on page 17, sketch C, showing the coordinates of the points where C crosses the x-axis.

(3)

a) $f(x) = \int f'(x)\,dx$

$= \int (6x^2 - 10x - 12)\,dx$

$= \frac{6x^3}{3} - \frac{10x^2}{2} - 12x + c$

$= 2x^3 - 5x^2 - 12x + c$

Passes through (5,65)

Given $f(5) = 65$

$\therefore\ 2 \times (5)^3 - 5(5)^2 - 12(5) + c = 65$

$\therefore\ 250 - 125 - 60 + c = 65$

$\therefore\ 65 + c = 65$

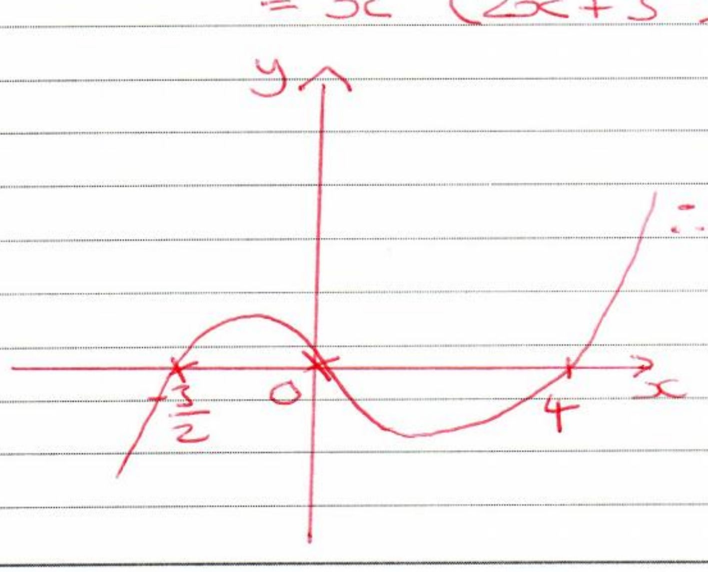
$\therefore\ c = 0$

$\therefore\ f(x) = 2x^3 - 5x^2 - 12x$

b) $f(x) = x(2x^2 - 5x - 12)$

$= x(2x+3)(x-4)$

c)



when $f(x) = 0$

$x(2x+3)(x-4) = 0$

$\therefore\ x = 0$ or $2x+3 = 0$ or $x - 4 = 0$

$\therefore\ x = 0,\ x = -\frac{3}{2},\ x = 4$

Leave blank

11. The gradient of a curve C is given by $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

a) $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}$

$= \frac{(x^2+3)(x^2+3)}{x^2}$

$= \frac{x^4 + 3x^2 + 3x^2 + 9}{x^2} = \frac{x^4 + 6x^2 + 9}{x^2}$

$\therefore \frac{dy}{dx} = x^2 + 6 + 9x^{-2}$

b) Integrate $\frac{dy}{dx}$ to get y

$\therefore y = \int (x^2 + 6x + 9x^{-2})\, dx$

$y = \frac{x}{3} + 6x + \frac{9x^{-1}}{-1} + c$

$y = \frac{x^3}{3} + 6x - \frac{9}{x} + c$

We are given (3, 20) is on C so using

$20 = \frac{3^3}{3} + (6 \times 3) - \frac{9}{3} + c$

$20 = 9 + 18 - 3 + c$

$20 - 9 - 18 + 3 = c$

$c = -4$

So $y = \frac{x^3}{3} + 6x - \frac{9}{x} - 4$

May 2011

6. Given that $\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^{p}+3x^{q}$,

(a) write down the value of p and the value of q.

(2)

Given that $\dfrac{dy}{dx}=\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

Leave blank

a) $\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$

$= 6x^{\frac{1}{2}} + 3x^{2}$

where $p = \frac{1}{2}$ $q = 2$

b) $\dfrac{dy}{dx} = 6x^{\frac{1}{2}} + 3x^{2}$

Integrating

$y = \dfrac{2}{3}\times 6x^{\frac{3}{2}} + \dfrac{3x^{3}}{3} + c$

$y = 4x^{\frac{3}{2}} + x^{3} + c$

when $y = 90$, $x = 4$

$90 = 4\times(4^{\frac{1}{2}})^{3} + 4^{3} + c$

$90 = 32 + 64 + c$

$c = -6$

$y = 4x^{\frac{3}{2}} + x^{3} - 6$

Leave blank

7. The point $P\ (4, -1)$ lies on the curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and

$$\mathrm{f}'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

(a) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

(b) Find $\mathrm{f}(x)$.

(4)

b) $f'(x) = \frac{1}{2}x - 6x^{-\frac{1}{2}} + 3$

$f(x) = \frac{1}{2\times 2}x^2 - \frac{6x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c$

$f(x) = \frac{1}{4}x^2 - 12x^{\frac{1}{2}} + 3x + c$

at $P(4,-1)$, $x = 4$

$-1 = \frac{1}{4}\times 4^2 - 12\times 4^{\frac{1}{2}} + 3\times 4 + c$

$-1 = 4 - 24 + 12 + c$

$c = -1 - 4 + 24 - 12$

$c = 7$

$f(x) = \frac{1}{4}x^2 - 12x^{\frac{1}{2}} + 3x + 7$

a) at $x = 4$

$f'(4) = \frac{1}{2}\times 4 - \frac{6}{\sqrt{4}} + 3$

$f'(4) = 2 - 3 + 3 = 2$

Gradient at $P = 2$

$P(4,-1)$ $y - y_1 = m(x - x_1)$

$y - (-1) = 2(x - 4)$

$y + 1 = 2x - 8$

$y = 2x - 9$ is equation of tangent

P 4 0 6 8 4 A 0 1 4 2 4

C1 Jan 2013

Leave blank

8.
$$\frac{dy}{dx} = -x^3 + \frac{4x-5}{2x^3}, \quad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

(6)

$$\frac{dy}{dx} = -x^3 + 2x^{-2} - \frac{5}{2}x^{-3}$$

Integrating

$$y = -\frac{x^4}{4} + \frac{2x^{-1}}{-1} - \frac{5}{2}\frac{x^{-2}}{-2} + c$$

$$y = -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + c$$

at $x = 1, y = 7$

$$7 = -\frac{1}{4}\times 1^4 - \frac{2}{1} + \frac{5}{4\times 1^2} + c$$

$$7 = -\frac{1}{4} - 2 + \frac{5}{4} + c$$

$$7 + \frac{1}{4} - \frac{5}{4} + 2 = c$$

$$7 - 1 + 2 = c$$

$$c = 8$$

So

$$y = -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + 8$$

CI MAY 2013

Leave blank

9.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0$$

(a) Show that $\quad f'(x) = 9x^{-2} + A + Bx^2,$

where A and B are constants to be found.

(3)

(b) Find $f''(x)$.

(2)

Given that the point $(-3, 10)$ lies on the curve with equation $y = f(x)$,

(c) find $f(x)$.

(5)

a) $f'(x) = \dfrac{(3-x^2)(3-x^2)}{x^2}$

$f'(x) = \dfrac{9 - 3x^2 - 3x^2 + x^4}{x^2} = \dfrac{9 - 6x^2 + x^4}{x^2}$

$f'(x) = 9x^{-2} - 6 + x^2$ in form required

where $A = -6$ and $B = 1$

b) $f''(x) = -18x^{-3} + 2x$ (differentiating)

c) Get $f(x)$ by integrating $f'(x)$

$f(x) = \dfrac{9}{-1}x^{-1} - 6x + \dfrac{x^3}{3} + c$

When $x = -3$, $f(x) = 10$

$10 = \dfrac{-9}{-3} - 6(-3) + \dfrac{(-3)^3}{3} + c$

$10 = +3 + 18 - 9 + c$

$10 - 3 - 18 + 9 = c$

$c = -2$

$f(x) = -9x^{-1} - 6x + \frac{1}{3}x^3 - 2$

P 4 1 8 0 2 A 0 1 8 2 8