Jan 07

Question Number	Scheme		Marks
5. (a)	$\sin x + \cos y = 0.5 \quad (\text{ eqn } *)$		
	$\left\{\frac{\cancel{dx}}{\cancel{dx}}\times\right\} \quad \cos x - \sin y\frac{dy}{dx} = 0 \quad (\text{ eqn } \#)$	Differentiates implicitly to include $\pm\sin y\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
	$\frac{dy}{dx} = \underline{\frac{\cos x}{\sin y}}$	$\underline{\frac{\cos x}{\sin y}}$	A1 **cso**
			[2]
(b)	$\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$	Candidate realises that they need to solve 'their numerator' = 0 …or candidate sets $\frac{dy}{dx} = 0$ in their (eqn #) and attempts to solve the resulting equation.	M1 $\sqrt{}$
	giving $\underline{x = -\frac{\pi}{2}}$ or $\underline{x = \frac{\pi}{2}}$	both $\underline{x = -\frac{\pi}{2}, \frac{\pi}{2}}$ or $\underline{x = \pm 90^\circ}$ or awrt $\underline{x = \pm 1.57}$ required here	A1
	When $x = -\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5$ When $x = \frac{\pi}{2}$, $\sin\left(\frac{\pi}{2}\right) + \cos y = 0.5$	Substitutes either their $x = \frac{\pi}{2}$ or $x = -\frac{\pi}{2}$ into eqn $*$	M1
	$\Rightarrow \cos y = 1.5 \Rightarrow$ y has no solutions $\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$	Only one of $y = \underline{\frac{2\pi}{3}}$ or $\underline{-\frac{2\pi}{3}}$ or $\underline{120^\circ}$ or $\underline{-120^\circ}$ or awrt $\underline{-2.09}$ or awrt $\underline{2.09}$	A1
	In specified range $(x, y) = \underline{\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)}$ and $\underline{\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)}$	Only exact coordinates of $\underline{\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)}$ and $\underline{\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)}$ **Do not award this mark if candidate states other coordinates inside the required range.**	A1
			[5]
			7 marks

5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

a) $x = -8$ $\quad (-8)^3 - 4y^2 = 12(-8)y$

$-512 - 4y^2 = -96y$

$0 = 4y^2 - 96y + 512$

$0 = y^2 - 24y + 128$

$0 = (y-8)(y-16)$

$y = 8,\ y = 16$

$(-8, 8)$ $\quad (-8, 16)$

b) Implicit differentiation.

$3x^2 - 8y\frac{dy}{dx} = 12x\frac{dy}{dx} + 12y$

$3x^2 - 12y = 12x\frac{dy}{dx} + 8y\frac{dy}{dx}$

$3x^2 - 12y = \frac{dy}{dx}(12x + 8y)$

$\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}$

Leave blank

$$\frac{dy}{dx} = \frac{3(-8)^2 - 12(8)}{12(-8) + 8(8)}$$

at $(-8, 8)$

$$= -3$$

and $$\frac{dy}{dx} = \frac{3(-8)^2 - 12(16)}{12(-8) + 8(16)}$$

at $(-8, 16)$

$$= 0.$$

Leave blank

1. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

a) Implicit differentiation

$$2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx}(2y-3) = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2y-3}$$

b) When $y=3$ substitute into original equation to find x =

$$9 - 9 = x^3 + 8$$

$$x^3 = -8$$

$$x = -2$$

Substitute $x=-2$ and $y=3$ into $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3(-2)^2}{2(3)-3}$$

$$= 4$$

3. The curve C has equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \le x \le \frac{\pi}{4}, \quad 0 \le y \le \frac{\pi}{6}.$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(*b*) Find the value of y at P.

(3)

(*c*) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)

3a) $-2\sin 2x - 3\sin 3y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$

(b) $x = \pi/6$

$\cos(2\pi/6) + \cos 3y = 1$

$\cos 3y = 1/2$

$3y = \pi/3$

$y = \pi/9$

(c) At $x = \pi/6$ $\frac{dy}{dx} = -\frac{2}{3}$

substitute into

$y - \pi/9 = -\frac{2}{3}(x - \pi/6)$

$6x + 9y - 2\pi = 0$

C4 Jan 2012

Leave blank

1. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.
The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P.

(5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax+by+c=0$, where a, b and c are integers.

(3)

Differentiate w.r.t. x

$$2 + 6y\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} = 8x$$

At $(-1,1)$ $\quad 2 + 6\frac{dy}{dx} - 6 + 3\frac{dy}{dx} = -8$

$$9\frac{dy}{dx} = -4$$

$$\frac{dy}{dx} = -\frac{4}{9}$$

(b) $\quad \frac{dy}{dx}_{\text{normal}} = \frac{9}{4}$

$$y - 1 = \frac{9}{4}(x - (-1))$$

$$y - 1 = \frac{9}{4}(x+1)$$

$$4y - 4 = 9x + 9$$

$$9x - 4y + 13 = 0.$$

P 4 0 0 8 5 A 0 2 2 8

Question Number	Scheme	Marks
2.	$2x+\left(2x\frac{dy}{dx}+2y\right)-6y\frac{dy}{dx}=0$	M1 (A1) A1
	$\frac{dy}{dx}=0 \Rightarrow x+y=0$ or equivalent	M1
	Eliminating either variable and solving for at least one value of x or y.	M1
	$y^2-2y^2-3y^2+16=0$ or the same equation in x	
	$y=\pm 2$ or $x=\pm 2$	A1
	$(2,-2),(-2,2)$	A1
		[7]
	Note: $\frac{dy}{dx}=\frac{x+y}{3y-x}$	
	Alternative	
	$3y^2-2xy-(x^2+16)=0$	
	$y=\frac{2x\pm\sqrt{(16x^2+192)}}{6}$	
	$\frac{dy}{dx}=\frac{1}{3}\pm\frac{1}{3}.\frac{8x}{\sqrt{(16x^2+192)}}$	M1 A1± A1
	$\frac{dy}{dx}=0 \Rightarrow \frac{8x}{\sqrt{(16x^2+192)}}=\pm 1$	M1
	$64x^2=16x^2+192$	
	$x=\pm 2$	M1 A1
	$(2,-2),(-2,2)$	A1
		[7]

June 06

edexcel

Question Number	Scheme	Marks
Aliter		
1.	$\left\{\frac{\cancel{dx}}{\cancel{dx}}\times\right\}$ $6x\frac{dx}{dy} - 4y + 2\frac{dx}{dy} - 3 = 0$ — Differentiates implicitly to include either $\pm kx\frac{dx}{dy}$ or $\pm 2\frac{dx}{dy}$. (Ignore $\left(\frac{dx}{dy} = \right)$.) Correct equation.	M1 A1
Way 2	$\left\{\frac{dx}{dy} = \frac{4y+3}{6x+2}\right\}$ — *not necessarily required.*	
	At (0, 1), $\frac{dx}{dy} = \frac{4+3}{0+2} = \frac{7}{2}$ — Substituting x = 0 & y = 1 into an *equation* involving $\frac{dx}{dy}$; to give $\frac{7}{2}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$ — Uses m(**T**) or $\frac{dx}{dy}$ to 'correctly' find m(**N**). Can be ft using "$-1.\frac{dx}{dy}$".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$ — $y - 1 = m(x - 0)$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient'; or uses $y = mx + 1$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient';	M1;
	N: 7x + 2y – 2 = 0 — Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe **cso**
		7 marks

June 06

edexcel

Question Number	Scheme		Marks
Aliter **1. Way 3**	$2y^2+3y-3x^2-2x-5=0$		
	$\left(y+\frac{3}{4}\right)^2-\frac{9}{16}=\frac{3x^2}{2}+x+\frac{5}{2}$		
	$y=\sqrt{\left(\frac{3x^2}{2}+x+\frac{49}{16}\right)}-\frac{3}{4}$		
	$\frac{dy}{dx}=\frac{1}{2}\left(\frac{3x^2}{2}+x+\frac{49}{16}\right)^{-\frac{1}{2}}(3x+1)$	Differentiates using the chain rule; Correct expression for $\frac{dy}{dx}$.	M1; A1 oe
	At (0, 1), $\frac{dy}{dx}=\frac{1}{2}\left(\frac{49}{16}\right)^{-\frac{1}{2}}=\frac{1}{2}\left(\frac{4}{7}\right)=\frac{2}{7}$	Substituting x = 0 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1 A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$
	Either **N**: $y-1=-\frac{7}{2}(x-0)$ or **N**: $y=-\frac{2}{7}x+1$	$y-1=m(x-0)$ with 'their tangent or normal gradient'; or uses $y=mx+1$ with 'their tangent or normal gradient'	M1
	N: 7x + 2y – 2 = 0	Correct equation in the form $'ax+by+c=0'$, where a, b and c are integers.	A1 oe
			[7]
			7 marks

~~June 06~~ June 06
~~Jan 06~~

edexcel

June 2006
6666 Pure Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1.	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\times\right\}$ $6x - 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) Correct equation.	M1 A1
	$\left\{\frac{dy}{dx} = \frac{6x+2}{4y+3}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient';	M1;
	N: 7x + 2y − 2 = 0	Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe **cso**
			[7]
			7 marks

Beware: $\frac{dy}{dx} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question. So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

Beware: The final accuracy mark is for completely correct solutions. If a candidate flukes the final line then they must be awarded A0.

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) = ∞, but obtains M0 if they write $y - 1 = \infty(x - 0)$. If they write, however, **N**: x = 0, then can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 1 = 0(x - 0)$ or y = 1.

Beware: The final **cso** refers to the whole question.

Leave blank

4. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)

a) $6x - 2y\frac{dy}{dx} + \left(y + x\frac{dy}{dx}\right) = 0$

$6x - \frac{dy}{dx}(2y - x) + y = 0$

$\frac{dy}{dx}(2y - x) = 6x + y$

$\frac{dy}{dx} = \frac{6x + y}{2y - x}$

$\frac{dy}{dx} = \frac{8}{3}$

given.

$\frac{8}{3} = \frac{6x + y}{2y - x}$

$16y - 8x = 18x + 3y$

$13y = 26x$

$y = 2x$

$y - 2x = 0$

Question 3 continued

b) At p and q $y = 2x$

Substituting into original

$$3x^2 - (2x)^2 + x(2x) = 4$$

$$x^2 = 4$$

$$x = \pm 2$$

Coordinates : $(2, 4)$

$(-2, -4)$

(Total 8 marks)

Leave blank

Q3

H 3 0 4 2 7 A 0 9 2 8

4. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P on C has coordinates $(0, 1)$.

(b) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Implicit differentiation

$$e^{-2x}\frac{dy}{dx} - 2ye^{-2x} = 2 + 2y\frac{dy}{dx}$$

(collect $\frac{dy}{dx}$ on one side)

$$e^{-2x}\frac{dy}{dx} - 2y\frac{dy}{dx} = 2 + 2ye^{-2x}$$

$$\frac{dy}{dx}\left(e^{-2x} - 2y\right) = 2 + 2ye^{-2x}$$

$$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$$

b) $P(0,1)$ $\quad \frac{dy}{dx} = -4$

tangent.

$$\frac{dy}{dx}_{\text{normal}} = \frac{1}{4}$$

$$(y - 1) = \frac{1}{4}(x - 0)$$

$$y = \tfrac{1}{4}x + 1$$

$$x - 4y + 4 = 0$$

Leave blank

3. A curve C has equation

$$2^x + y^2 = 2xy.$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).

(7)

3

$$2^x \ln 2 + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$$

$$2^x \ln 2 - 2y = 2x\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$2^x \ln 2 - 2y = \frac{dy}{dx}(2x - 2y)$$

$$\frac{dy}{dx} = \frac{2^x \ln 2 - 2y}{2x - 2y}$$

At (3, 2)

$x = 3$

$y = 2$

$$\frac{dy}{dx} = \frac{2^3 \ln 2 - 4}{6 - 4}$$

$$= \frac{8\ln 2 - 4}{2}$$

$$\frac{dy}{dx} = 4\ln 2 - 2$$

C4 June 2011

Leave blank

5. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0,\ y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x + 2x\left(\frac{1}{x}\right)$$

↖ product rule

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x + 2$$

At $x=2$ $\quad \frac{1}{y}\frac{dy}{dx} = 2\ln 2 + 2$

need y!

from original $\quad \ln y = 2x\ln x$

at $x=2$ $\quad \ln y = 4\ln 2$

$$\ln y = \ln 2^4$$

$$\ln y = \ln 16$$

$$y = 16$$

substitute $y=16$

$$\frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$$

$$\frac{dy}{dx} = 16(2\ln 2 + 2)$$

P 3 8 1 6 0 A 0 1 2 2 4

Leave blank

5. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$.

(7)

a) Find $\frac{dy}{dx}$

$$48y^2\frac{dy}{dx} + 9x^2\frac{dy}{dx} + 18xy - 54 = 0$$

$$\frac{dy}{dx}(48y^2 + 9x^2) = 54 - 18xy$$

$$\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2}$$

(b) $$\frac{54 - 18xy}{48y^2 + 9x^2} = 0$$

$$54 - 18xy = 0$$

$$54 = 18xy$$

$$3 = xy$$

$$y = \frac{3}{x}$$

Substitute $y = 3/x$ into original equation.

Leave blank

Question 5 continued

$$16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0$$

$$\frac{432}{x^3} + 27x - 54x = 0$$

$$\frac{432}{x^3} - 27x = 0$$

multiply by x^3

$$432 - 27x^4 = 0$$

$$27x^4 = 432$$

$$x^4 = 16$$

$$x = \pm 2$$

$(2, 3/2)$ and $(-2, -3/2)$

C4 June 2013

Leave blank

7. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

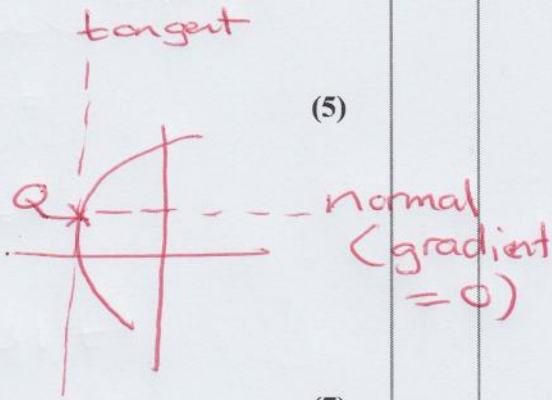
(a) Find $\frac{dy}{dx}$ in terms of x and y. **(5)**

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x coordinate of Q is negative,

(b) use your answer to part (a) to find the coordinates of Q. **(7)**



a) $2x + 4x\frac{dy}{dx} + 4y + 2y\frac{dy}{dx} = 0$

$4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$

$\frac{dy}{dx}(4x+2y) = -2x-4y$

$\frac{dy}{dx} = \frac{-2x-4y}{4x+2y} = \frac{-2(x+2y)}{2(2x+y)}$

$\frac{dy}{dx} = -\frac{x+2y}{2x+y}$

b) as tangent parallel to y-axis, gradient of normal through Q is zero

gradient of tangent $= -\frac{x+2y}{2x+y}$

so gradient of normal $= \frac{2x+y}{x+2y}$

$\therefore\ 0 = \frac{2x+y}{x+2y} \Rightarrow 2x+y=0$

$y = -2x$

C4 June 2013

7b continued

sub $y = -2x$ in curve equation

$$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$$

$$x^2 - 8x^2 + 4x^2 + 27 = 0$$

$$-3x^2 + 27 = 0$$

$$3x^2 = 27$$

$$x^2 = \frac{27}{3} = 9$$

$$x = 3 \text{ or } -3$$

Part a) says x-coord of Q is negative

$$y = -2x - 3$$

$$y = 6$$

Q is $(-3, 6)$