

2. (a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

Leave blank

(1)

- (b) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)

$$\begin{array}{r} 3 \ 6 \\ 3 \sqrt{108} \end{array}$$

a) $\sqrt{108} = \sqrt{36 \times 3} = 6\sqrt{3}$



b) $(2 - \sqrt{3})(2 - \sqrt{3})$

$$= 4 - 2\sqrt{3} - 2\sqrt{3} + 3 = 7 - 4\sqrt{3}$$

(where $b = 7$
 $c = 4$)

Q2

(Total 4 marks)



6. (a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found.

(2)

- (b) Find $\int (4 + 3\sqrt{x})^2 dx$.

(3)

$$\begin{aligned} \text{a)} \quad & (4 + 3\sqrt{x})(4 + 3\sqrt{x}) \\ &= 16 + 12\sqrt{x} + 12\sqrt{x} + 9x \\ &= 16 + 24\sqrt{x} + 9x \end{aligned} \quad (\text{where } k = 24)$$

$$\text{b)} \quad \int (4 + 3\sqrt{x})^2 dx$$

$$\begin{aligned} &= \int (16 + 24\sqrt{x} + 9x) dx \\ &= 16x + \frac{24x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{9x^2}{2} + C \\ &= 16x + \frac{2}{3} \times 24x^{\frac{3}{2}} + \frac{9x^2}{2} + C \\ &= 16x + 16x^{\frac{3}{2}} + \frac{9x^2}{2} + C \end{aligned}$$

Q6

(Total 5 marks)



2. (a) Write down the value of $16^{\frac{1}{4}}$.

for power $\frac{3}{4}$ (1)

- fourth root
 - then cube
- (2)

a) $16^{\frac{1}{4}} = 2$

b) $(16x^{12})^{\frac{3}{4}} = (2x^3)^3$
 $= 8x^9$

Q2

(Total 3 marks)



1. (a) Write down the value of $125^{\frac{1}{3}}$.

(1)

(b) Find the value of $125^{-\frac{2}{3}}$.

(2)

a) 5

b) $125^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{125})^2} = \frac{1}{5^2}$
 $= \frac{1}{25}$

Q1

(Total 3 marks)



3. Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$.

(2)

$$\begin{aligned} & (\sqrt{7} + 2)(\sqrt{7} - 2) \\ &= 7 - 2\sqrt{7} + 2\sqrt{7} - 4 \\ &= 3 \end{aligned}$$

Q3

(Total 2 marks)



1. (a) Find the value of $16^{-\frac{1}{4}}$

(2)

- (b) Simplify $x(2x^{-\frac{1}{4}})^4$

(2)

$$\text{a)} \quad 16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{2}$$

$$\begin{aligned} \text{b)} \quad & x(2x^{-\frac{1}{4}})^4 \\ &= x \times 2^4 \times (x^{-\frac{1}{4}})^4 \\ &= x \times 16 \times x^{-1} \\ &= 16 \end{aligned}$$

Q1

(Total 4 marks)



3. Simplify

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$$

giving your answer in the form $p+q\sqrt{3}$, where p and q are rational numbers.

(4)

$$\frac{(5-2\sqrt{3})}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{5\sqrt{3} + 5 - 2\sqrt{3}\times\sqrt{3} - 2\sqrt{3}}{3 - 1 - \sqrt{3} + \sqrt{3}}$$

$$= \frac{5\sqrt{3} + 5 - 6 - 2\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3} - 1}{2}$$

$$= -\frac{1}{2} + \frac{3}{2}\sqrt{3}$$

(in form $p+q\sqrt{3}$)



2. (a) Simplify

$$\sqrt{32} + \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Simplify

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}}$$

giving your answer in the form $b\sqrt{2} + c$, where b and c are integers.

(4)

$$\begin{aligned} a) \quad \sqrt{32} + \sqrt{18} &= \sqrt{16} \times \sqrt{2} + \sqrt{9} \times \sqrt{2} \\ &= 4\sqrt{2} + 3\sqrt{2} \\ &= 7\sqrt{2} \end{aligned}$$

$$\begin{aligned} b) \quad \frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}} &= \frac{7\sqrt{2}}{(3 + \sqrt{2})} \times \frac{(3 - \sqrt{2})}{(3 - \sqrt{2})} \\ &= \frac{21\sqrt{2} - 14}{9 - 3\sqrt{2} + 3\sqrt{2} - 2} = \frac{21\sqrt{2} - 14}{7} \\ &= 3\sqrt{2} - 2 \end{aligned}$$

in form $b\sqrt{2} + c$

$$\begin{aligned} b &= 3 \\ c &= -2 \end{aligned}$$



1. Simplify $(3 + \sqrt{5})(3 - \sqrt{5})$.

$$(3 + \sqrt{5})(3 - \sqrt{5})$$

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O

$$= 9 - 3\sqrt{5} + 3\sqrt{5} - 5$$
$$= 4$$

(2)

Q1

(Total 2 marks)



2. (a) Find the value of $8^{\frac{4}{3}}$.

(2)

(b) Simplify $\frac{15x^{\frac{4}{3}}}{3x}$.

(2)

a) $8^{\frac{4}{3}} = (\sqrt[3]{8})^4$

$$\begin{aligned} &= 2^4 \\ &= 16 \end{aligned}$$

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m$$

b) $\frac{15x^{\frac{4}{3}}}{3x} = 5x^{\frac{1}{3}}$

$$x^{\frac{4}{3}} \div x^1 = x^{\frac{1}{3}}$$

Q2

(Total 4 marks)



1. Simplify

(a) $(3\sqrt{7})^2$ (1)

(b) $(8+\sqrt{5})(2-\sqrt{5})$ (3)

a) $(3\sqrt{7})^2 = 3^2 \times (\sqrt{7})^2 = 9 \times 7 = 63$

b) $(8+\sqrt{5})(2-\sqrt{5})$

$$= 16 - 8\sqrt{5} + 2\sqrt{5} - 5$$

$$= 11 - 6\sqrt{5}$$

Q1

(Total 4 marks)



2. Given that $32\sqrt{2} = 2^a$, find the value of a .

(3)

$$32 \times \sqrt{2}$$

$$= 2^5 \times 2^{\frac{1}{2}}$$

$$= 2^{\frac{11}{2}}$$

$$a = \frac{11}{2}$$

Q2

(Total 3 marks)



6. (a) Expand and simplify $(4 + \sqrt{3})(4 - \sqrt{3})$.

(2)

- (b) Express $\frac{26}{4+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

(2)

a) $(4 + \sqrt{3})(4 - \sqrt{3})$

$$= 16 - 4\sqrt{3} + 4\sqrt{3} - 3$$
$$= 13$$

b) $\frac{26}{4 + \sqrt{3}} \times \frac{(4 - \sqrt{3})}{(4 - \sqrt{3})}$

$$= \frac{104 - 26\sqrt{3}}{13} = 8 - 2\sqrt{3}$$

in form $a + b\sqrt{3}$

where $a = 8$, $b = -2$



May 2011

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1. Find the value of

(a) $25^{\frac{1}{2}}$

(1)

(b) $25^{-\frac{3}{2}}$

(2)

a)

5

b) $\frac{1}{(25^{\frac{1}{2}})^3} = \frac{1}{5^3} = \frac{1}{125}$

Q1

(Total 3 marks)



2. (a) Evaluate $(32)^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)

a) $(32^{\frac{1}{5}})^3 = 2^3 = 8$

b)
$$\frac{1}{(25x^4)^{\frac{1}{2}}} = \frac{1}{5x^2}$$

$$= \frac{2}{5x^2}$$

Q2

(Total 4 marks)



P 4 0 6 8 4 A 0 3 2 4

3. Show that $\frac{2}{\sqrt{12} - \sqrt{8}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. (5)

$$\frac{2}{(\sqrt{12} - \sqrt{8})} \times \frac{(\sqrt{12} + \sqrt{8})}{(\sqrt{12} + \sqrt{8})}$$

$$= \frac{2\sqrt{12} + 2\sqrt{8}}{12 - 8}$$

$$= \frac{2\sqrt{12} + 2\sqrt{8}}{4}$$

$$= \frac{2\sqrt{4}\sqrt{3} + 2\sqrt{4}\sqrt{2}}{4}$$

$$= \frac{4\sqrt{3} + 4\sqrt{2}}{4} = \frac{4(\sqrt{3} + \sqrt{2})}{4}$$

$$= \sqrt{3} + \sqrt{2}$$

in form $\sqrt{a} + \sqrt{b}$



2. Express 8^{2x+3} in the form 2^y , stating y in terms of x .

(2)

$$\begin{aligned} & 8^{2x+3} \\ &= (2^3)^{2x+3} \\ &= 2^{6x+9} \end{aligned}$$

so $y = 6x + 9$

Q2

(Total 2 marks)



3. (i) Express

$$(5 - \sqrt{8})(1 + \sqrt{2})$$

in the form $a + b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80} + \frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)

(i) $(5 - \sqrt{8})(1 + \sqrt{2})$

$$\begin{aligned} &= 5 + 5\sqrt{2} - \sqrt{8} - \sqrt{16} \\ &= 5 + 5\sqrt{2} - \sqrt{4}\sqrt{2} - 4 \\ &= 5 + 5\sqrt{2} - 2\sqrt{2} - 4 \\ &= 1 + 3\sqrt{2} \end{aligned}$$

in form $a + b\sqrt{2}$
where $a = 1$, $b = 3$

(ii) $\sqrt{80} + \frac{30}{\sqrt{5}} = \sqrt{16}\sqrt{5} + \frac{30\sqrt{5}}{\sqrt{5}\sqrt{5}}$

$$\begin{aligned} &= 4\sqrt{5} + \frac{30\sqrt{5}}{5} \\ &= 4\sqrt{5} + 6\sqrt{5} \\ &= 10\sqrt{5} \end{aligned}$$



1. Simplify

$$\frac{7+\sqrt{5}}{\sqrt{5}-1}$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers.

(4)

$$\left(\frac{7+\sqrt{5}}{\sqrt{5}-1} \right) \times \left(\frac{\sqrt{5}+1}{\sqrt{5}+1} \right) = \frac{7\sqrt{5} + 7 + 5 + \sqrt{5}}{5 + \sqrt{5} - \sqrt{5} - 1}$$

$$= \frac{8\sqrt{5} + 12}{4} = 2\sqrt{5} + 3$$

$$= \underline{\underline{3 + 2\sqrt{5}}}$$

in form $a + b\sqrt{5}$

($a = 3, b = 2$)

Q1

(Total 4 marks)



3. (a) Find the value of $8^{\frac{5}{3}}$

(2)

(b) Simplify fully $\frac{(2x^{\frac{1}{2}})^3}{4x^2}$

(3)

a) $(8^{\frac{1}{3}})^5 = 2^5 = \underline{\underline{32}}$

b) $\frac{(2x^{\frac{1}{2}})^3}{4x^2} = \frac{8x^{\frac{3}{2}}}{4x^2} = \underline{\underline{2x^{-\frac{1}{2}}}}$

