

1. A stone is thrown vertically upwards with speed  $16 \text{ m s}^{-1}$  from a point  $h$  metres above the ground. The stone hits the ground 4 s later. Find

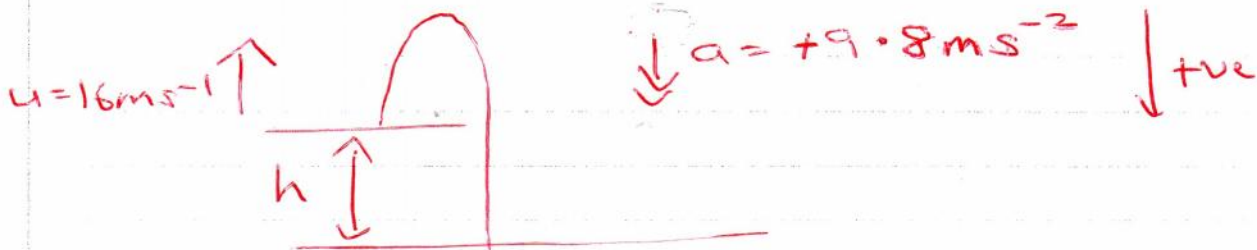
(a) the value of  $h$ .



(3)

(b) the speed of the stone as it hits the ground.

(3)



$$s = h \text{ m}, u = -16 \text{ m s}^{-1}, v = ?, a = 9.8 \text{ m s}^{-2}, t = 4 \text{ sec}$$

$$s = ut + \frac{1}{2}at^2$$

$$h = (-16 \times 4) + \frac{1}{2} \times 9.8 \times (4^2)$$

$$h = -64 + 78.4$$

$$h = \underline{\underline{14.4 \text{ m}}}$$

$$b) v^2 = u^2 + 2as$$

$$v^2 = (-16)^2 + 2 \times 9.8 \times 14.4$$

$$v^2 = 256 + 282.24$$

$$v^2 = 538.24$$

$$v = \underline{\underline{23.2 \text{ m s}^{-1}}}$$



5. A ball is projected vertically upwards with speed  $21 \text{ m s}^{-1}$  from a point  $A$ , which is  $1.5 \text{ m}$  above the ground. After projection, the ball moves freely under gravity until it reaches the ground. Modelling the ball as a particle, find

(a) the greatest height above  $A$  reached by the ball,

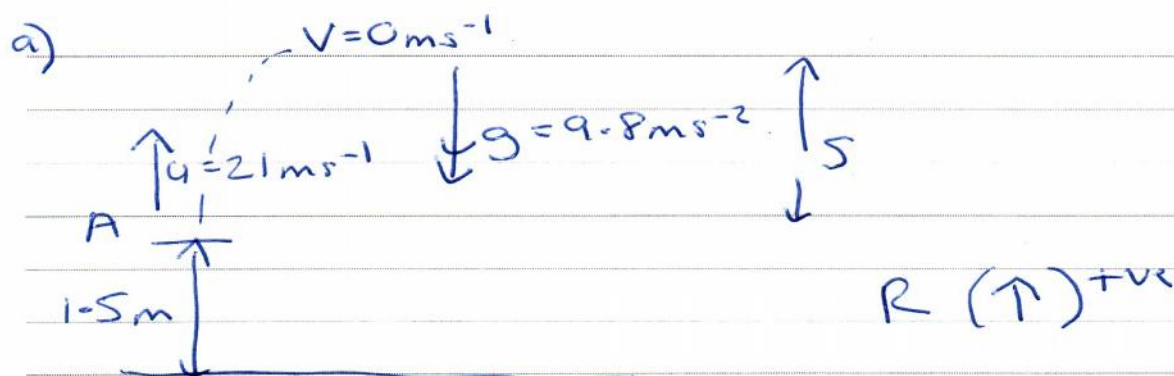
(3)

(b) the speed of the ball as it reaches the ground,

(3)

(c) the time between the instant when the ball is projected from  $A$  and the instant when the ball reaches the ground.

(4)



$$s = ? , u = 21 \text{ m s}^{-1}, v = 0 \text{ m s}^{-1}, a = -9.8 \text{ m s}^{-2}$$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$\text{Height above } A = s = \frac{0^2 - 21^2}{2 \times -9.8} = \frac{-441}{-19.6} = 22.5 \text{ m}$$

b) So greatest height =  $22.5 + 1.5 = 24 \text{ m}$  above ground

$R(\downarrow)^{+ve}$   $s = 24 \text{ m}, u = 0 \text{ m s}^{-1}, a = 9.8 \text{ m s}^{-2}, v = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 9.8 \times 24$$

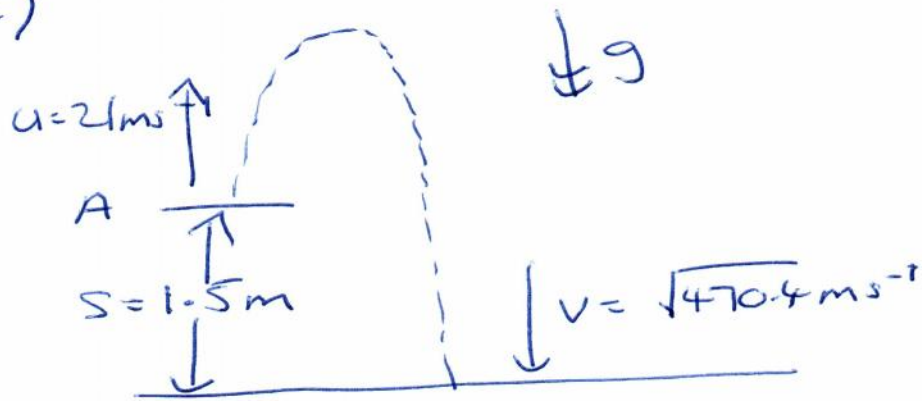
$$v^2 = 470.4$$

$$v = \sqrt{470.4} = 21.688707$$

$$= 21.7 \text{ m s}^{-1} \text{ (3 sf)}$$



Sc)



R ( $\uparrow$ )<sup>+</sup>ve

$$v = \sqrt{-470.4}\text{ms}^{-1}, u = 21\text{ms}^{-1}, a = -9.8\text{ms}^{-2}, t = ?$$

$$v = u + at$$

$$-\sqrt{470.4} = 21 + (-9.8)t$$

$$9.8t = 21 + \sqrt{470.4}$$

$$t = \frac{21 + \sqrt{470.4}}{9.8}$$

$$t = 4.3559905$$

$$t = 4.4 \text{ seconds (1dp)}$$

2. A firework rocket starts from rest at ground level and moves vertically. In the first 3 s of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration  $a \text{ m s}^{-2}$ . Find

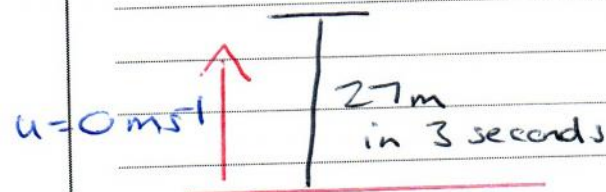
(a) the value of  $a$ , (2)

(b) the speed of the rocket 3 s after it has left the ground. (2)

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

(c) Find the height of the rocket above the ground 5 s after it has left the ground. (4)

a)



$$\begin{aligned} a &= ? \\ t &= 3 \text{ seconds} \\ s &= 27 \text{ m} \\ u &= 0 \\ v &= ? \end{aligned}$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ 27 &= 0 \times 3 + \frac{1}{2} \times a \times 3^2 \\ 27 &= 0 + \frac{9}{2}a \\ a &= \frac{27 \times 2}{9} \\ a &= 6 \text{ m s}^{-2} \end{aligned}$$

b)  $v = ?$ ,  $t = 3 \text{ secs}$ ,  $a = 6 \text{ m s}^{-2}$ ,  $u = 0 \text{ m s}^{-1}$

$$\begin{aligned} v &= u + at \\ v &= 0 + 6 \times 3 \\ v &= 18 \text{ m s}^{-1} \end{aligned}$$

Speed after 3 seconds is  $18 \text{ m s}^{-1}$



JAN 2008

## Equations of motion

2c)

$$u = 18 \text{ ms}^{-1}$$

+ve

$$g = 9.8 \text{ ms}^{-2}$$

$$s = ? \quad (\text{starting level})$$

$$t = 2 \text{ seconds} \quad (\text{start clock again, and time from 3 to 5} = 2 \text{ seconds})$$

$$a = -9.8 \text{ ms}^{-2}$$

$$u = 18 \text{ ms}^{-1}$$

27m

$$s = ut + \frac{1}{2}at^2$$

$$s = 18 \times 2 + \frac{1}{2}(-9.8) \times 2^2$$

$$s = 36 - 19.6$$

$$s = 16.4 \text{ m}$$

As we started motion from height of 27m

$$\text{Height reached is } 27 + 16.4 = 43.4 \text{ m} \\ = 43 \text{ m (2sf)}$$

5. A stone is projected vertically upwards from a point  $A$  with speed  $u \text{ m s}^{-1}$ . After projection the stone moves freely under gravity until it returns to  $A$ . The time between the instant that the stone is projected and the instant that it returns to  $A$  is  $3\frac{4}{7}$  seconds.

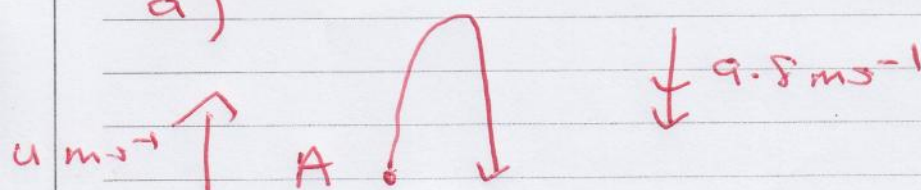
Modelling the stone as a particle,

(a) show that  $u = 17\frac{1}{2}$ , (3)

(b) find the greatest height above  $A$  reached by the stone, (2)

(c) find the length of time for which the stone is at least  $6\frac{3}{5}$  m above  $A$ . (6)

a)



R (↑)

$$s = 0 \text{ m}$$

$$u = u \text{ m s}^{-1}$$

$$v = ?$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = 3\frac{4}{7}$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 3\frac{4}{7}u + \frac{1}{2} \times -9.8 \times \left(3\frac{4}{7}\right)^2$$

$$62.5 = 3\frac{4}{7}u$$

$$u = 17.5 \text{ m s}^{-1}$$

R (↑)

b)

$$s = ?$$

$$u = 17.5 \text{ m s}^{-1}$$

$$v = 0 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = ?$$

$$v^2 = u^2 + 2as$$

$$0^2 = 17.5^2 + 2 \times -9.8 \times s$$

$$s = \frac{17.5^2}{2 \times 9.8} = 15.625 \text{ m}$$

Greatest height above  $A = 15.6 \text{ m}$  (3sf)



M1 Jan 2012

5 c)  $s = 6.6 \text{ m}$   
 $u = 17.5 \text{ m}$   
 $v = ?$   
 $a = -9.8 \text{ ms}^{-2}$   
 $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$6.6 = 17.5t + \frac{1}{2} \times -9.8 \times t^2$$

$$6.6 = 17.5t - 4.9t^2$$

$$4.9t^2 - 17.5t + 6.6 = 0$$

Quadratic formula to get 2  
times when height is 6.6m  
 $a = 4.9$     $b = -17.5$  ,  $c = 6.6$

$$t = \frac{17.5 \pm \sqrt{17.5^2 - 4 \times 4.9 \times 6.6}}{2 \times 4.9}$$

$$t = \frac{17.5 + \sqrt{176.89}}{9.8} \quad \text{or} \quad t = \frac{17.5 - \sqrt{176.89}}{9.8}$$

$$t = 3.1428571 \text{ s} \quad \text{or} \quad t = 0.4285714 \text{ s}$$

$$\begin{aligned} \text{Time above} &= 3.1428571 - 0.4285714 \\ &= 2.7142857 \\ &= 2.71 \text{ seconds (3sf)} \end{aligned}$$

3. A train moves along a straight track with constant acceleration. Three telegraph poles are set at equal intervals beside the track at points  $A$ ,  $B$  and  $C$ , where  $AB = 50$  m and  $BC = 50$  m. The front of the train passes  $A$  with speed  $22.5 \text{ m s}^{-1}$ , and 2 s later it passes  $B$ . Find

- (a) the acceleration of the train, (3)
- (b) the speed of the front of the train when it passes  $C$ , (3)
- (c) the time that elapses from the instant the front of the train passes  $B$  to the instant it passes  $C$ . (4)

$$22.5 \text{ m s}^{-1}$$



- a) Find acceleration between  $A$  and  $B$   
 $s = 50 \text{ m}$ ,  $u = 22.5 \text{ m s}^{-1}$ ,  $v = ?$ ,  $a = ?$ ,  $t = 2 \text{ s}$

$$s = ut + \frac{1}{2} at^2$$

$$50 = 22.5 \times 2 + \frac{1}{2} \times 2^2 \times a$$

$$50 = 45 + 2a$$

$$2a = 50 - 45$$

$$2a = 5$$

$$a = 2.5 \text{ m s}^{-2}$$

- b)  $s = 100 \text{ m}$ ,  $u = 22.5 \text{ m s}^{-1}$ ,  $v = ?$ ,  $a = 2.5 \text{ m s}^{-2}$ ,  $t = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 22.5^2 + 2 \times 2.5 \times 100$$

$$v^2 = 506.25 + 500$$

$$v^2 = 1006.25$$

$$v = 31.721444$$

$$v = 31.7 \text{ m s}^{-1} \quad (3 \text{ sf})$$

- c)  $v = u + at$  (journey from  $A$  to  $C$ )  
 $31.721444 = 22.5 + 2.5t$   
 $t = \frac{31.721444 - 22.5}{2.5} = 3.6885$

$\therefore$  Journey from  $B$  to  $C = 3.6885 - 2$   
 $= 1.69 \text{ seconds}$   
 $(3 \text{ sf})$

1. Three posts  $P$ ,  $Q$  and  $R$ , are fixed in that order at the side of a straight horizontal road. The distance from  $P$  to  $Q$  is 45 m and the distance from  $Q$  to  $R$  is 120 m. A car is moving along the road with constant acceleration  $a \text{ m s}^{-2}$ . The speed of the car, as it passes  $P$ , is  $u \text{ m s}^{-1}$ . The car passes  $Q$  two seconds after passing  $P$ , and the car passes  $R$  four seconds after passing  $Q$ . Find

- (i) the value of  $u$ ,

- (ii) the value of  $a$ .

$$\begin{array}{ccc} t=0 & t=2 & t=6 \\ \downarrow & \downarrow & \downarrow \\ u \text{ m s}^{-1} & & \end{array} \quad (7)$$



Journey from  $P$  to  $Q$

$$s = 45 \text{ m}, u = u, v = ?, a = a, t = 2 \text{ sec}$$

$$s = ut + \frac{1}{2}at^2$$

$$45 = 2u + \frac{1}{2}a \times 2^2$$

$$45 = 2u + 2a \quad (1)$$

Journey from  $P$  to  $R$

$$s = 165 \text{ m}, u = u, v = ?, a = a, t = 6 \text{ sec}$$

$$s = ut + \frac{1}{2}at^2$$

$$165 = 6u + \frac{1}{2}a \times 36$$

$$165 = 6u + 18a \quad (2)$$

Solve (1) and (2) simultaneously

$$(1) \times 3 \text{ gives } 135 = 6u + 6a \quad (3)$$

$$165 = 6u + 18a \quad (2)$$

$$(2) - (3)$$

$$30 = 12a$$

$$a = \frac{30}{12} = 2.5$$

Put  $a = 2.5$  in (1)

$$45 = 2 \times 2.5 + 2u$$

$$45 - 5 = 2u$$

$$40 = 2u$$

$$u = 20$$



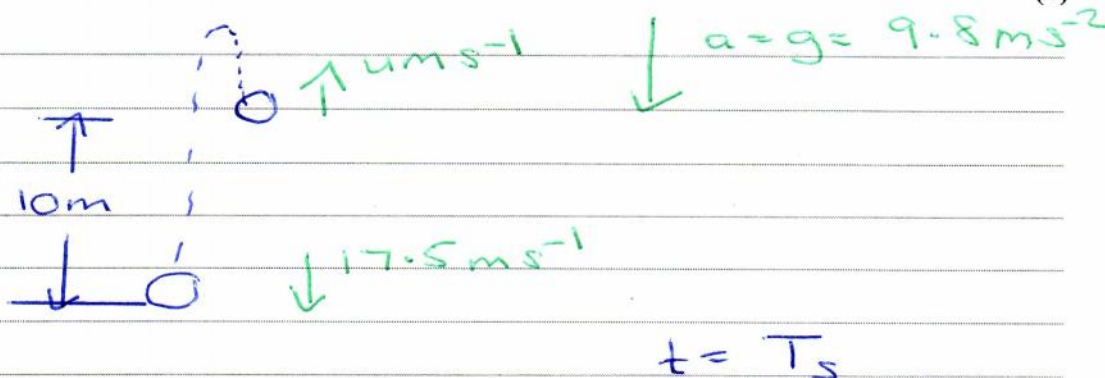
2. At time  $t = 0$ , a particle is projected vertically upwards with speed  $u \text{ m s}^{-1}$  from a point 10 m above the ground. At time  $T$  seconds, the particle hits the ground with speed  $17.5 \text{ m s}^{-1}$ . Find

(a) the value of  $u$ ,

(3)

(b) the value of  $T$ .

(4)



a)  $\uparrow$

$$s = -10 \text{ m}, u = ?, v = -17.5 \text{ m s}^{-1},$$

$$a = -9.8 \text{ m s}^{-2}, t = T_s$$

use  $v^2 = u^2 + 2as$

$$(-17.5)^2 = u^2 + 2(-9.8)(-10)$$

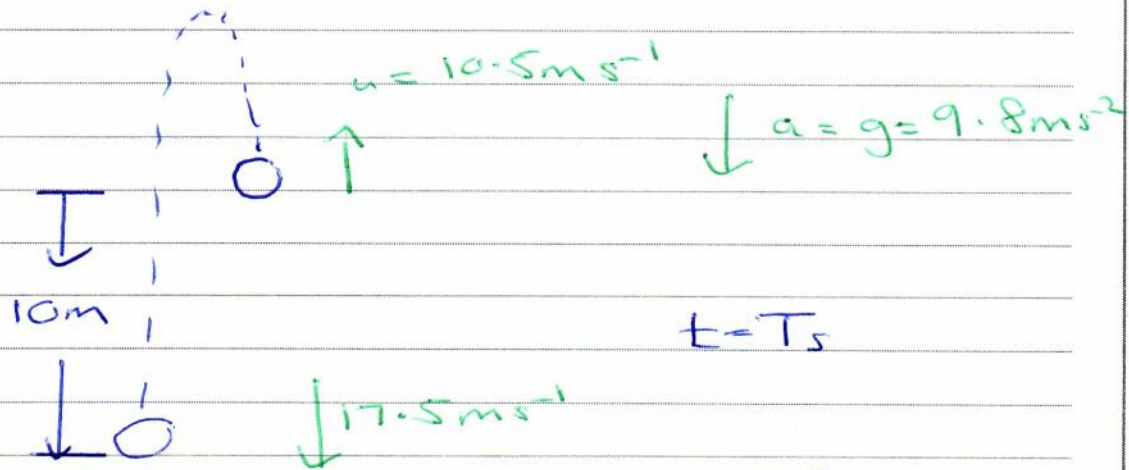
$$\therefore 306.25 = u^2 + 196$$

$$\therefore u^2 = 110.25$$

$$u = 10.5 \text{ m s}^{-1}$$



Question 2 continued



b)  $\uparrow$

$$s = -10 \text{ m}, u = 10.5 \text{ ms}^{-1}$$

$$v = -17.5 \text{ ms}^{-1}, a = -9.8 \text{ ms}^{-2}$$

$$t = T_s$$

Equations  
to we could be

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{(u+v)t}{2}$$

$$v = u + at$$

using  $v = u + at$

$$\therefore -17.5 = 10.5 + (-9.8)T$$

$$\therefore -17.5 = 10.5 - 9.8T$$

$$\therefore 9.8T = 28$$

$$\therefore T = \frac{28}{9.8}$$

$$T = 2.857 \dots$$

$$= 2.9 \text{ s (1dp)}$$

Q2

(Total 7 marks)

May 2010

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6. A ball is projected vertically upwards with a speed of  $14.7 \text{ m s}^{-1}$  from a point which is  $49 \text{ m}$  above horizontal ground. Modelling the ball as a particle moving freely under gravity, find

(a) the greatest height, above the ground, reached by the ball,

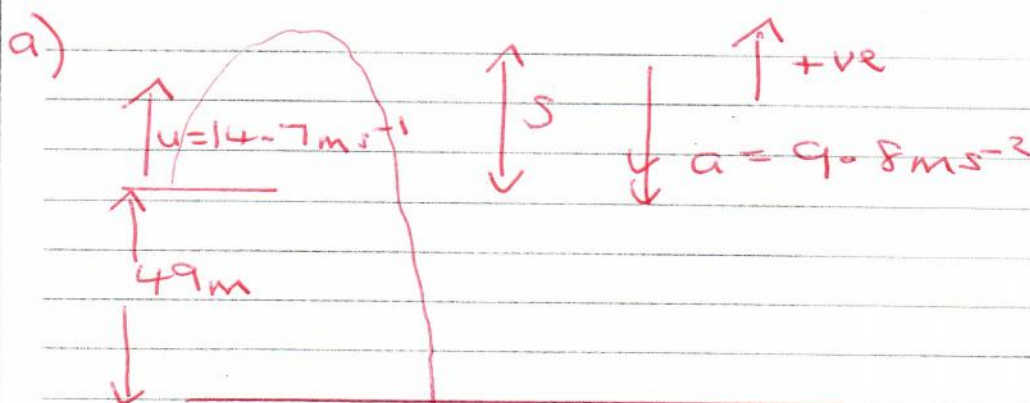
(4)

(b) the speed with which the ball first strikes the ground,

(3)

(c) the total time from when the ball is projected to when it first strikes the ground.

(3)



greatest height when  $v = 0$   
 $s = ?$ ,  $u = 14.7 \text{ m s}^{-1}$ ,  $v = 0 \text{ m s}^{-1}$   
 $a = -9.8 \text{ m s}^{-2}$ ,  $t = ?$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - (14.7)^2}{(2)(-9.8)}$$

$$s = 11.025 \text{ m}$$

$$\therefore \text{Greatest height} = 49 + 11.025$$

$$= 60.025 \text{ m}$$



6b)

R (↓) +ve

May 2010

$$s = 60.025$$

$$u = 0, v = ?, a = 9.8 \text{ m s}^{-2}$$

$$t = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 9.8 \times 60.025$$

$$v^2 = 1176.49$$

$$v = 34.3 \text{ m s}^{-1}$$

R (↑) +ve

c)  $s = -49 \text{ m}, u = 14.7 \text{ m s}^{-1}, a = -9.8 \text{ m s}^{-2},$   
 $v = -34.3 \text{ m s}^{-1}, t = ?$

$$v = u + at$$

$$t = \frac{v - u}{a}$$

$$t = \frac{-34.3 - 14.7}{-9.8}$$

$$t = 5 \text{ seconds}$$

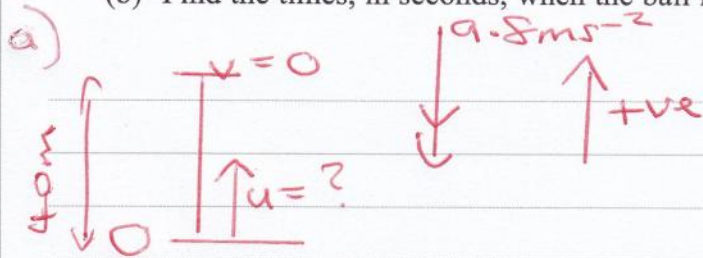
1. At time  $t = 0$  a ball is projected vertically upwards from a point  $O$  and rises to a maximum height of 40 m above  $O$ . The ball is modelled as a particle moving freely under gravity.

(a) Show that the speed of projection is  $28 \text{ m s}^{-1}$ .

(3)

(b) Find the times, in seconds, when the ball is 33.6 m above  $O$ .

(5)



$$s = 40 \text{ m}$$

$$u = ?$$

$$v = 0 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = ?$$

$$v^2 = u^2 + 2as$$

$$u^2 = v^2 - 2as$$

$$u^2 = 0^2 - 2 \times (-9.8) \times 40$$

$$u^2 = 784$$

$$u = 28 \text{ m s}^{-1}$$

as required

b)

$$s = 33.6 \text{ m}$$

$$u = 28 \text{ m s}^{-1}$$

$$v = ?$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$33.6 = 28t + \frac{1}{2} \times (-9.8) \times t^2$$

$$4.9t^2 - 28t + 33.6 = 0$$

using quadratic formula

$$a = 4.9 \quad b = -28 \quad c = 33.6$$

$$t = \frac{28 \pm \sqrt{(-28)^2 - 4 \times 4.9 \times 33.6}}{2 \times 4.9}$$



May 2011

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Question 1 continued

1b)

$$t = \frac{28 + 11.2}{9.8}$$

$$\text{or } t = \frac{28 - 11.2}{9.8}$$

$$t = 4 \text{ seconds}$$

$$t = 1.71 \text{ seconds} \\ (2dp)$$

Q1

(Total 8 marks)



5. A particle  $P$  is projected vertically upwards from a point  $A$  with speed  $u \text{ m s}^{-1}$ . The point  $A$  is  $17.5 \text{ m}$  above horizontal ground. The particle  $P$  moves freely under gravity until it reaches the ground with speed  $28 \text{ m s}^{-1}$ .

(a) Show that  $u = 21$

(3)

At time  $t$  seconds after projection,  $P$  is  $19 \text{ m}$  above  $A$ .

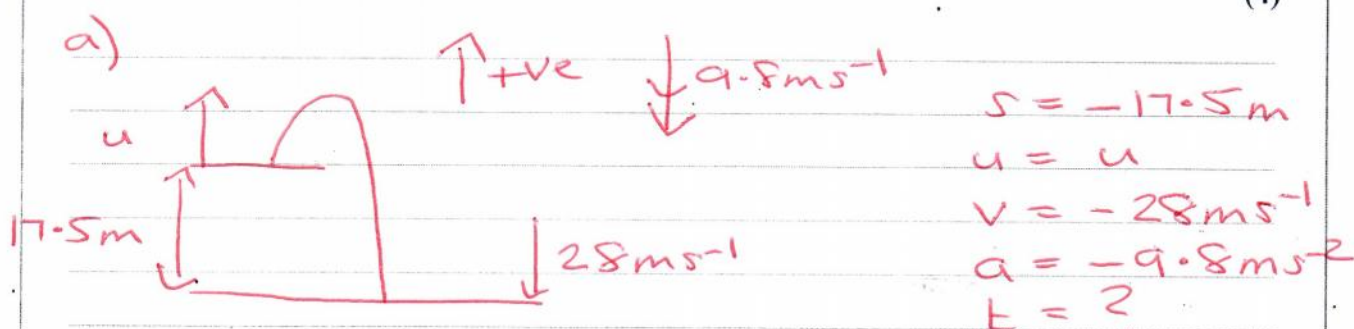
(b) Find the possible values of  $t$ .

(5)

The ground is soft and, after  $P$  reaches the ground,  $P$  sinks vertically downwards into the ground before coming to rest. The mass of  $P$  is  $4 \text{ kg}$  and the ground is assumed to exert a constant resistive force of magnitude  $5000 \text{ N}$  on  $P$ .

(c) Find the vertical distance that  $P$  sinks into the ground before coming to rest.

(4)



$$v^2 = u^2 + 2as$$

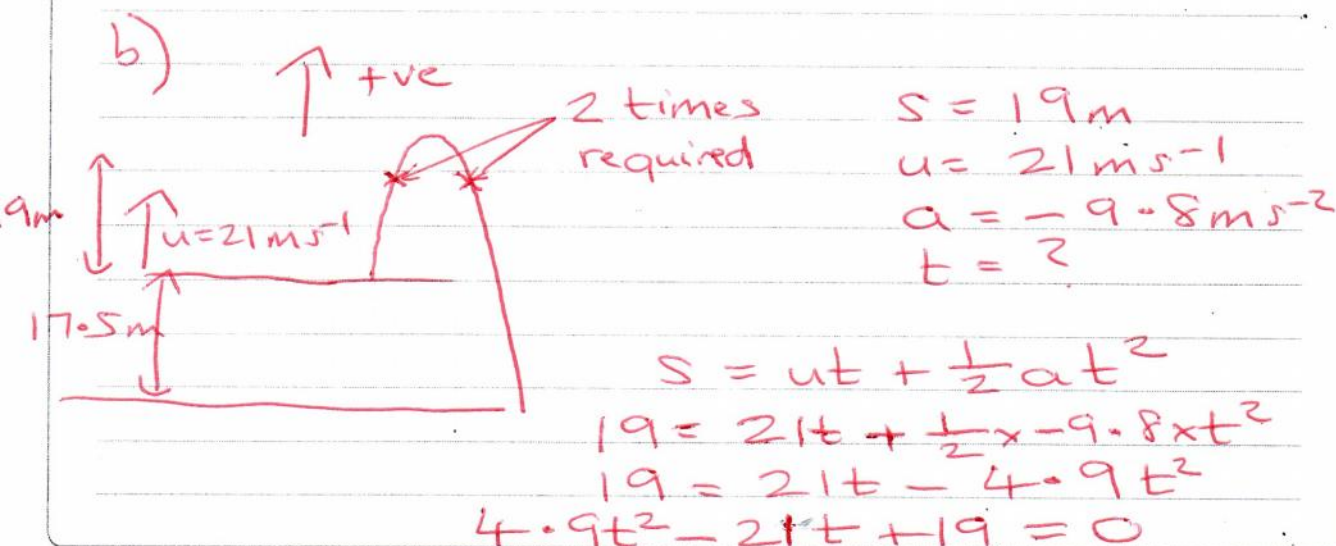
$$u^2 = v^2 - 2as$$

$$u^2 = (-28)^2 - 2 \times -9.8 \times -17.5$$

$$u^2 = 784 - 343$$

$$u^2 = 441$$

$$u = 21 \quad (\text{as required})$$



MI - MAY 2012

5b (continued)

quadratic formula

$$t = \frac{21 \pm \sqrt{(-21)^2 - 4 \times 4.9 \times 19}}{2 \times 4.9}$$

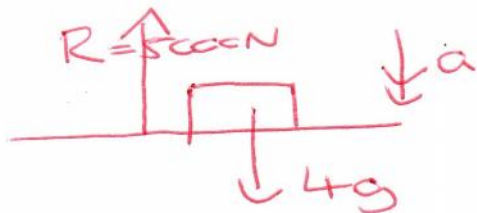
$$t = \frac{21 + \sqrt{68}}{9.8} \quad \text{or} \quad t = \frac{21 - \sqrt{68}}{9.8}$$

$$t = 2.9880114$$

$$t = 1.2977029$$

Values of  $t$  are  $t = 2.99$  (3sf)  
 $t = 1.30$  (3sf)

5c)



Equation of motion  
when particle hits  
ground

$R (\downarrow)$

$$4 \times a = 4g - 5000$$

$$a = \frac{4g - 5000}{4}$$

$$a = -1240.2 \text{ ms}^{-2}$$

( $\downarrow$ ) +ve

$$s = ?$$

$$u = 28 \text{ ms}^{-1}$$

$$v = 0 \text{ ms}^{-1}$$

$$a = -1240.2 \text{ ms}^{-2}$$

$$t = ?$$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - 28^2}{2 \times -1240.2} = 0.316078$$

Distance sinks = 0.316 m (3sf)

M1 MAY 2013

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2. A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of  $2 \text{ m s}^{-2}$ . By modelling the cable as being light and inextensible, find

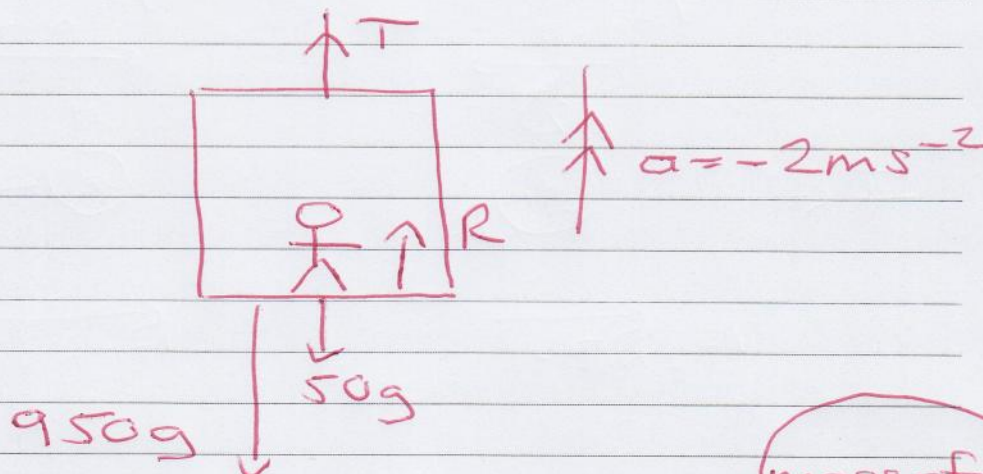
(a) the tension in the cable,

(3)

(b) the magnitude of the force exerted on the woman by the floor of the lift.

(3)

a)



a)  $R$  ( $\uparrow$ ) for lift  $m \times a$

$$T - 1000g = 1000 \times -2$$

$$T - 9800 = -2000$$

$$T = 7800 \text{ N}$$

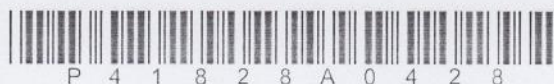
b)

$R$  ( $\uparrow$ )

$$R - 50g = 50 \times -2$$

$$R = -100 + 490$$

$$R = 390 \text{ N}$$



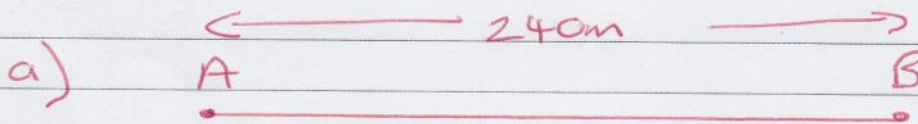
4. A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point  $A$  with speed  $u \text{ m s}^{-1}$ , ( $u < 34$ ), and 10 seconds later passes a point  $B$  with speed  $34 \text{ m s}^{-1}$ . Given that  $AB = 240 \text{ m}$ , find

(a) the value of  $u$ ,

(3)

(b) the time taken for the lorry to move from  $A$  to the mid-point of  $AB$ .

(6)



$$u \text{ m s}^{-1}$$

$$S = 240 \text{ m}$$

$$u = u \text{ m s}^{-1}$$

$$v = 34 \text{ m s}^{-1}$$

$$a = ?$$

$$t = 10 \text{ s}$$

$$S = \frac{(u+v)t}{2}$$

$$2 \times 240 = 10(u + 34)$$

$$\frac{480}{10} = u + 34$$

$$u = 48 - 34$$

$$\underline{u = 14}$$

b) mid point is 120m

$$S = 120 \text{ m}$$

$$u = 14 \text{ m s}^{-1}$$

$$v = ?$$

$$a = 2 \text{ m s}^{-2}$$

$$t = ?$$

from a)  
need acceleration

$$v = u + at$$

$$a = \frac{v-u}{t}$$

$$a = \frac{34-14}{10} = 2 \text{ m s}^{-2}$$

$$S = ut + \frac{1}{2}at^2$$

$$120 = 14t + \frac{1}{2} \times 2 \times t^2$$

$$120 = 14t + t^2$$

$$0 = t^2 + 14t - 120$$

$$0 = (t+20)(t-6)$$

$$t = -20 \text{ or } t = 6$$

$$\underline{t = 6 \text{ seconds}}$$

