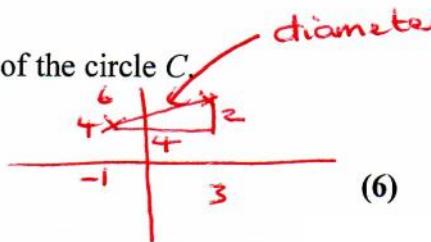


3. The line joining the points $(-1, 4)$ and $(3, 6)$ is a diameter of the circle C .

Find an equation for C .



Mid point of the diameter is
the centre of the circle

$$\left(\frac{-1+3}{2}, \frac{4+6}{2} \right) = (1, 5) \text{ is the centre}$$

$$r = \sqrt{\frac{(3-(-1))^2 + (6-4)^2}{2}} = \sqrt{\frac{(3+1)^2 + 2^2}{2}}$$

$$r = \sqrt{\frac{4^2 + 2^2}{2}} = \sqrt{\frac{20}{2}}$$

$$\therefore r^2 = \frac{\sqrt{20} \times \sqrt{20}}{2} = \frac{\sqrt{400}}{4} = \frac{20}{4} = 5$$

\therefore Equation of C is

$$(x-a)^2 + (y-b)^2 = r^2$$

where (a, b) is the centre of
the circle and r is the
radius

Equation of C is

$$(x-1)^2 + (y-5)^2 = 5$$



8. A circle C has centre $M(6, 4)$ and radius 3.

- (a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2. \quad (2)$$

Figure 3

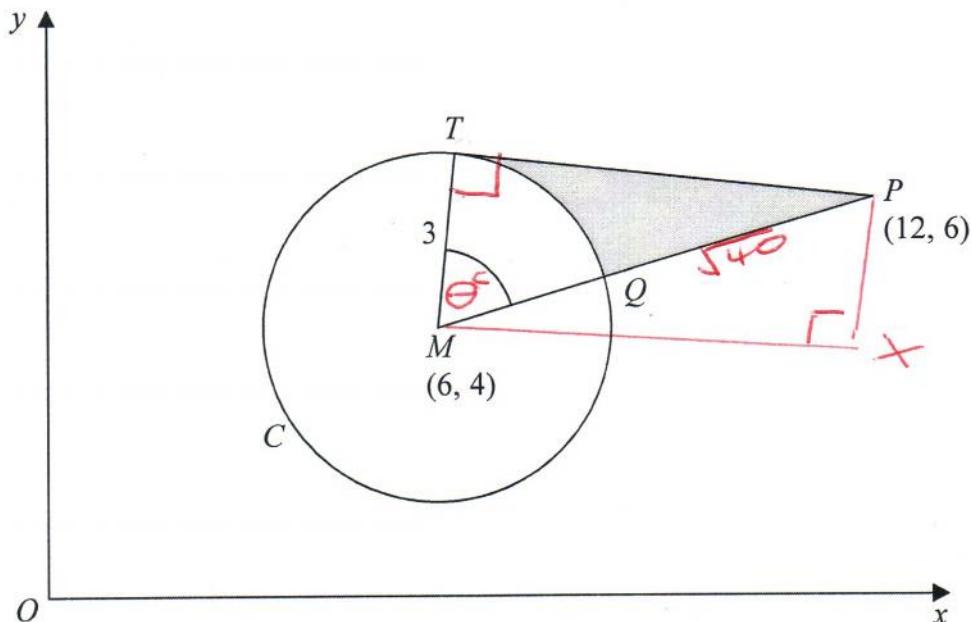


Figure 3 shows the circle C . The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q .

- (b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP , QP and the arc TQ , as shown in Figure 3.

- (c) Find the area of the shaded region TPQ . Give your answer to 3 decimal places.

(5)

a) $(x - a)^2 + (y - b)^2 = r^2$

The equation of circle is
 $(x - 6)^2 + (y - 4)^2 = 3^2$

b) $MP^2 = MX^2 + PX^2$
 $MP^2 = (12 - 6)^2 + (6 - 4)^2$
 $MP^2 = 6^2 + 2^2$
 $MP^2 = 40$
 $MP = \sqrt{40}$



85) continued

$$\cos \theta = \frac{3}{\sqrt{40}}$$

$$\therefore \theta = \cos^{-1} \frac{3}{\sqrt{40}}$$

(make sure calculator is in radians)

$$\therefore \theta = 1.076580 \dots \text{ radians}$$

$$\therefore \theta = 1.0766 \text{ radians } (4 \text{ dp})$$

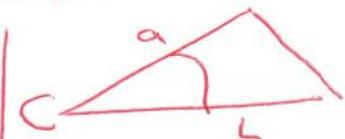
c) Work out area of $\triangle MTP$ then take away area of sector

Area $\triangle MTP$

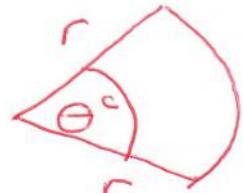
$$= \frac{1}{2} (3) \sqrt{40} \sin 1.076580 \dots$$
$$= 8.351645 \dots \text{ units}^2$$

Area Sector TMQ

$$= \frac{1.076580}{2\pi} \times \pi \times 3^2$$
$$= 4.844611 \dots \text{ unit}^2$$



$$\text{Area} = \frac{1}{2} ab \sin C$$



$$\text{Area} = \frac{\theta}{2\pi} \times \pi r^2$$
$$= \frac{1}{2} \theta r^2$$

Area TPC shaded

$$= 8.351645 \dots - 4.844611 \dots$$
$$= 3.5070 \dots$$
$$= 3.507 \text{ units}^2 \quad (3 \text{ dp})$$

5.

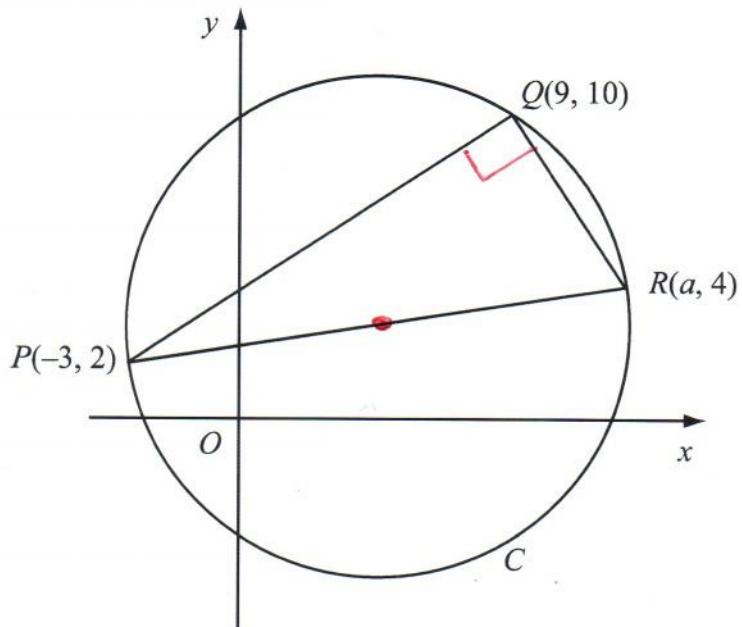
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Figure 2

The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C , as shown in Figure 2. Given that PR is a diameter of C ,

(a) show that $a = 13$,

(3)

(b) find an equation for C .

(5)

a) As PR is a diameter, Angle \hat{PQR} is 90°
(angle in semicircle)

So lines PQ and QR are perpendicular
gradient of $PQ = \frac{10-2}{9-(-3)} = \frac{8}{12} = \frac{2}{3}$

\therefore as QR is perpendicular to PQ

$$\text{gradient of } QR = -\frac{3}{2}$$

$$-\frac{3}{2} = \frac{10-4}{9-a}$$

$$-\frac{3}{2}(9-a) = 2(6)$$

$$-27 + 3a = 12$$

$$3a = 39$$

$$a = 13 \quad (\text{as required})$$

5b)

First find circle centre coordinates
(mid point of PR)

Mid point x-coord is $\frac{1}{2}(-3+13) = 5$
y-coord is $\frac{1}{2}(4+2) = 3$
Centre of C is (5, 3)

Equation of circle needs length of radius

$$\text{Length PR} = \sqrt{(4-2)^2 + (13-3)^2}$$

$$PR = \sqrt{260}$$

$$\text{radius} = \frac{1}{2} \times PR = \frac{\sqrt{260}}{2} = 8.0622577$$

Equation of circle is

$$(x-a)^2 + (y-b)^2 = r^2$$

where (a, b) is circle centre and r is radius

So equation of C is

$$(x-5)^2 + (y-3)^2 = 8.0622577^2$$

$$(x-5)^2 + (y-3)^2 = 65$$

8.

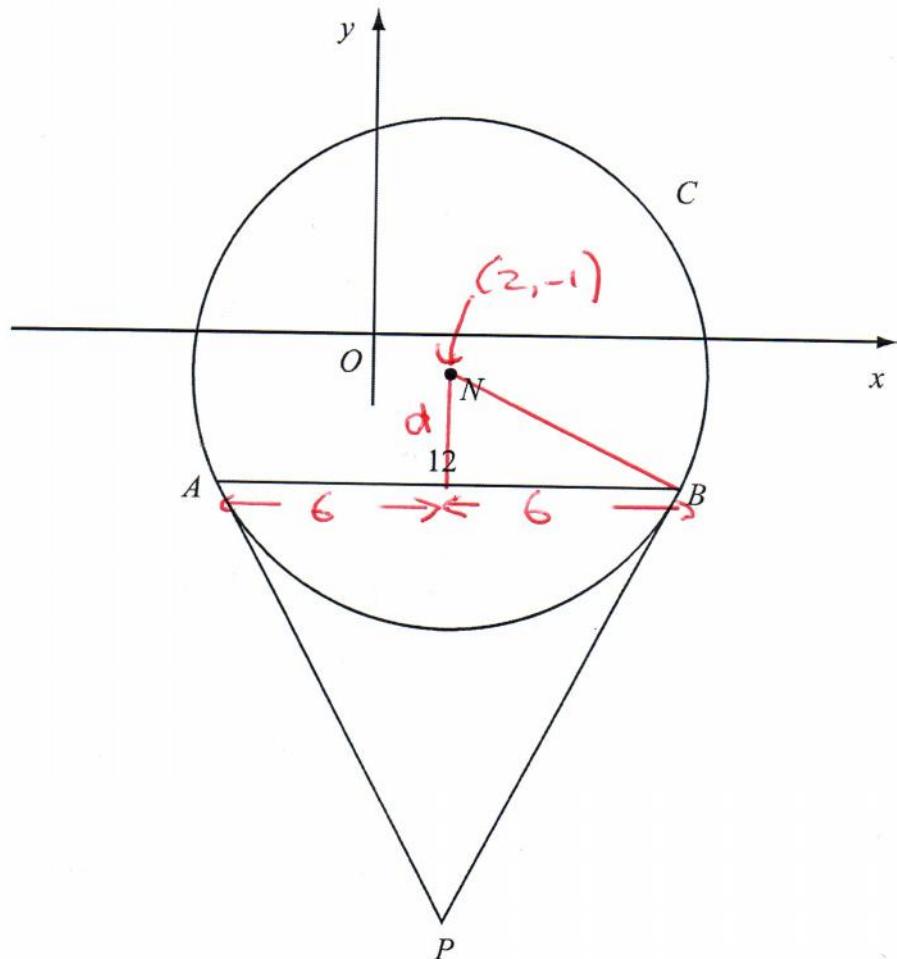
**Figure 3**

Figure 3 shows a sketch of the circle C with centre N and equation

$$(x - 2)^2 + (y + 1)^2 = \frac{169}{4}$$

- (a) Write down the coordinates of N . (2)

- (b) Find the radius of C . (1)

The chord AB of C is parallel to the x -axis, lies below the x -axis and is of length 12 units as shown in Figure 3.

- (c) Find the coordinates of A and the coordinates of B . (5)

- (d) Show that angle $ANB = 134.8^\circ$, to the nearest 0.1 of a degree. (2)

The tangents to C at the points A and B meet at the point P .

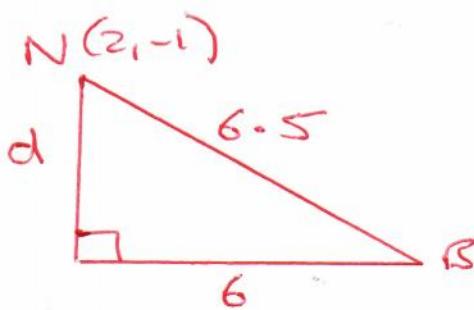
- (e) Find the length AP , giving your answer to 3 significant figures. (2)



8a) Centre N is $(2, -1)$

b) radius = $\sqrt{\frac{169}{4}} = 6.5$ units

c) x - coordinate for B
is $2 + 6 = 8$



Use Pythagoras
to find
distance
 d from
N to line
 AB

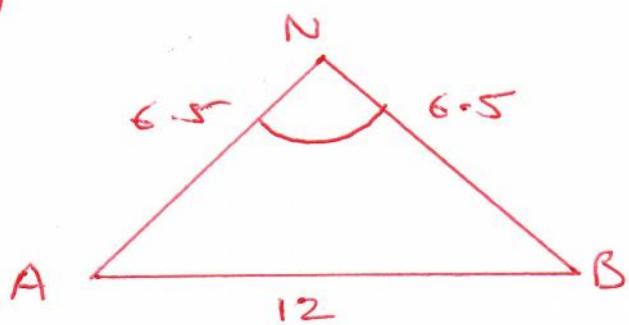
$$d = \sqrt{6.5^2 - 6^2}$$

$$d = 2.5 \text{ units}$$

\therefore Coordinates of A are $(2-6, -1-2.5)$
 $= (-4, -3.5)$

Coordinates of B are $(2+6, -1-2.5)$
 $= (8, -3.5)$

d)



$$\cos N = \frac{6.5^2 + 6.5^2 - 12^2}{2 \times 6.5 \times 6.5}$$

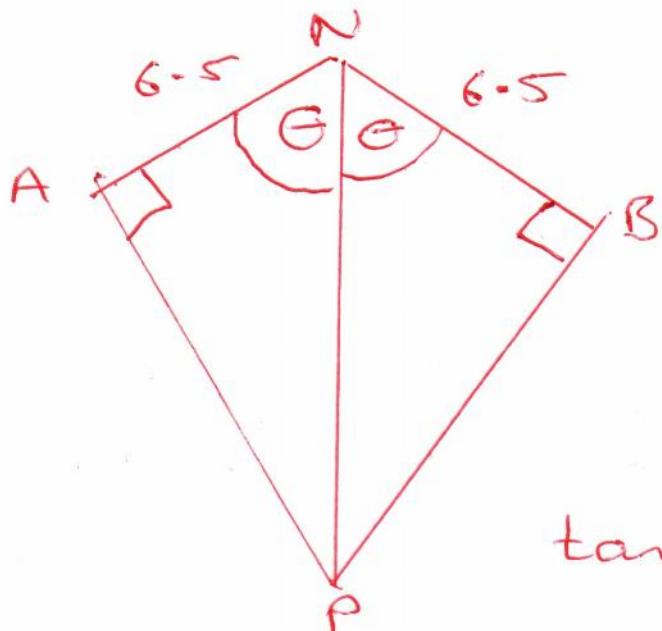
$$\cos N = -0.704142$$

$$N = 134.76027$$

$$= 134.8^\circ$$

to nearest 0.1°

8e



$$AP = BP \\ (\text{both tangent})$$

We found angle
 2θ in part d)

$$\therefore \theta = 67.380135^\circ$$

$$\tan \theta = \frac{AP}{6.5}$$

$$\begin{aligned} AP &= 6.5 \times \tan 67.380135^\circ \\ &= 15.6 \text{ units (3sf)} \end{aligned}$$

9. The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C ,

- (a) show that the centre of C has coordinates $(3, 6)$, (1)

- (b) find an equation for C . (4)

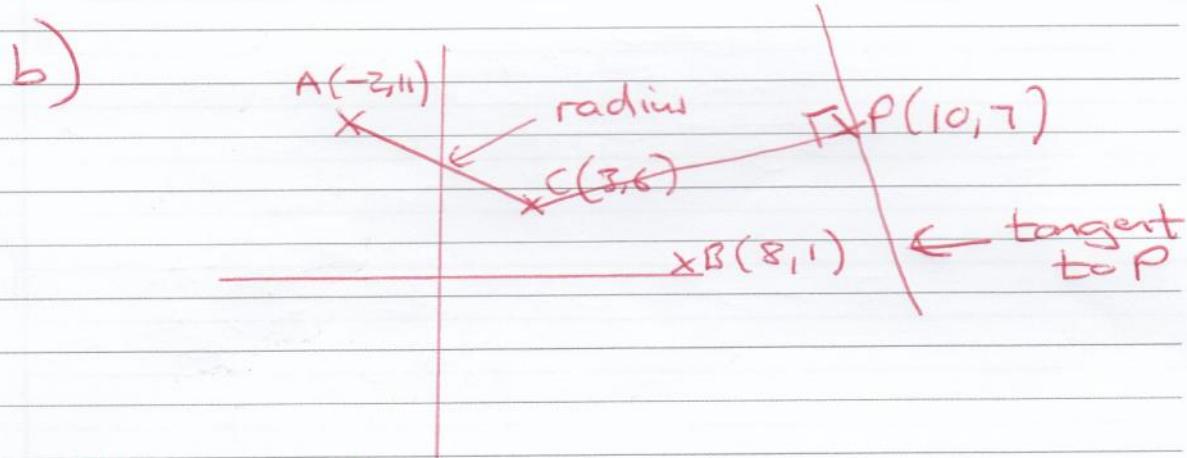
- (c) Verify that the point $(10, 7)$ lies on C . (1)

- (d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants. (4)

a) C is mid point of AB

$$C = \left(\frac{-2+8}{2}, \frac{11+1}{2} \right) = \left(\frac{6}{2}, \frac{12}{2} \right) = (3, 6)$$

as required



Radius of circle = AC

$$\begin{aligned} \text{Length } AC &= \sqrt{(11-6)^2 + (-2-3)^2} \\ &= \sqrt{25+25} \\ &= \sqrt{50} \end{aligned}$$

Equation of circle with centre $(3, 6)$ and radius length $\sqrt{50}$

is $(x-3)^2 + (y-6)^2 = (\sqrt{50})^2$
 $(x-3)^2 + (y-6)^2 = 50$



Question 9 continued

c) put $x = 10$ and $y = 7$ in equation of circle

$$(10-3)^2 + (7-6)^2 = 50$$

$$7^2 + 1^2 = 50$$

$$49 + 1 = 50$$

So the point $(10, 7)$ is on C

d)

CP is a radius where C is centre,
 P is point $(10, 7)$

$$\text{Gradient of } CP = \frac{7-6}{10-3} = \frac{1}{7}$$

\therefore Gradient of tangent to P = -7

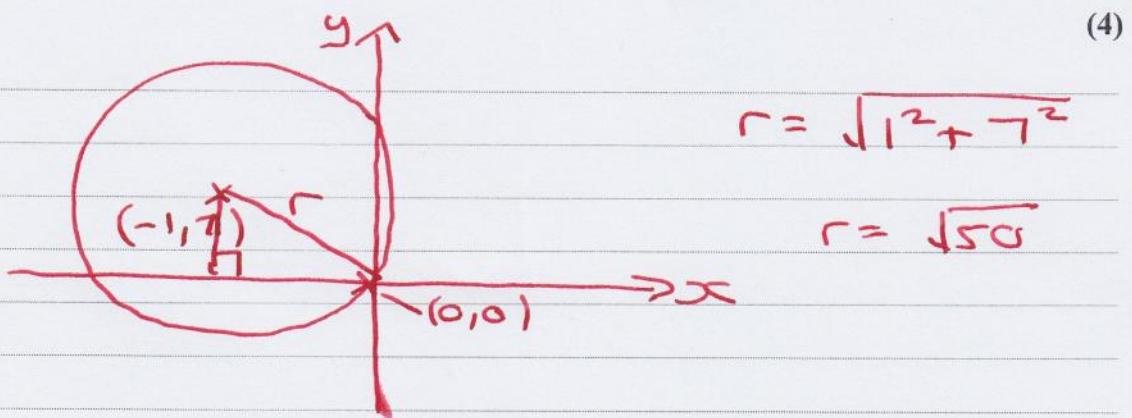
$$y - y_1 = m(x - x_1)$$

$$y - 7 = -7(x - 10)$$

$$y - 7 = -7x + 70$$

$y = -7x + 77$ is equation of tangent at $(10, 7)$

2. A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C .



Equation of circle is
 $(x + a)^2 + (y + b)^2 = r^2$

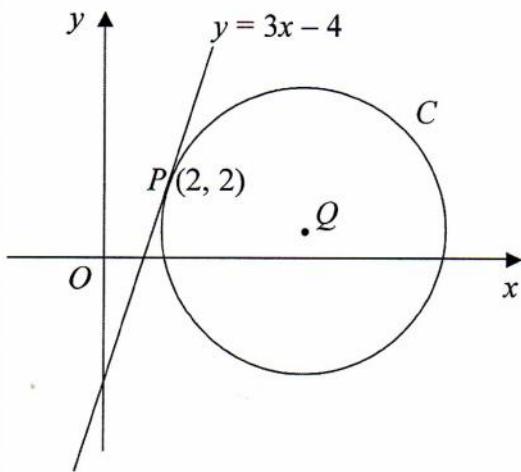
(centre of circle is $(-1, 7)$)

$$(x + 1)^2 + (y - 7)^2 = 50$$



7.

Figure 1



The line $y = 3x - 4$ is a tangent to the circle C , touching C at the point $P(2, 2)$, as shown in Figure 1.

The point Q is the centre of C .

- (a) Find an equation of the straight line through P and Q .

(3)

Given that Q lies on the line $y = 1$,

- (b) show that the x -coordinate of Q is 5,

(1)

- (c) find an equation for C .

(4)

a) Gradient of PQ is perpendicular to tangent

$$\text{Gradient of tangent} = 3$$

$$\therefore \text{gradient of (perpendicular)} PQ = -\frac{1}{3}$$

\therefore Equation of line through PQ is

$$y - y_1 = m(x - x_1) \quad \text{where } P(2, 2)$$

$$y - 2 = -\frac{1}{3}(x - 2)$$

$$3y - 6 = -(x - 2)$$

$$3y - 6 = -x + 2$$

$$3y = x + 8$$

b) put $y=1$ in above equation

$$\therefore 3(1) = -x + 8$$

$$x = 8 - 3 \quad \therefore x = 5$$



$$7(c) \text{ Radius } QP = \sqrt{(2-5)^2 + (2-1)^2}$$

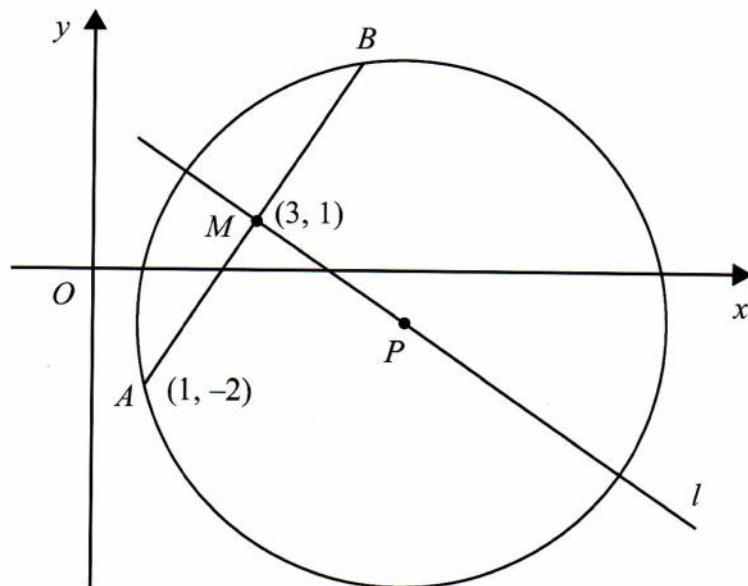
$$= \sqrt{9+1}$$

$$= \sqrt{10}$$

\therefore Equation of C is

$$(x-5)^2 + (y-1)^2 = 10$$

7.

**Figure 3**

The points A and B lie on a circle with centre P , as shown in Figure 3.
 The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
 The line l passes through the points M and P .

- (a) Find an equation for l .

(4)

Given that the x -coordinate of P is 6,

- (b) use your answer to part (a) to show that the y -coordinate of P is -1 ,

(1)

- (c) find an equation for the circle.

(4)

a) gradient of $AB = \frac{1 - (-2)}{3 - 1} = \frac{3}{2}$

\therefore gradient of L is $-\frac{2}{3}$ (as M is

mid point of AB and L is a diameter,
 they are perpendicular)

$$y - y_1 = m(x - x_1) \quad (\text{using } M(3, 1))$$

$$\therefore y - 1 = -\frac{2}{3}(x - 3)$$

$$y = -\frac{2}{3}(x - 3) + 1$$

$$y = -\frac{2}{3}x + 2 + 1$$

$$y = -\frac{2}{3}x + 3$$



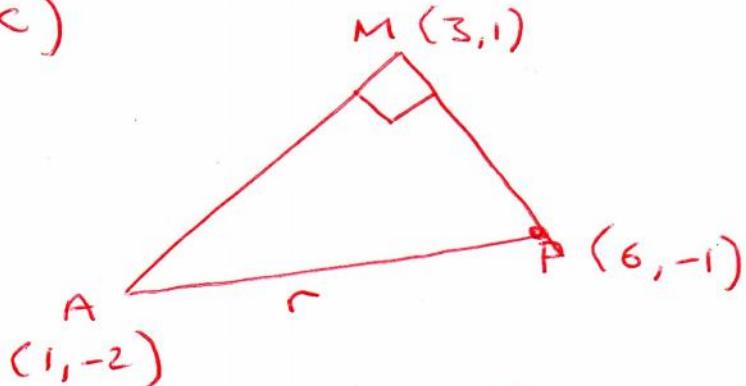
Tb) x coordinate is 6

$$y = -\frac{2}{3}(6) + 3$$

$$y = -4 + 3$$

$$y = -1 \quad (\text{as required})$$

c)



$$r^2 = (6-1)^2 + (-1-(-2))^2$$

$$r^2 = 5^2 + 1^2$$

$$r^2 = 26$$

Equation of circle

$(x-a)^2 + (y-b)^2 = r^2$ where (a,b)
is centre of circle
and r is the
radius

$$(x-6)^2 + (y-(-1))^2 = 26$$

$$(x-6)^2 + (y+1)^2 = 26$$

5. The circle C has centre $(3, 1)$ and passes through the point $P(8, 3)$.

(a) Find an equation for C .

(4)

(b) Find an equation for the tangent to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(5)

a) Equation of C is
of the form

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

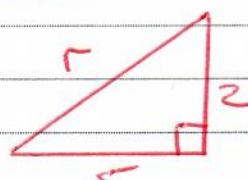
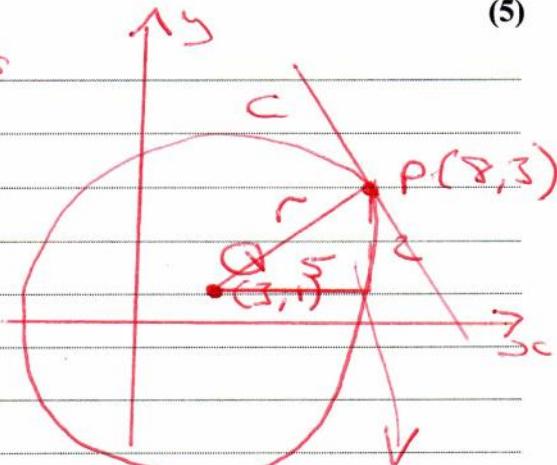
$$r^2 = (8-3)^2 + (3-1)^2$$

$$r^2 = 5^2 + 2^2$$

$$r^2 = 25 + 4$$

$$r^2 = 29$$

$$\therefore r = \sqrt{29}$$



\therefore Equation of circle C is
 $(x - 3)^2 + (y - 1)^2 = 29$

b) Have got y_1 and x_1 , but need gradient.

First find gradient of
Line PQ

$$= \frac{3-1}{8-3} = \frac{2}{5}$$

So gradient of tangent at P is $-\frac{5}{2}$
as it is perpendicular to PQ

Using $y - y_1 = m(x - x_1)$

\therefore Equation of tangent at P is

$$y - 3 = -\frac{5}{2}(x - 8)$$

$$\therefore 2y - 6 = -5(x - 8)$$

$$\therefore 2y - 6 = -5x + 40$$

$$\therefore 5x + 2y - 46 = 0$$



6. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

- (a) Find the centre and the radius of C .

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C .

- (b) Show that PQ is a diameter of C .

(2)

The point R lies on the positive y -axis and the angle $PRQ = 90^\circ$.

- (c) Find the coordinates of R .

(4)

$$\begin{aligned} a) \quad & (x-3)^2 - 9 + (y+2)^2 - 4 = 12 \\ & (x-3)^2 + (y+2)^2 = 12 + 9 + 4 \\ & (x-3)^2 + (y+2)^2 = 25 \end{aligned}$$

Equation of Circle

$$(x-a)^2 + (y-b)^2 = r^2$$

where (a, b) is circle centre
and radius $= r$

Centre is $(3, -2)$
and radius $= 5$

$$b) \quad PQ = \sqrt{(7-(-1))^2 + (-5-1)^2}$$

$$PQ = \sqrt{8^2 + (-6)^2}$$

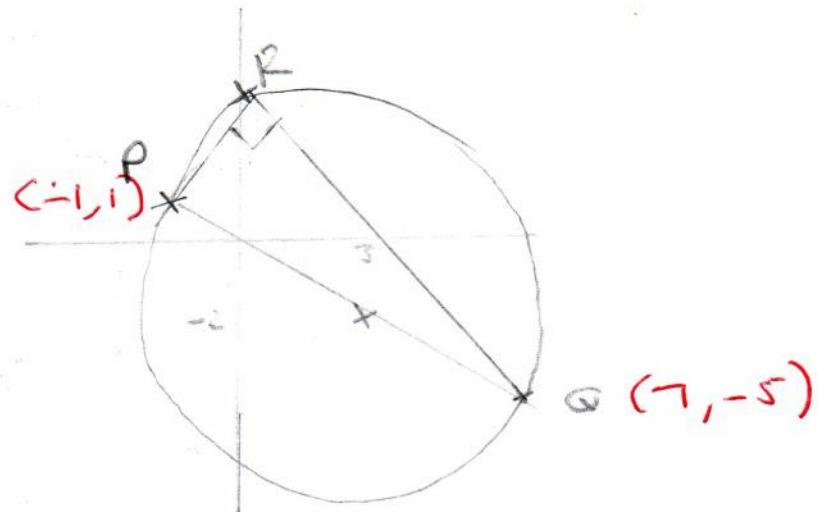
$$PQ = \sqrt{64 + 36}$$

$$PQ = \sqrt{100} = 10$$

As PQ is exactly twice the length of the radius it must be the diameter.



6c)



R must lie on the circle
as angle in semicircle = 90°
(PQ is diameter)

Initial equation of circle was

$$x^2 + y^2 - 6x + 4y = 12$$

on y -axis, $x = 0$

$$\text{so } 0^2 + y^2 - 0 + 4y = 12$$

$$y^2 + 4y - 12 = 0$$

$$(y+6)(y-2) = 0$$

Either $y = -6$ or $y = 2$

As we are told R lies on
positive y -axis

Coordinates of R are $(0, 2)$

10. The circle C has centre $A(2, 1)$ and passes through the point $B(10, 7)$.

(a) Find an equation for C .

(4)

The line l_1 is the tangent to C at the point B .

(b) Find an equation for l_1 .

(4)

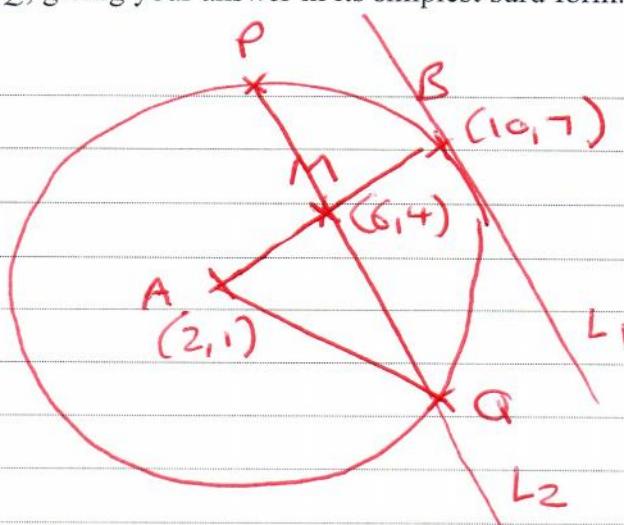
The line l_2 is parallel to l_1 and passes through the mid-point of AB .

Given that l_2 intersects C at the points P and Q ,

(c) find the length of PQ , giving your answer in its simplest surd form.

(3)

a)



$$\begin{aligned} \text{Length } AB &= \sqrt{(10-2)^2 + (7-1)^2} \\ &= \sqrt{8^2 + 6^2} \\ &= 10 \quad \text{← (radius of circle)} \end{aligned}$$

Equation of circle

$$\frac{(x-2)^2}{(x-2)^2} + \frac{(y-1)^2}{(y-1)^2} = 10^2$$

b) Gradient $AB = \frac{7-1}{10-2} = \frac{6}{8} = \frac{3}{4}$

\therefore Gradient of l_1 (perpendicular) is $-\frac{4}{3}$



10b) continued

Equation of L.

$$y - y_1 = m(x - x_1)$$

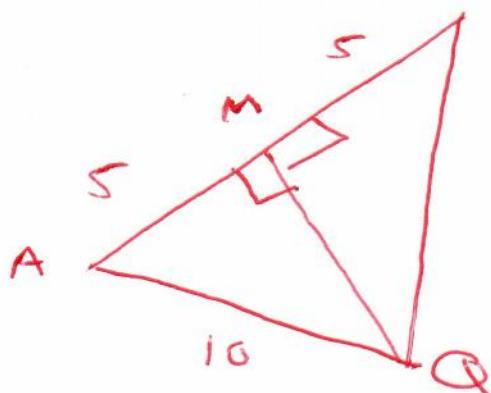
$$y - 7 = -\frac{4}{3}(x - 10)$$

$$3(y - 7) = -4(x - 10)$$

$$3y - 21 = -4x + 40$$

$$3y = -4x + 61$$

c) mid point of AB is $\left(\frac{10+2}{2}, \frac{7+1}{2}\right) = (6, 4)$



By Pythagoras

$$MQ = \sqrt{10^2 - 5^2}$$

$$PQ = 2 \times MQ$$

$$PQ = 2 \sqrt{10^2 - 5^2}$$

$$PQ = 2 \sqrt{100 - 25}$$

$$PQ = 2 \sqrt{75}$$

$$= 2 \sqrt{25} \sqrt{3}$$

$$= 2 \times 5 \times \sqrt{3}$$

$$= 10\sqrt{3} \text{ units}$$

4. The circle C has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0$$

Find

- (a) the coordinates of the centre of C ,

(2)

- (b) the radius of C ,

(2)

- (c) the coordinates of the points where C crosses the y -axis, giving your answers as simplified surds.

(4)

a) $x^2 + y^2 + 4x - 2y - 11 = 0$

$$x^2 + 4x + y^2 - 2y - 11 = 0$$

Completing the square

$$(x+2)^2 - 4 + (y-1)^2 - 1 - 11 = 0$$

$$(x+2)^2 + (y-1)^2 - 16 = 0$$

$$(x+2)^2 + (y-1)^2 = 16$$

Centre of circle is $(-2, 1)$

b) radius = $\sqrt{16} = 4$

c) at y -axis, $x = 0$

$$x^2 + y^2 + 4x - 2y - 11 = 0$$

$$y^2 - 2y - 11 = 0$$

$$a=1, b=-2, c=-11$$

$$y = \frac{-b \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2}$$



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Question 4 continued

$$y = \frac{2 + \sqrt{48}}{2} \quad \text{or} \quad y = \frac{2 - \sqrt{48}}{2}$$

$$y = \frac{2 + \sqrt{16}\sqrt{3}}{2} \quad \text{or} \quad y = \frac{2 - \sqrt{16}\sqrt{3}}{2}$$

$$y = \frac{2 + 4\sqrt{3}}{2} \quad \text{or} \quad y = \frac{2 - 4\sqrt{3}}{2}$$

$$y = 1 + 2\sqrt{3} \quad \text{or} \quad y = 1 - 2\sqrt{3}$$

Coordinates are

$$(0, 1 + 2\sqrt{3})$$

$$(0, 1 - 2\sqrt{3})$$



P 3 8 1 5 8 A 0 9 3 2

3.

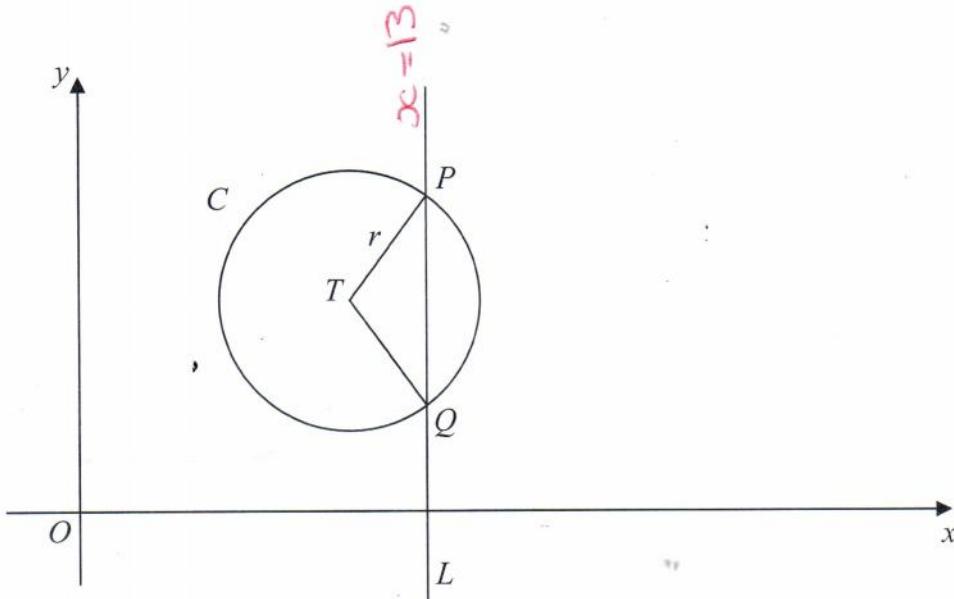


Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

- (a) Find the coordinates of the centre of C . (3)
- (b) Show that $r = 5$ (2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

- (c) Find the y coordinate of P and the y coordinate of Q . (3)

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,

- (d) find the perimeter of the sector PTQ . (3)

a) $x^2 - 20x + y^2 - 16y + 139 = 0$
 Completing the square
 $(x-10)^2 - 10^2 + (y-8)^2 - 8^2 + 139 = 0$
 $(x-10)^2 + (y-8)^2 = 100 + 64 - 139$
 $(x-10)^2 + (y-8)^2 = 25$

b) Centre of circle is $(10, 8)$
 $r^2 = 25$ so $r = 5$



Question 3 continued

c) put $x = 13$ into circle equation

$$13^2 + y^2 - 20 \times 13 - 16y + 139 = 0$$

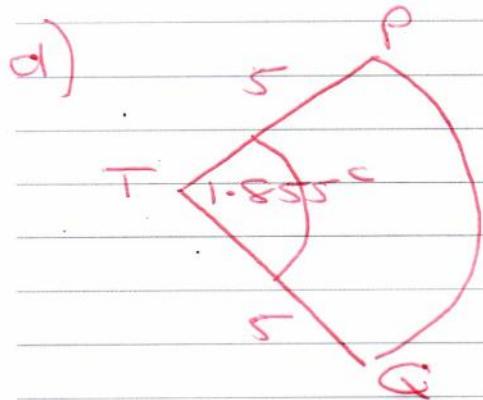
$$169 + y^2 - 260 - 16y + 139 = 0$$

$$y^2 - 16y + 48 = 0$$

$$(y - 12)(y - 4) = 0$$

$$y = 12 \text{ or } y = 4$$

Coordinate of P is (13, 12)
 Coordinate of Q is (13, 4)



$$\begin{aligned} \text{length } PQ &= r\theta \\ &= 5 \times 1.855 \\ &= 9.275 \end{aligned}$$

$$\begin{aligned} \text{Perimeter of PTQ} &= 5 + 5 + 9.275 \\ &= 19.275 \text{ units} \end{aligned}$$



5. The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M .

- (a) Find

- (i) the coordinates of the point M ,
- (ii) the radius of the circle C .

(5)

N is the point with coordinates $(25, 32)$.

- (b) Find the length of the line MN .

(2)

The tangent to C at a point P on the circle passes through point N .

- (c) Find the length of the line NP .

(2)

$$\begin{aligned} \text{a)} \quad & x^2 - 20x + y^2 - 24y + 195 = 0 \\ & (x-10)^2 - 10^2 + (y-12)^2 - 12^2 + 195 = 0 \\ & (x-10)^2 + (y-12)^2 - 100 - 144 + 195 = 0 \\ & (x-10)^2 + (y-12)^2 = 49 \end{aligned}$$

(i) M is $(10, 12)$

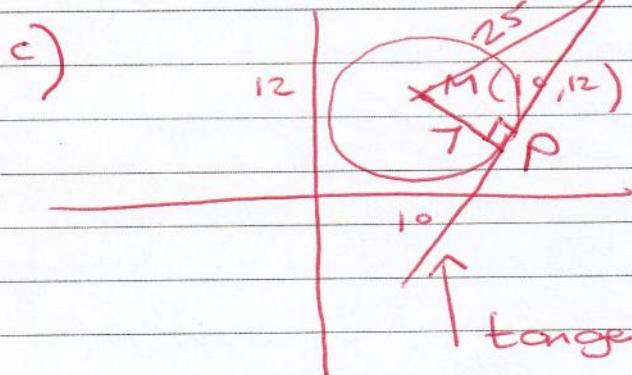
(ii) radius = 7

b) N is $(25, 32)$

$$MN = \sqrt{(25-10)^2 + (32-12)^2}$$

using
Pythagoras
Theorem

$$\underline{MN = 25}$$



$$\begin{aligned} PN &= \sqrt{25^2 - 7^2} \\ &= \underline{\underline{24}} \end{aligned}$$

tangent 90° to radius



10.

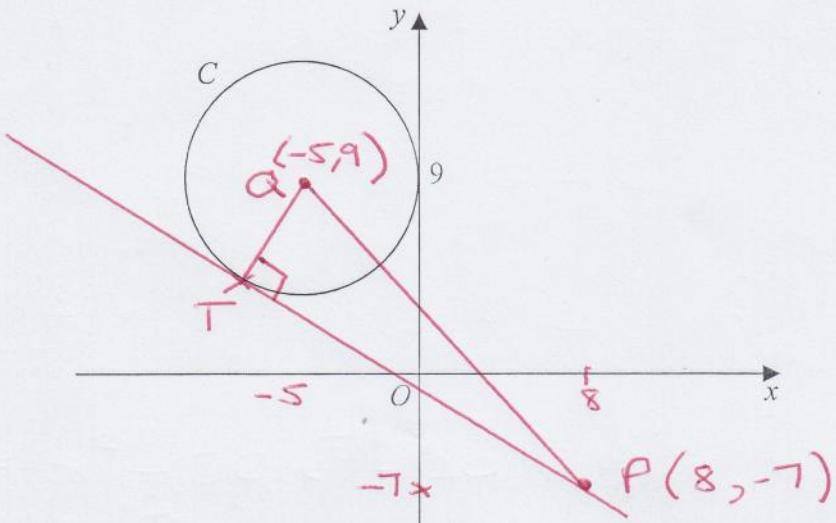


Figure 4

The circle C has radius 5 and touches the y -axis at the point $(0, 9)$, as shown in Figure 4.

- (a) Write down an equation for the circle C , that is shown in Figure 4.

(3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T .

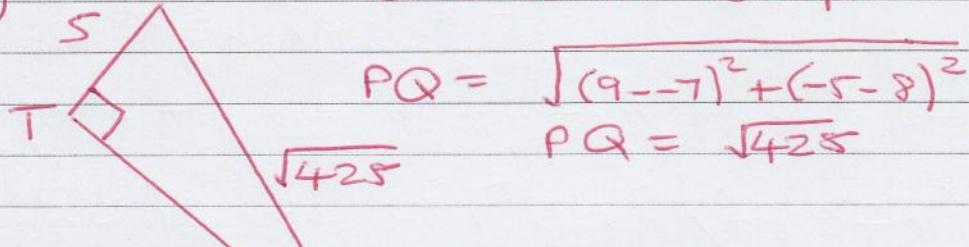
- (b) Find the length of PT .

(3)

a) centre is $(-5, 9)$ radius is 5

$$(x+5)^2 + (y-9)^2 = 25$$

b) Let centre be point Q



$$\begin{aligned} TP &= \sqrt{(\sqrt{425})^2 - 5^2} \\ &= \sqrt{400} \end{aligned}$$

$$\underline{\underline{TP = 20 \text{ units}}}$$

