9.

Figure 2

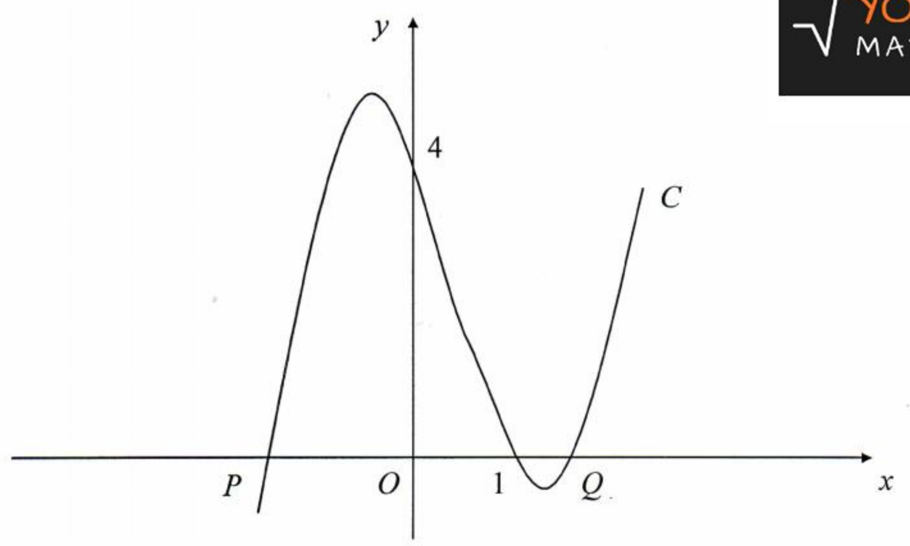


Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, $(1, 0)$ and Q, as shown in Figure 2.

(*a*) Write down the x-coordinate of P and the x-coordinate of Q.

(2)

(*b*) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$.

(3)

(*c*) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$.

(2)

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(*d*) Find the exact coordinates of R.

(5)

---

a) x coordinate for P is -2.
x coordinate for Q is 2

b) $y = (x-1)(x^2-4)$
$y = x^3 - 4x - x^2 + 4$
$y = x^3 - x^2 - 4x + 4$

$\frac{dy}{dx} = 3x^2 - 2x - 4$ as required

9c) At point $(-1, 6)$ find gradient

$$\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4$$
$$\frac{dy}{dx} = 3 + 2 - 4$$
$$\frac{dy}{dx} = 1$$

Equation of tangent

$$y - y_1 = m(x - x_1)$$
$$y - 6 = 1(x - (-1))$$
$$y - 6 = x + 1$$
$$y = x + 7$$ is equation of tangent as required

d) If parallel, gradient of tangent at R is 1

$$\therefore \quad 3x^2 - 2x - 4 = 1$$
$$3x^2 - 2x - 5 = 0$$
$$(3x - 5)(x + 1) = 0$$

Either $3x - 5 = 0$ or $x + 1 = 0$
$x = \frac{5}{3}$ $\quad x = -1$

To find coordinates of R

$$y = \left(\frac{5}{3} - 1\right)\left(\left(\frac{5}{3}\right)^2 - 4\right)$$
$$y = \left(\frac{2}{3}\right)\left(\frac{25}{9} - 4\right)$$
$$= \frac{2}{3}\left(\frac{25}{9} - \frac{36}{9}\right)$$
$$= \frac{2}{3}\left(-\frac{11}{9}\right) = -\frac{22}{27}$$

coordinates of R are $\left(\frac{5}{3}, -\frac{22}{27}\right)$

Leave blank

7. The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find $f(x)$. **(5)**

To get $f(x)$ from $f'(x)$ you need to integrate

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers. **(4)**

a) $f'(x) = 3x^2 - 6 - \frac{8}{x^2}$

$= 3x^2 - 6 - 8x^{-2}$

$f(x) = \int (3x^2 - 6 - 8x^{-2})\,dx$

$f(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1} + c$

$f(x) = x^3 - 6x + \frac{8}{x} + c$

We are given point $P(2,1)$ lies on C

$\therefore 1 = 2^3 - 6(2) + \frac{8}{2} + c$

$1 - 2^3 + 6(2) - \frac{8}{2} = c$

$1 - 8 + 12 - 4 = c$

$c = 1$

$\therefore f(x) = x^3 - 6x + \frac{8}{x} + 1$

b) We need the gradient at point P $(x = 2)$

gradient $= f'(x) = 3(2^2) - 6 \;\; \frac{8}{2^2}$

$= 12 - 6 - 2$

$= 4$

Using $y - y_1 = m(x - x_1)$ with $P(2,1)$

$y - 1 = 4(x - 2)$

$y - 1 = 4x - 8$

$y = 4x - 7$ is the equation of tangent in form $y = mx + c$

Leave blank

8. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, \quad x > 0.$

(a) Find an expression for $\frac{dy}{dx}$.

(3)

(b) Show that the point P (4, 8) lies on C.

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

(3)

a) $y = 4x + 3x^{\frac{3}{2}} - 2x^2$

$\frac{dy}{dx} = 4 + \frac{3}{2} \times 3x^{\frac{1}{2}} - 4x$

$\frac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$

b) $P(4,8)$

put $x=4$ in $y = 4x + 3x^{\frac{3}{2}} - 2x^2$

$y = 4 \times 4 + 3 \times (4)^{\frac{3}{2}} - 2(4^2)$

$y = 16 + (3 \times 8) - 2(16)$

$y = 16 + 24 - 32$

$y = 8$

$\therefore$ point $P(4,8)$ lies on C

c) gradient at $P(4,8)$ — $x=4$

$\frac{dy}{dx} = 4 + \frac{9}{2} 4^{\frac{1}{2}} - (4)(4)$

$= 4 + \frac{9}{\cancel{2}_1} \times \cancel{2}^1 - 16$

$= -3$

$\therefore$ Gradient of normal $= \frac{1}{3}$

8c) continued

Equation of normal (through $P(4,8)$)

$y - y_1 = m(x - x_1)$

$y - 8 = \frac{1}{3}(x - 4)$

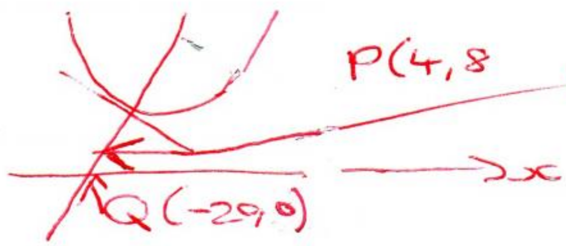
$\times$ through by 3

$3y - 24 = (x - 4)$

$3y = 24 \quad x - 4$

$3y = x + 20$ as required

d)



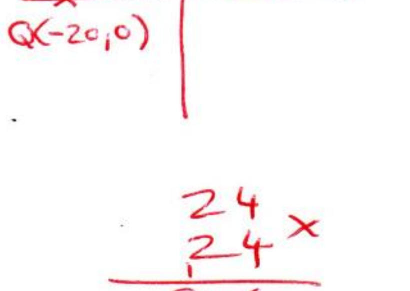
Normal cuts x-axis at Q where $y = 0$

$\therefore \; 3 \times 0 = x + 20$

$x = -20$

point Q is $(-20, 0)$

P is $(4, 8)$



Length PQ (Pythagoras)

$= \sqrt{(4 - -20)^2 + (8 - 0)^2}$

$= \sqrt{24^2 + 8^2}$

$= \sqrt{576 + 64}$

$= \sqrt{640}$

$= \sqrt{64} \times \sqrt{10}$

Length PQ $= 8\sqrt{10}$

$$\begin{array}{r} 24 \\ \times \; 24 \\ \hline 96 \\ 480 + \\ \hline 576 \end{array}$$

$$\begin{array}{r} 576 \\ 64 + \\ \hline 640 \end{array}$$

Leave blank

9. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $f(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

a) $f'(x) = 4x - 6x^{\frac{1}{2}} + 8x^{-2}$

To get $f(x)$ we need to integrate

$f(x) = \frac{4x^2}{2} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{8x^{-1}}{-1} + c$

$f(x) = 2x^2 - 4x^{\frac{3}{2}} - 8x^{-1} + c$

$f(x) = 2x^2 - 4x^{\frac{3}{2}} - \frac{8}{x} + c$

Point $P(4,1)$ lies on C, so we this with $f(x) = 1$, $x = 4$

$1 = 2(4)^2 - 4(4)^{\frac{3}{2}} - \frac{8}{4} + c$

$1 = 32 - 32 - 2 + c$

$c = 3$

$\therefore f(x) = 2x^2 - 4x^{\frac{3}{2}} - \frac{8}{x} + 3$

b) gradient at $P(4,1)$

$f'(x) = (4 \times 4) - (6 \times \sqrt{4}) + \frac{8}{4^2}$

$f'(x) = 16 - 12 + \frac{1}{2}$

$f'(x) = 4\frac{1}{2} = \frac{9}{2}$

$\therefore$ gradient of normal at P is $-\frac{2}{9}$

Equation of normal is $y - y_1 = m(x - x_1)$

$y - 1 = -\frac{2}{9}(x - 4)$

$y - 1 = -\frac{2}{9}x + \frac{8}{9}$

$y = -\frac{2}{9}x + \frac{17}{9}$

Leave blank

11. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0.$$ $y = 9 - 4x - 8x^{-1}$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB.

(4)

a) find y coordinate on P

$y = 9 - 4(2) - \frac{8}{2} = 9 - 8 - 4 = -3$

$\frac{dy}{dx} = -4 + 8x^{-2}$

$\frac{dy}{dx} = -4 + \frac{8}{x^2}$

at P, $x = 2$

$\frac{dy}{dx} = -4 + \frac{8}{2^2} = -4 + \frac{8}{4} = -2$

Find equation of tangent using

$y - y_1 = m(x - x_1)$ using $P(2, -3)$

$y - (-3) = -2(x - 2)$

$y + 3 = -2x + 4$

$y = -2x + 1$ as required

b) Equation of normal at $P(2, -3)$

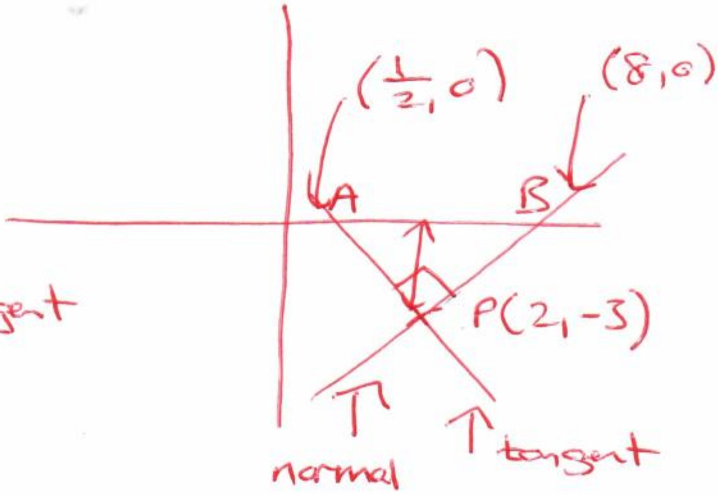
Gradient of normal is $\frac{1}{2}$

$y - (-3) = \frac{1}{2}(x - 2)$

$y + 3 = \frac{1}{2}x - 1$

$y = \frac{1}{2}x - 4$

11c)



To find point A

$y = 0$, $x = ?$

using equation of tangent

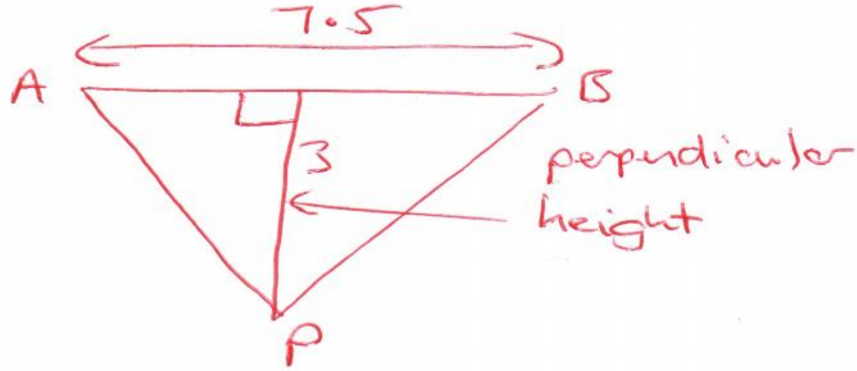
$$0 = -2x + 1$$
$$x = \frac{1}{2}$$

To find point B

$y = 0$, $x = ?$

using equation of normal

$$0 = \frac{1}{2}x - 4$$
$$x = 8$$



$$\text{Area of triangle } ABH = \frac{1}{2} \times 7.5 \times 3$$
$$= \frac{1}{2} \times \frac{15}{2} \times 3$$
$$= \frac{45}{4} \text{ units squared}$$

C1 JAN 2011

11. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0$$

(a) Find $\frac{dy}{dx}$.

(4)

(b) Show that the point $P(4, -8)$ lies on C.

(2)

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

a) $y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + 8x^{-1} + 30$

$\frac{dy}{dx} = \frac{3}{2}x^2 - \frac{3}{2} \times 9x^{\frac{1}{2}} - 8x^{-2}$

$\frac{dy}{dx} = \frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$

b) $y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30$

at $x = 4$

$y = \frac{1}{2}(4^3) - 9(4^{\frac{3}{2}}) + \frac{8}{4} + 30$

$y = 32 - 9 \times 8 + 2 + 30$

$y = 32 - 72 + 2 + 30$

$y = -8$

So point $P(4, -8)$ lies on C

c) at $x = 4$, $\frac{dy}{dx} = \frac{3}{2}(4^2) - \frac{27}{2}(4^{\frac{1}{2}}) - \frac{8}{4^2}$

$\frac{dy}{dx} = 24 - 27 - \frac{1}{2} = -3\frac{1}{2} = -\frac{7}{2}$

$\therefore$ Gradient of normal $= \frac{2}{7}$

$y - y_1 = m(x - x_1)$

$y - (-8) = \frac{2}{7}(x - 4)$

$y + 8 = \frac{2}{7}x - \frac{8}{7}$

$7y + 56 = 2x - 8$

$2x - 7y - 64 = 0$ is equation of normal

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10.

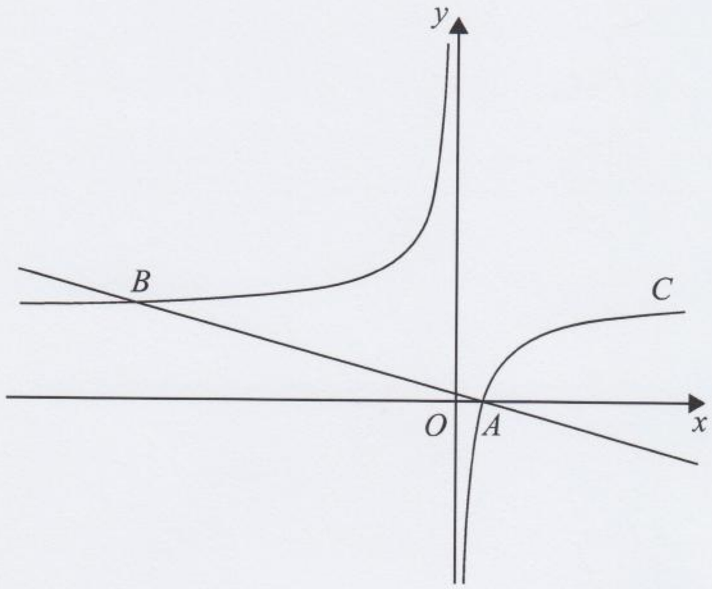


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \quad x \neq 0$$

The curve crosses the x-axis at the point A.

(a) Find the coordinates of A.

(1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0$$

(6)

The normal to C at A meets C again at the point B, as shown in Figure 2.

(c) Find the coordinates of B.

(4)

a) x-axis $\quad y = 0$

$0 = 2 - \frac{1}{x}$

$\frac{1}{x} = 2$

$x = \frac{1}{2}$

C1 Jan 2012

10 b) $y = 2 - x^{-1}$

$\frac{dy}{dx} = x^{-2} = \frac{1}{x^2}$

at $x = \frac{1}{2}$, $\frac{dy}{dx} = \frac{1}{(\frac{1}{2})^2} = \frac{1}{\frac{1}{4}} = 4$

gradient of normal to C at A $= -\frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{4}(x - \frac{1}{2})$ (x by 8)

$8y = -2(x - \frac{1}{2})$

$8y = -2x + 1$

$2x + 8y - 1 = 0$ is equation of normal

c) $2x + 8(2 - \frac{1}{x}) - 1 = 0$

$2x + 16 - \frac{8}{x} - 1 = 0$ (x by x)

$2x^2 + 16x - 8 - x = 0$

$2x^2 + 15x - 8 = 0$

$(2x - 1)(x + 8) = 0$

$x = \frac{1}{2}$ or $x = -8$

$y = 2 - \frac{1}{-8}$

$y = 2\frac{1}{8}$

Coordinates of B are $(-8, 2\frac{1}{8})$

10. The curve C has equation $y = x^2(x-6) + \frac{4}{x},\ x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$.

(4)

(b) Show that the tangents to C at P and Q are parallel.

(5)

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

a) when $x = 1$, $y = 1^2(1-6) + \frac{4}{1}$

$= -1$

$\therefore P(1, -1)$

when $x = 2$, $y = 2^2(2-6) + \frac{4}{2}$

$= -14$

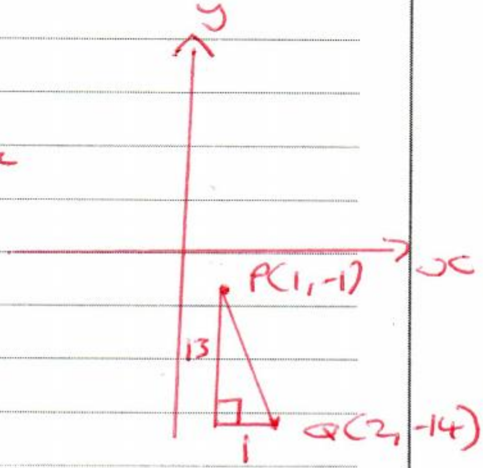
$\therefore Q(2, -14)$

$\therefore PQ = \sqrt{(2-1)^2 + (-1--14)^2}$

$= \sqrt{1^2 + 13^2}$

$= \sqrt{1 + 169}$

$= \sqrt{170}$



b) $y = x^2(x-6) + \frac{4}{x}$

$\therefore y = x^3 - 6x^2 + 4x^{-1}$ then differentiate to find gradient of C

$\therefore \frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}$

$= 3x^2 - 12x - \frac{4}{x^2}$

When $x = 1$ (x co-ord at P), $\frac{dy}{dx} = 3\times(1)^2 - 12(1) - \frac{4}{(1)^2}$

$= 3 - 12 - 4$

$= -13$

When $x = 2$ (x co-ord of Q), $\frac{dy}{dx} = 3\times(2)^2 - 12(2) - \frac{4}{(2)^2}$

$= 12 - 24 - 1$

$= -13$

$\therefore$ Tangents are parallel at P and Q as they have the same gradient -13

10 c)

At P, gradient of the normal $= \frac{1}{13}$

$y - y_1 = m(x - x_1)$

at P

$(1, -1)$

$x_1 = 1, \; y_1 = -1$

Gradient at P was -13

Perpendicular gradient $= \frac{1}{13}$

$\therefore$ Equation of normal at P is

$y - (-1) = \frac{1}{13}(x - 1)$

$\therefore 13y + 13 = x - 1$

$\therefore x - 13y - 14 = 0$

9. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

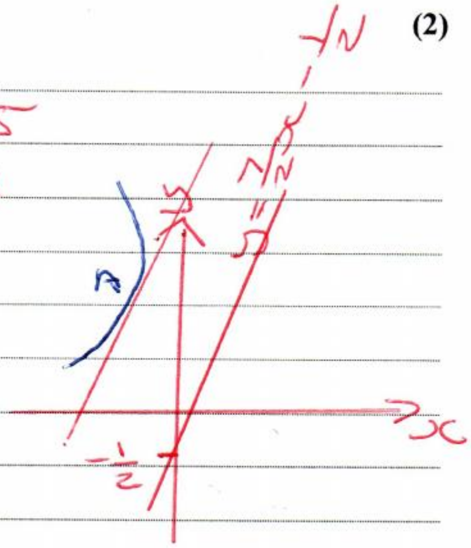
(a) Find $\frac{dy}{dx}$. **(2)**

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k, **(4)**

(c) the value of the y-coordinate of A. **(2)**



a) $y = kx^3 - x^2 + x - 5$

$\frac{dy}{dx} = 3kx^2 - 2x + 1$

b) At point A, gradient to curve is parallel to line $y = \frac{7}{2}x - \frac{1}{2}$, so it has gradient $\frac{7}{2}$

At A, x co-ordinate is $-\frac{1}{2}$

$2y - 7x + 1 = 0$

$2y = 7x - 1$

$y = \frac{7}{2}x - \frac{1}{2}$

when $x = -\frac{1}{2}$, $\frac{dy}{dx} = \frac{7}{2}$

$\therefore \frac{7}{2} = 3k\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1$

$\therefore \frac{7}{2} = \frac{3}{4}k + 1 + 1$

$\therefore \frac{7}{2} = \frac{3}{4}k + 2$ $\times$ both sides by 4 to get rid of fractions

$\therefore 14 = 3k + 8$

$3k = 6$

$k = 2$

c) at A $x = -\frac{1}{2}$, $k = 2$

$y = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 5$

$y = -\frac{1}{4} - \frac{1}{4} - \frac{1}{2} - 5 = -6$ so $y = -6$

Leave blank

10. The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $\mathrm{f}'(x) = 2x + \dfrac{3}{x^2}$,

(a) find $\mathrm{f}(x)$.

(5)

(b) Verify that $\mathrm{f}(-2) = 5$.

(1)

(c) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

a) $f'(x) = 2x + 3x^{-2}$

Integrate to get $f(x)$

$f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1} + c$

$f(x) = x^2 - \frac{3}{x} + c$

passes through $(3, 7\frac{1}{2})$

$7\frac{1}{2} = 3^2 - \frac{3}{3} + c$

$7\frac{1}{2} = 9 - 1 + c$

$7\frac{1}{2} = 8 + c$

$c = -\frac{1}{2}$

$f(x) = x^2 - \frac{3}{x} - \frac{1}{2}$

b) $f(-2) = (-2)^2 - \frac{3}{(-2)} - \frac{1}{2}$

$= 4 + \frac{3}{2} - \frac{1}{2}$

$= 5$ as required

c) Use $f'(x) = 2x + \frac{3}{x^2}$ at point $(-2, 5)$ to get gradient

$f'(x) = 2(-2) + \frac{3}{(-2)^2} = -4 + \frac{3}{4} = -3\frac{1}{4} = \frac{-13}{4}$

10c (continued)

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{13}{4}(x - (-2))$$

$$y - 5 = -\frac{13}{4}(x + 2)$$

× through by 4

$$4y - 20 = -13x - 26$$

$$13x + 4y + 6 = 0$$

in form $ax + by + c = 0$

May 2010

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11. The curve C has equation $y = f(x)$, $x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on C, find

(a) $f(x)$, **(5)**

(b) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

a) $f'(x) = 3x - 5x^{-\frac{1}{2}} - 2$

$f(x) = \frac{3x^2}{2} - \frac{5x^{\frac{1}{2}}}{\frac{1}{2}} - 2x + c$

$f(x) = \frac{3}{2}x^2 - 10x^{\frac{1}{2}} - 2x + c$

$f(x) = 5$ when $x = 4$

$\therefore 5 = \frac{3}{2}(4^2) - 10(4^{\frac{1}{2}}) - 2(4) + c$

$5 = 24 - 20 - 8 + c$

$5 - 24 + 20 + 8 = c$

$c = 9$

$\therefore f(x) = \frac{3}{2}x^2 - 10x^{\frac{1}{2}} - 2x + 9$

b) gradient at $C = \frac{dy}{dx}$

$\frac{dy}{dx} = 3(4) - \frac{5}{\sqrt{4}} - 2$

$= 12 - \frac{5}{2} - 2$

$= 7.5 = \frac{15}{2}$

$y - y_1 = m(x - x_1)$ using $m = \frac{15}{2}$ and $P(4,5)$

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11 b) continued

$$y - 5 = \frac{15}{2}(x - 4)$$

$$2(y-5) = 15(x-4)$$

$$2y - 10 = 15x - 60$$

$$0 = 15x - 2y + 10 - 60$$

$$0 = 15x - 2y - 50$$

in form $ax + by + c = 0$

where $a = 15$

$b = -2$

$c = -50$

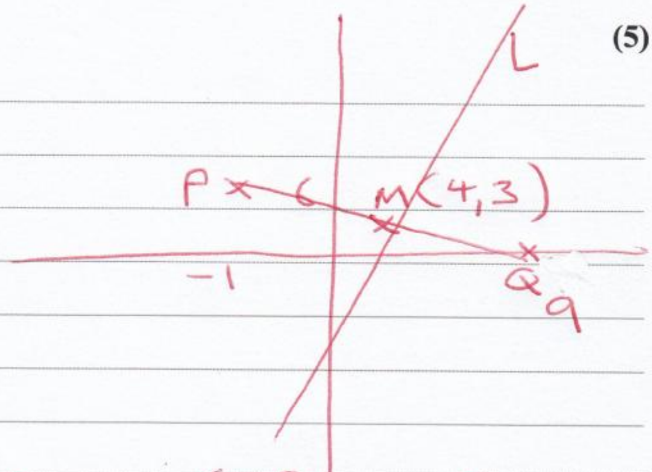
Leave blank

3. The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)



Gradient of $PQ = \frac{6-0}{-1-9} = \frac{6}{-10} = -\frac{3}{5}$

Mid point of $PQ = \left(\frac{-1+9}{2}, \frac{6+0}{2}\right)$

$= (4, 3)$

Gradient of L is $\frac{5}{3}$

Equation for l is (use Mid-Point M)

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{5}{3}(x - 4)$

$y - 3 = \frac{5}{3}x - \frac{20}{3}$

$3y - 9 = 5x - 20$

$0 = 5x - 3y - 11$

where $a = 5$, $b = -3$, $c = 11$

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5. The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B. **(2)**

Given that O is the origin,

(c) find the area of the triangle OAB. **(2)**

a) L_1 $y = -2x + 3$, gradient $= -2$

l_2 gradient is perpendicular $= \frac{1}{2}$

$y - y_1 = m(x - x_1)$ using $(5, 6)$

$y - 6 = \frac{1}{2}(x - 5)$ $\times$ by 2

$2(y - 6) = x - 5$

$2y - 12 = x - 5$

$0 = x - 2y - 5 + 12$

$x - 2y + 7 = 0$ is equation of l_2

b) x-axis at A, $y = 0$

$x + 7 = 0$

$\underline{\underline{x = -7}}$ x-coord of A

y-axis at B, $x = 0$

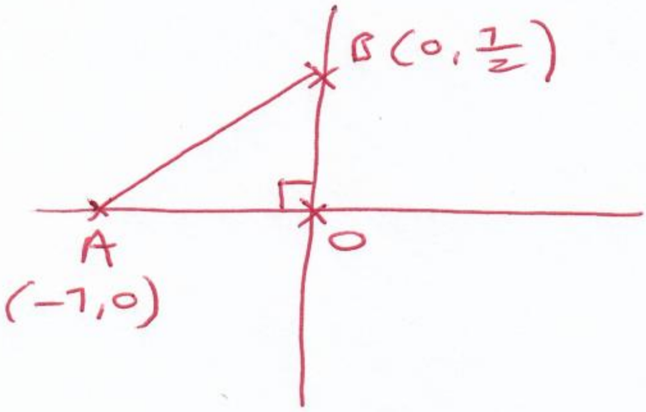
$-2y + 7 = 0$

$2y = 7$

$\underline{\underline{y = \frac{7}{2}}}$ y-coord of B

C1 Jan 2012

5c)



Area $\triangle AOB = \frac{1}{2} \times 7 \times \frac{7}{2}$

$= \frac{49}{4}$ square units

C1 Jan 2013

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11. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$

(c) Find the coordinates of Q.

(5)

a) $y = 2x - 8x^{\frac{1}{2}} + 5$

$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}}$

b) $P \left(x = \frac{1}{4}\right)$ gradient

$\frac{dy}{dx} = 2 - \frac{4}{\sqrt{x}} = 2 - \frac{4}{\sqrt{\frac{1}{4}}}$

$= 2 - \frac{4}{\frac{1}{2}} = 2 - 8 = -6$

y-coord at P

$y = 2 \times \frac{1}{4} - 8 \times \sqrt{\frac{1}{4}} + 5$

$y = \frac{1}{2} - 8 \times \frac{1}{2} + 5$

$y = \frac{1}{2} - 4 + 5 = \frac{3}{2}$ P is $\left(\frac{1}{4}, \frac{3}{2}\right)$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - \frac{3}{2} = -6\left(x - \frac{1}{4}\right)$

$y - \frac{3}{2} = -6x + \frac{\cancel{6}^{3}}{\cancel{4}_{2}}$

$y = -6x + \frac{3}{2} + \frac{3}{2}$

$y = -6x + \frac{6}{2}$

$y = -6x + 3$

C1 Jan 2013

11c) $2x - 3y + 18 = 0$

$2x + 18 = 3y$

$\frac{2}{3}x + \frac{18}{3} = y$

gradient of tangent at Q is $\frac{2}{3}$

use $\frac{dy}{dx} = \frac{2}{3}$ to find x-coordinate

$\frac{2}{3} = 2 - \frac{4}{\sqrt{x}}$

$\frac{4}{\sqrt{x}} = 2 - \frac{2}{3}$

$\frac{4}{\sqrt{x}} = \frac{4}{3}$

So $\sqrt{x} = 3$

$x = 9$

Put $x = 9$ in equation for y

$y = 2 \times 9 - 8 \times \sqrt{9} + 5$

$y = 18 - 8 \times 3 + 5$

$y = 18 - 24 + 5$

$y = -1$

Coordinates of Q are $\underline{(9, -1)}$

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11.

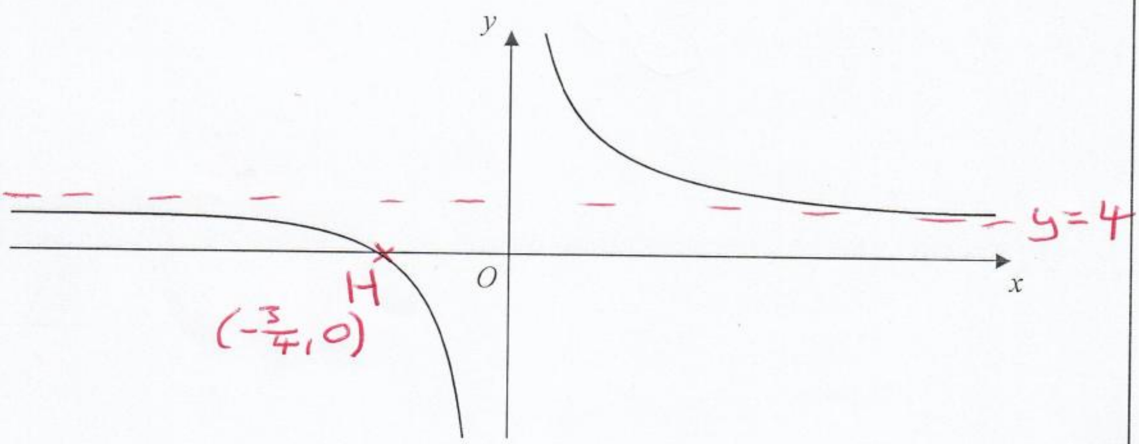

Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis. **(1)**

(b) Give the equations of the asymptotes to H. **(2)**

(c) Find an equation for the normal to H at the point $P(-3, 3)$. **(5)**

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd. **(3)**

a) $0 = \frac{3}{x} + 4$

$-4 = \frac{3}{x}$ so $x = -\frac{3}{4}$

H is $\underline{\underline{(-\frac{3}{4}, 0)}}$

b) $\underline{\underline{x = 0}}$, $\underline{\underline{y = 4}}$

c) $y = 3x^{-1} + 4$

$\frac{dy}{dx} = -3x^{-2} = -\frac{3}{x^2}$

at $x = -3$, $\frac{dy}{dx} = \frac{-3}{(-3)^2} = -\frac{1}{3}$

Gradient at P is $-\frac{1}{3}$, so gradient of normal at P is 3

P 4 1 8 0 2 A 0 2 4 2 8

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11 c) continued

$y - y_1 = m(x - x_1)$ using $m = 3$
$(-3, 3)$

$y - 3 = 3(x - -3)$

$y - 3 = 3x + 9$

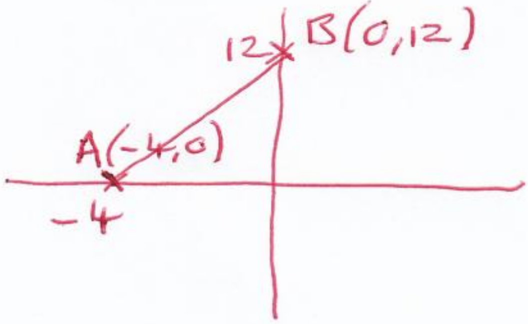
$y = 3x + 12$ is equation of normal

d) at A, $y = 0$, $0 = 3x + 12$

$x = -4$

at B, $x = 0$ $y = 3 \times 0 + 12$

$y = 12$



$AB = \sqrt{(12-0)^2 + (0--4)^2}$

$= \sqrt{12^2 + 4^2}$

$= \sqrt{144 + 16}$

$= \sqrt{160}$

$= \sqrt{16} \times \sqrt{10}$

$AB = 4\sqrt{10}$ units